# Maxwell’s Demon and Landauer’s Principle

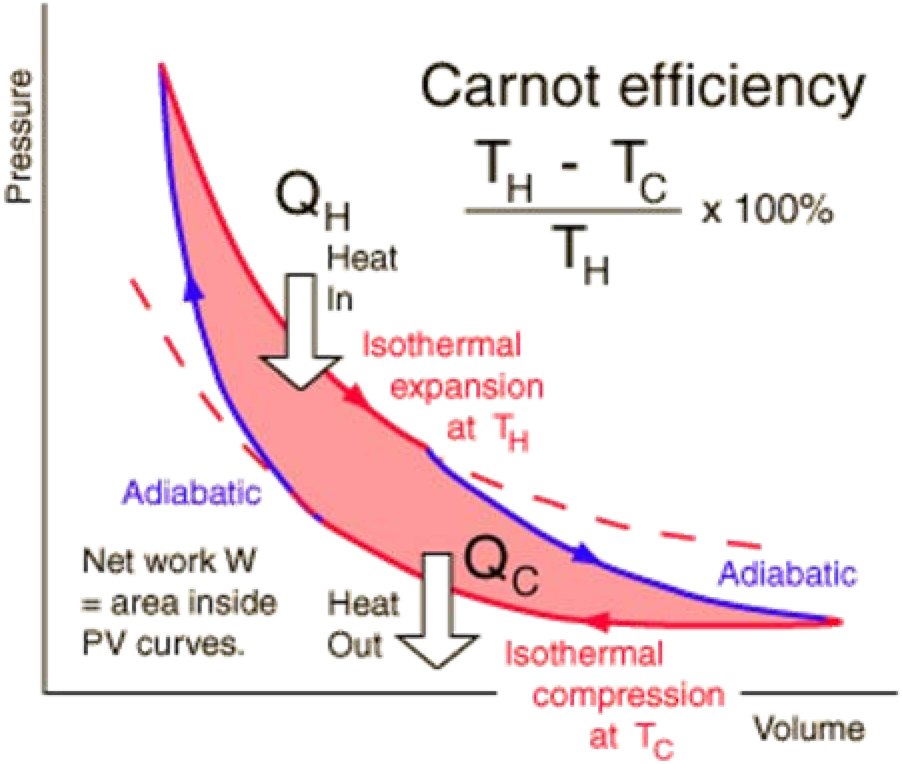
## The thermodynamics of quantum information theory

# Why Do We Care about Maxwell's Demon?

- Brief(!) introduction to thermodynamics
- How Maxwell's Demon violates almighty 2nd Law
- Landauer's Genius and likewise his Principle
- Generalized entropy – information is physical!
- Classical and quantum information erasure
- Implications for communication

