

Maxwell's Demon and Landauer's Principle:

The thermodynamics of quantum information theory

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I. Background

Maxwell was one of those scientists whose brilliance was limited only by the length of his life. He brought together the deceptively disparate subjects of electricity, magnetism, and light into an elegant formalism whose beauty is still remarkable and tremendously useful to the discerning student of science and engineering today. However, his genius was not limited to laying down a set of electromagnetic equations on which thousands of pages have expounded, he is also one of the fathers of classical thermodynamics and statistical mechanics. As such, Maxwell is probably most famous for his demon and its proclivity for egregiously violating the unbreakable second law of thermodynamics. Sadly, Maxwell passed away long before his demon was exorcized properly.

II. Introduction to Thermodynamics and Statistical Mechanics

Thermodynamics deals with averages and absolutes that, at first blush, should likely only apply to uncountably large ensembles of particles and systems at very high temperatures. However, upon further inspection (and a good study of statistical mechanics), one finds that the laws that arise from thermodynamics must also apply at the single-particle and very low-temperature limit. These laws are typically referred to as the First and Second Laws of thermodynamics and are often stated thus:

First Law of Thermodynamics: If work, W , is added to (subtracted from) a system, there must be a corresponding increase (decrease) in energy, Q , of that system:

$$W = \Delta Q$$

Second Law of Thermodynamics: The entropy of a closed system will never

decrease: $\Delta S = \frac{\Delta Q}{T}$ where ΔS is the change in entropy, ΔQ is the change in heat,

and T is the temperature of the system.

In other words, energy is always conserved and things tend to get more disordered with time.

The concepts contained in the first two laws of thermodynamics are usually best explained in terms of one of Maxwell's predecessor's ubiquitous engines: the Carnot engine. A Carnot engine is a simple machine used to convert heat into work and thus exemplifies the first law: any work you extract from something must come with a corresponding loss of heat. A Carnot engine can also be used backwards: adding heat to a system results in a corresponding loss of energy available for work. Unfortunately, the second law is not particularly evident from such statements and can practically be overlooked in the ideal case where all heat is converted into usable work and vice versa. This ideal case is dubbed a "cyclic" process and results in no overall change in entropy.

As we all know, little in life is ideal and so most processes are not cyclic and therefore result in an increase in entropy. This can be understood in terms of an imperfect Carnot engine – instead of converting all heat into usable work, some heat is lost to the environment. A perfect example of such a system is a car: cars get hot and that heat cannot be used to propel the car further but rather is simply dissipated into the air around us. If we then tried to extract the work from the now moving vehicle and convert it back into heat, we'd find the final temperature of the system would be lower than the initial. The second law is a description of nature's tendency to do just this: for non-cyclic processes, energy is lost due to inefficiencies and entropy increases as a result.

In order to decrease the entropy of a local system, we must use energy that will yield a corresponding increase in entropy of the total system.

III. Maxwell's Demon

Maxwell's demon comes in many forms but the central perplexing issue that is always included is his ability to somehow circumvent the second law of thermodynamics by lowering a system's entropy and extracting work from it in the process. Most arguments against this demon come from an energetic standpoint: he must get the energy from somewhere, right? Quantum mechanics seemed to provide an answer that was sadly turned on its head by a new formulation of Maxwell's demon provided by Leo Szilard. Coincidentally, Szilard did discover a way to seemingly extract energy from "nothing" with his secret patent of the atomic bomb – his discovery of which was crucial in the months leading up to the Manhattan Project.

Szilard's version of Maxwell's demon is deceptively simple: put a single particle into a rectangular box with a demon, a piston, a tiny weight, and a heat bath. The demon measures the location of the particle and with that information brings the box into contact with the heat bath and allows the particle to do work on a piston with the thermal energy it gains from the bath and raises the tiny weight some distance. The piston is then removed and the demon can start over. It then appears as if the demon is extracting heat from the bath and converting it into work, thus decreasing the entropy of the bath with no corresponding increase in entropy anywhere else. Figure 1 shows how Maxwell's demon extracts usable work from *and* decreases the entropy of a heat bath.