# A Brief Introduction to Thermodynamics

- Carnot's Perfect Heat Engine




$$\frac{Q_C}{T_C} = \frac{Q_H}{T_H}$$

- Reality: Work and Entropy

Change in Entropy

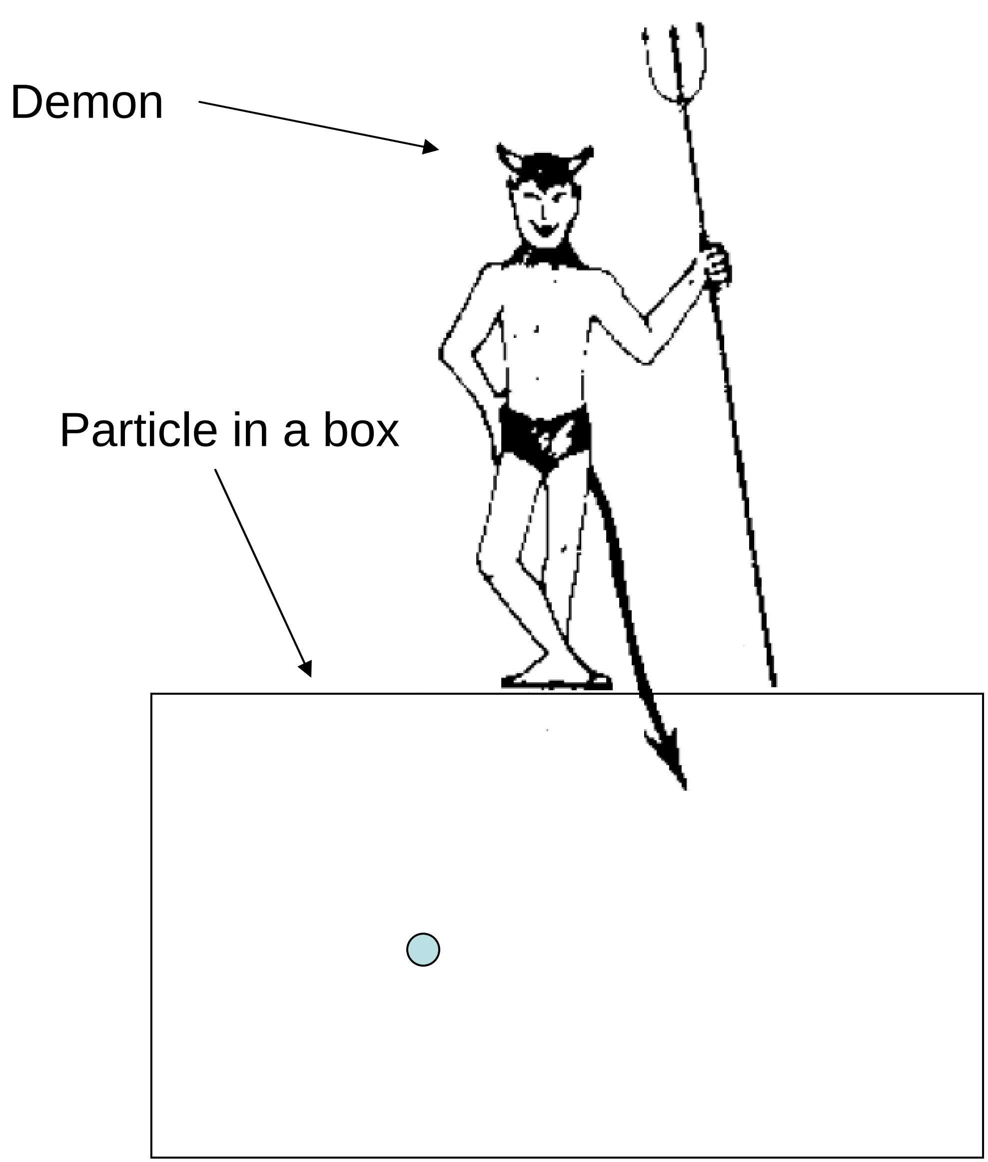
$$\Delta S = \frac{\Delta Q}{T}$$

Heat Added to System

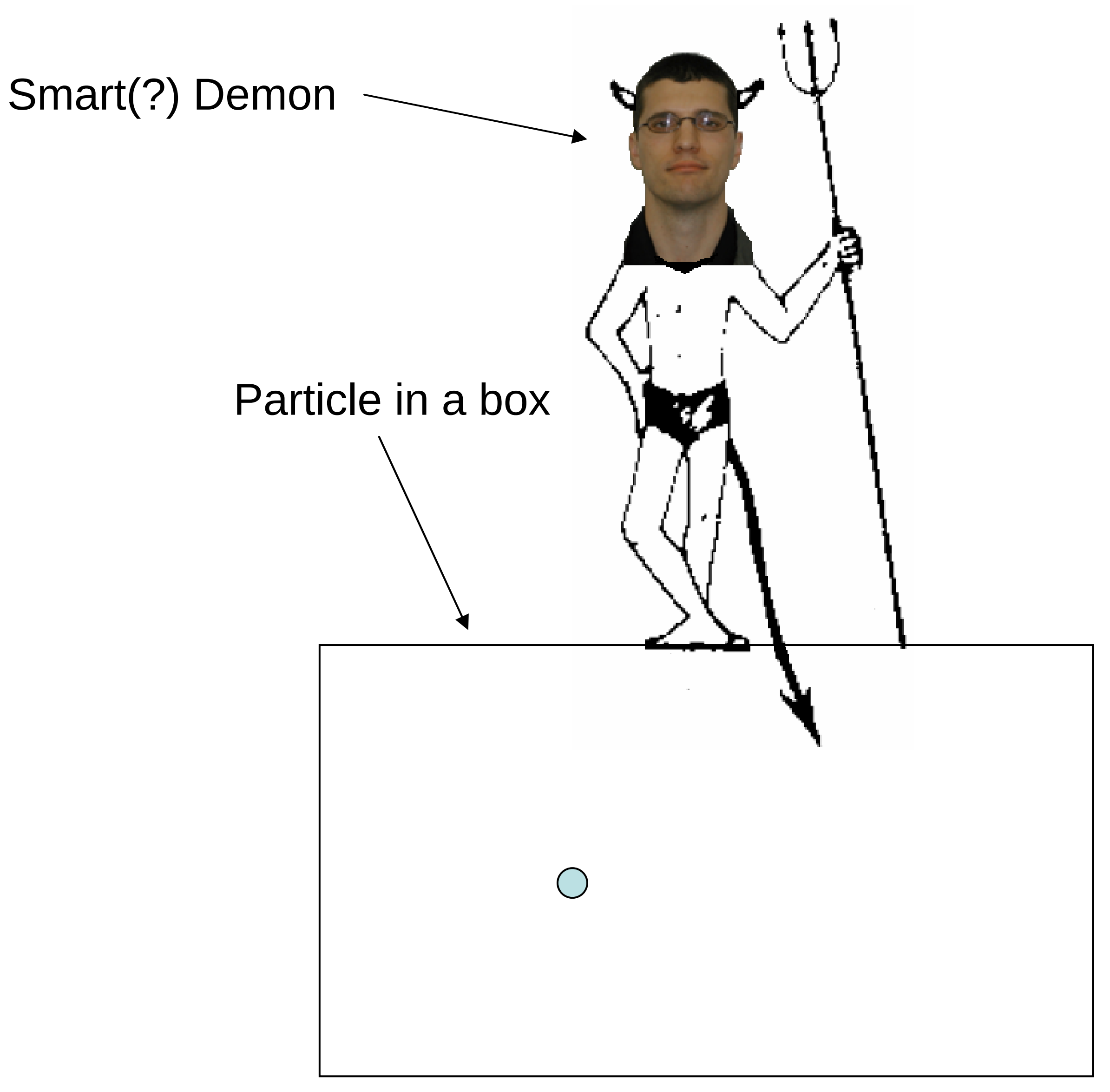
$$W = \Delta Q = Q_H - Q_C$$

Work Extracted

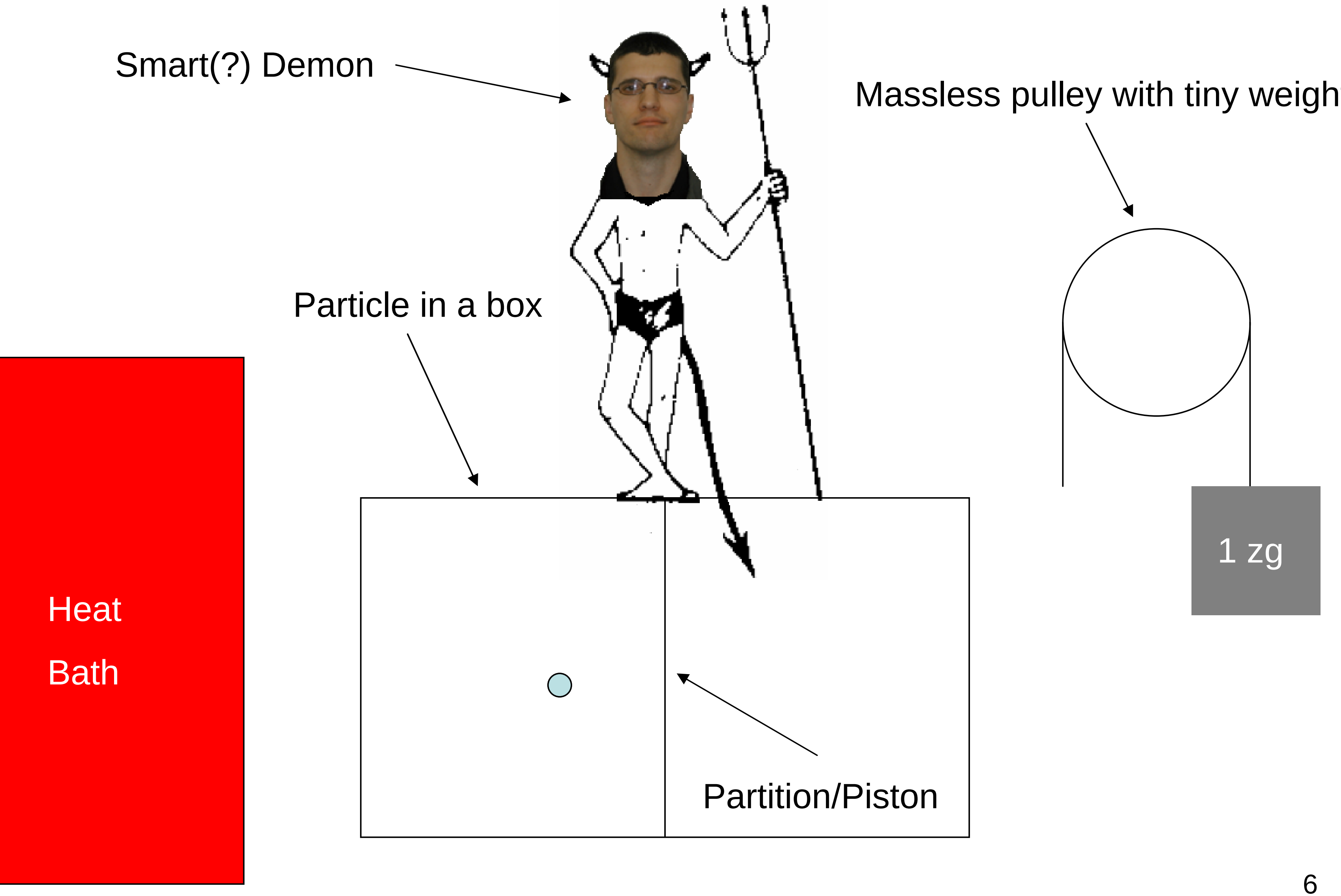
# Maxwell's Demon – What's the Problem?



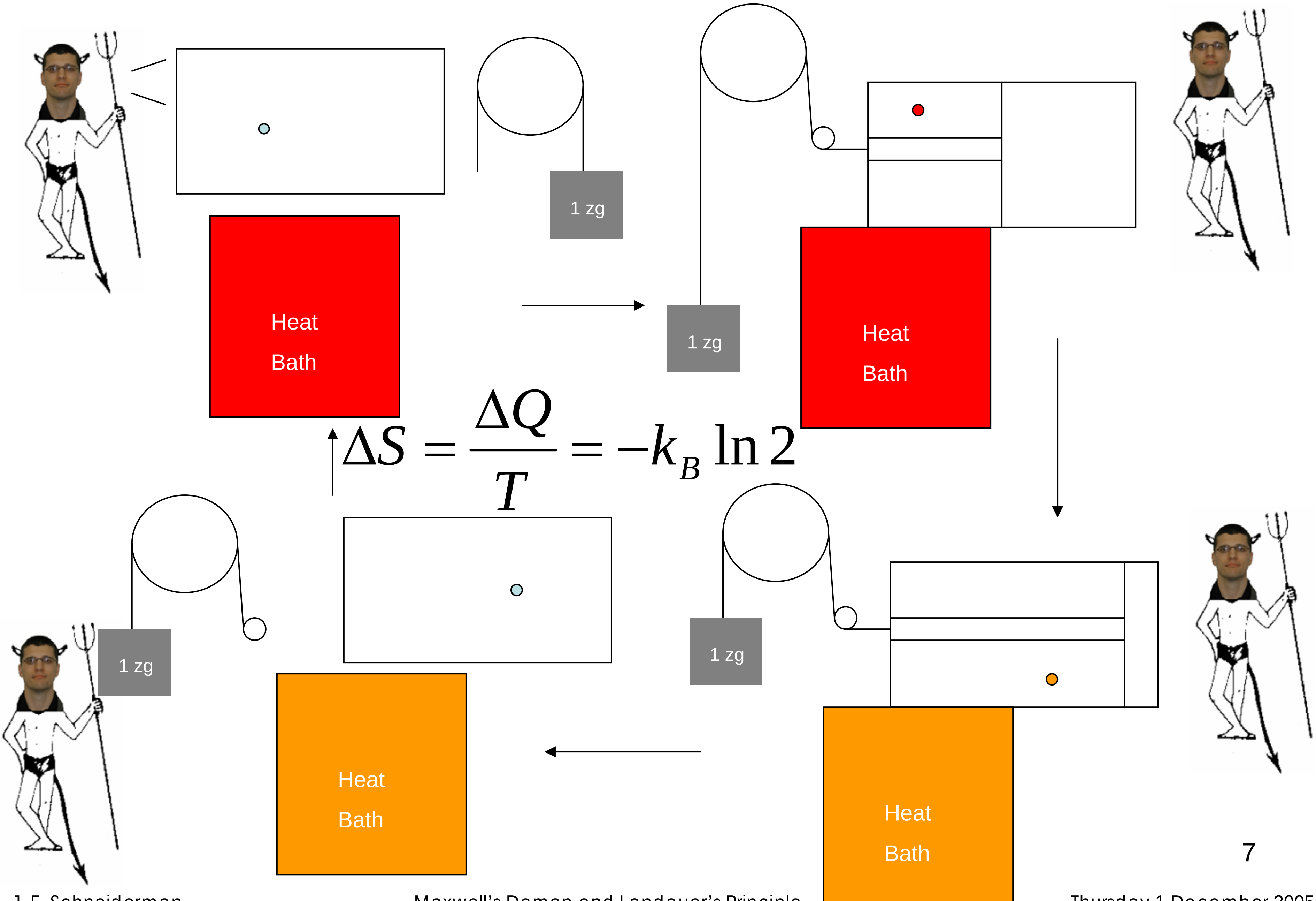
# Maxwell's Demon – What's the Problem?