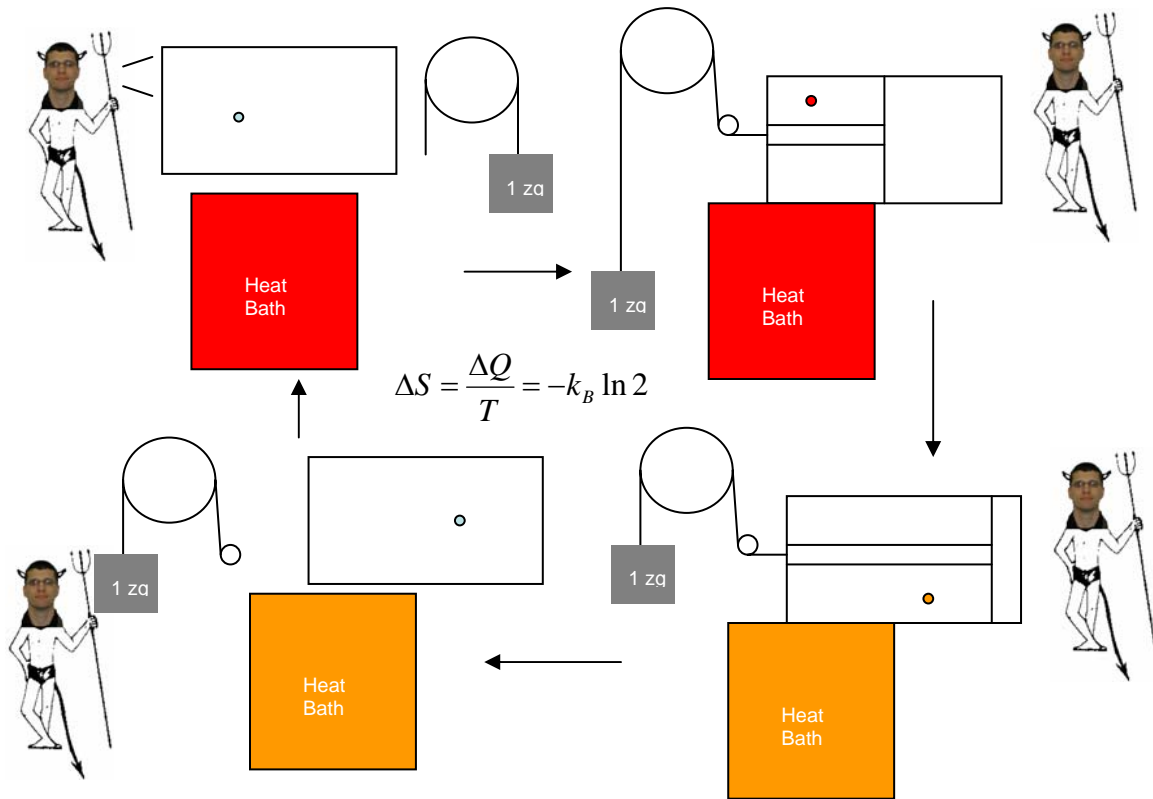


Figure 1: Maxwell’s demon at work.

As mentioned above, physicists attempted to explain the paradox created by Maxwell’s Demon in terms of energy costs associated with measurement of the particle. Quantum mechanics even seemed to provide some insight with its requirement on minimum uncertainty. Unfortunately, these solutions did not hold up under scrutiny, and their shortcomings were most apparent when they were applied to Szilard’s formulation of Maxwell’s demon. The demon would not have to gain any information about the particle’s speed (or energy) nor a good measure of its location – only its relative location inside the box.

IV. Landauer’s Erasure Principle

It wasn't until the end of the McCarthy era in 1961 that Rolf Landauer was finally able to truly exorcise Maxwell's demon. Landauer realized that information was necessarily physical: in order for it to be useful, we must be able to record and extract information from some physical system. It had recently been discovered that it cost no energy to copy information, but what about erasing? Landauer's principle summarizes the ideal case for storing information in the simplest two-level system and thus applies to ALL information storage devices. The result: erasing a bit of information results in a minimum energy cost of $W = k_B T \ln 2$ per bit, where k_B is Boltzmann's constant and $\ln 2$ comes from binary encoding. Likewise, for every bit erased, there is a corresponding minimum increase in entropy of the system by $\Delta S = k_B \ln 2$. Figure 2 is an example of such an erasure procedure for a box like that of Maxwell's demon.

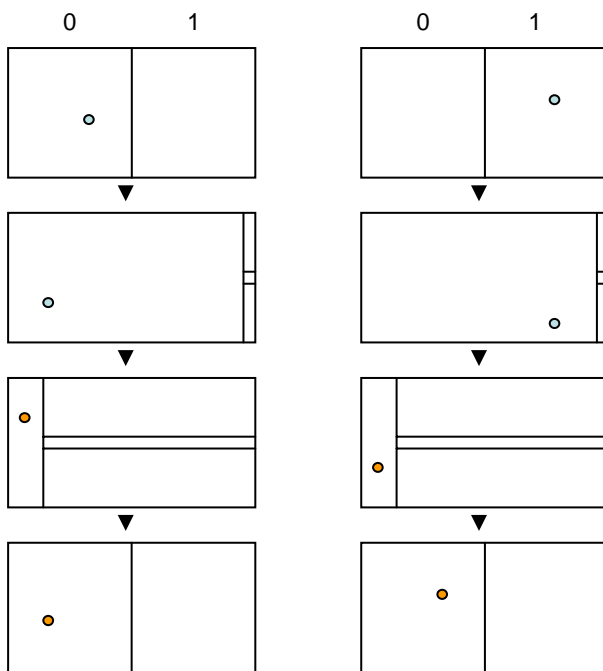


Figure 2: Entropy of erasure. Note: procedure must work for both states *without* measurement of state of system.

IV. Shannon's Entropy vs. (Thermodynamic) Entropy

Parallels between statistical mechanics and information theory became evident with Claude Shannon's thorough exposition of the field of classical information and what it meant for something to contain information. Shannon's measure of information was suggestively dubbed "Shannon's Entropy" in homage to its pioneer as well as its physical cousin, thermodynamic entropy (or, to most physicists, simply "entropy"). The obvious questions are thus: What of this parallel? Why are they both considered entropy when one has to do with physical systems and the other with information? The answer lies in the fact that information is physical and must therefore be included in any measurement, especially those pertaining to limits on (thermodynamic) entropy.

Maxwell's demon provides a natural example of the need for unification of entropies. The exorcism occurs through inclusion of the demon's "recording apparatus" in the total system. When the demon runs out of blank (low-entropy) recording room, he must start erasing bits which, due to Landauer's Principle, costs him work and increases his entropy. If we create a new measure of physical information with natural entropic units, $I = k_B n \ln 2$, where n is the number of bits stored, it is easy to include it in a new generalized entropy. The work required to erase n is then simply nI . The heretofore artificial separation between thermodynamic and information entropies is removed. In terms of Maxwell's demon, for every cycle of his process, he gains a bit of information which must be erased: the decrease in entropy of (and work extracted from) the system are balanced out perfectly with the increase in entropy of (and work required to erase) the information stored in the demon's brain!