# Maxwell’s Demon – What’s the Problem?



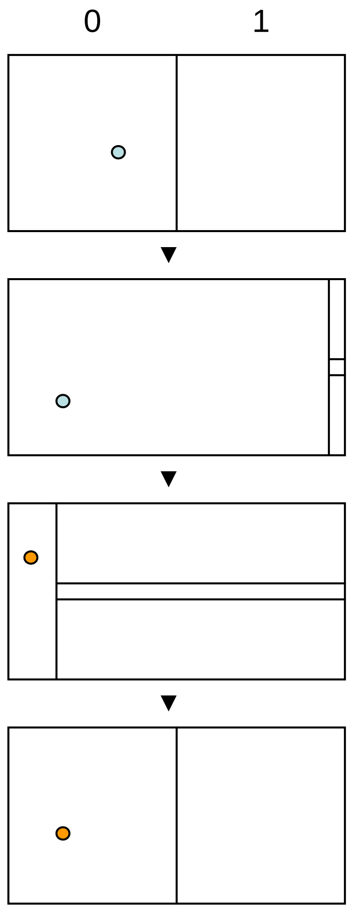
# Maxwell's Demon – What's the Problem?




$$\Delta S = \frac{\Delta Q}{T} = -k_B \ln 2$$

# Landeaur's Principle – Information and Entropy

- Many noticed parallel between thermodynamic entropy and Shannon's entropy – but what of it?
- Landeaur investigated physical cost of information erasure:

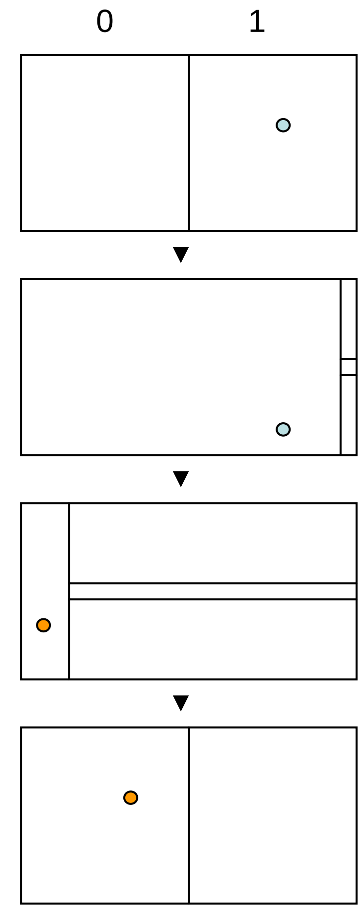



-Must ensure erasure of both 0 and 1

-Cannot know what state was before erasure

-Entropy decrease of system is $k_B ln2$

-Energy cost is AT LEAST $k_B T ln2$



# Generalized Entropy

- Redefine information in terms of physical parameters: $I = nk_B \ln 2$

Physical Information → $I$

Number of bits stored in system → $n$

Boltzmann's Constant → $k_B$

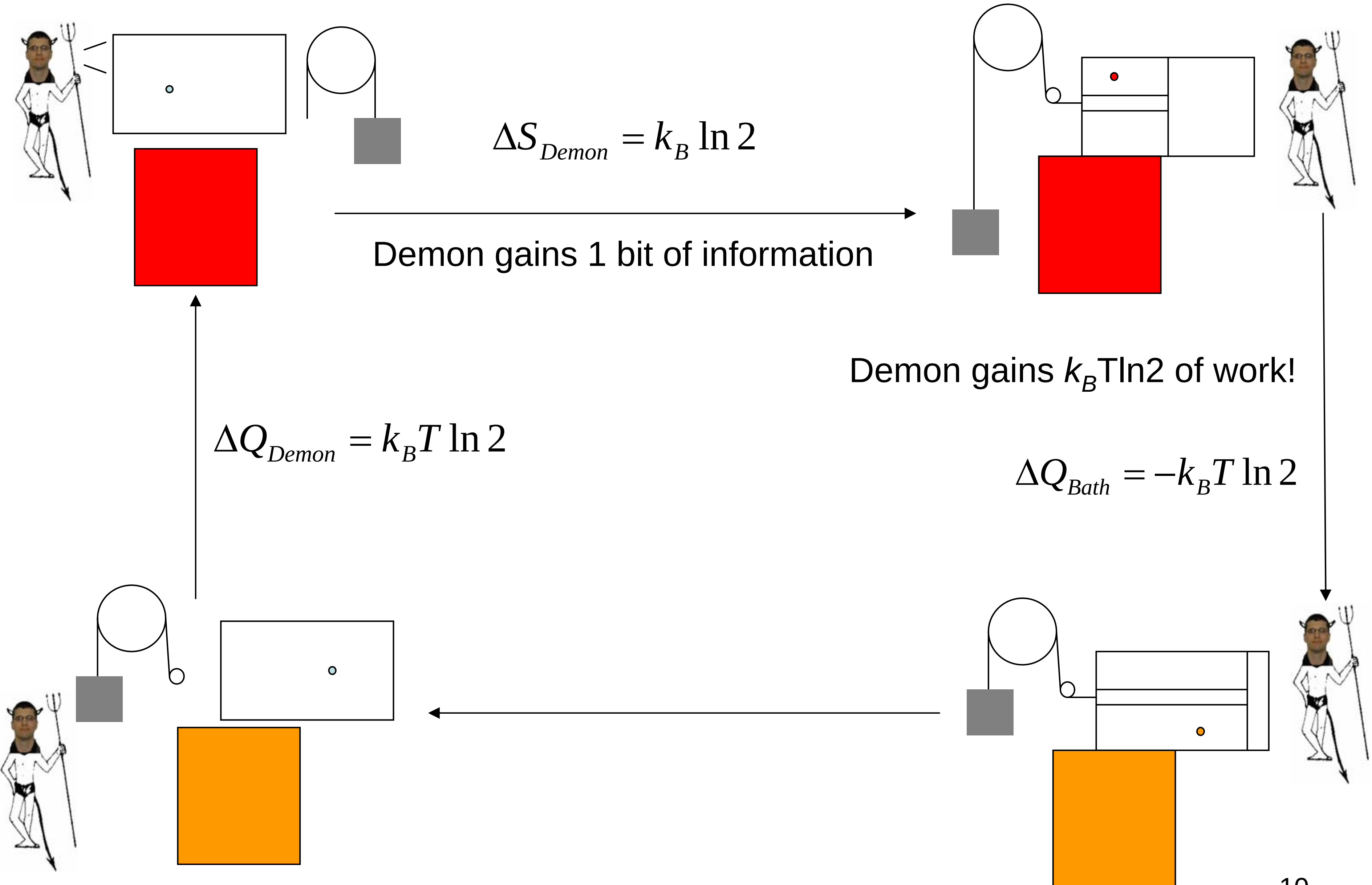
Binary encoding → $\ln 2$

- Unit of information is then thermodynamic entropy

- Work needed to erase can then be extracted from relation:

$$W = T\Delta S \qquad \Delta S = nk_B \ln 2 = I \qquad W = TI$$

# The Demon Demystified – Include Him in System!



# Can the Mind Be a Quantum Computer?

- Methods of erasure of quantum information
  - Measurement and rotation
  - Thermalization
- Landauer's Principle
  - Both must obey… or else!
  - Necessary calculations

# Methods of Quantum Information Erasure:

- Measurement and rotation on mixture of pure states
  - Perform projective measurement on system – classical information is extracted

$$\rho \xrightarrow{POVM} \sum_i p_i |e_i\rangle + I_{Classical}$$

  - Use information gained to perform appropriate rotation of state to reset it

$$U_i |e_i\rangle \longrightarrow |e_0\rangle$$

  - Must erase classical record of state – use Landauer's Principle as usual

$$\Delta Q = nk_B T \ln 2 \qquad \langle n \rangle = H(\{p_i\}) \longrightarrow \langle \Delta Q \rangle = k_B T \ln 2 H(\{p_i\})$$

  - Work used is minimized if measurement performed in eigenbasis of $\rho$:

von Neumann Entropy

$$\Delta Q = k_B T \ln 2 S(\rho)$$

# Methods of Quantum Information Erasure:

- Thermal randomization of mixture of pure states
  - Notes:
    - pure states have zero thermodynamic entropy
    - Ignorance of mixture provides information entropy
  - System contacts heat bath and thermalizes

$$\rho = \sum_i p_i |e_i\rangle \xrightarrow{Thermalization} \omega$$

  - System entropy increases by von Neumann entropy of $\omega$:

$$\Delta S_{System} = k_B \ln 2 S(\omega)$$

  - Bath entropy decreases by change of energy of system:

$$\Delta S_{Bath} = -\frac{U_{final} - U_{initial}}{T}$$

  - Use expectation value of energy of initial and final states:

$$\Delta S_{Bath} = -\frac{tr\{\omega H\} - tr\{\rho H\}}{T} = -\frac{tr\{(\omega - \rho) H\}}{T}$$

  - Use thermodynamic approximation to Hamiltonian:

$$H = -k_B T \ln(Z\omega)$$

Partition function

# Erasure of Quantum Information (cont'd)

- Landauer's Principle must still be obeyed
  - Expand entropy of bath:

$$\Delta S_{Bath} = k_B tr\{(\omega-\rho)\ln(Z\omega)\} = k_B tr\{(\omega-\rho)\ln\omega\} + k_B tr\{(\omega-\rho)\ln(Z)\}$$

  - Note second term drops as $tr\{\omega\} = tr\{\rho\} = 1$ and $\ln(Z)$ can be taken out of the trace
  - Expanding the first term:

$$\Delta S_{Bath} = k_B tr\{\omega\ln\omega\} - k_B tr\{\rho\ln\omega\} = -k_B \ln 2 S(\omega) - k_B tr\{\rho\ln\omega\}$$

  - Looking at the total entropy as the sum of the system and bath entropies:

$$\Delta S_{Total} = \Delta S_{System} - \Delta S_{Bath} = k_B \ln 2 S(\omega) - k_B \ln 2 S(\omega) - k_B tr\{\rho\ln\omega\}$$

  - Which can be minimized if we set the temperature of the heat bath so that the thermal equilibrium state of the system is $\rho$:

$$\min\{\Delta S_{Total}\} = -k_B tr\{\rho\ln\rho\} = S(\rho)$$

# Quantum Erasure: Conclusions

- Methods of erasing quantum information both obey Landauer's Principle

- Fact that erasure of quantum information can yield a minimum system-wide change of entropy equal to von Neumann entropy indicates only way to quantify classical information contained in a mixed quantum state is the von Neumann entropy