Performance of a Propagation Delay Tolerant ALOHA Protocol for Underwater Wireless Networks

Joon Ahn and Bhaskar Krishnamachari

Ming Hsieh Department of Electrical Engineering
Viterbi School of Engineering
University of Southern California, Los Angeles, CA 90089
joonahn@usc.edu, bkrishna@usc.edu

Abstract. Underwater wireless networks have recently gained a great deal of attention as a topic of research. Although there have been several recent studies on the performance of medium access control (MAC) protocols for such networks, they are mainly based on simulations, which can be insufficient for understanding the fundamental behavior of systems. In this work we show a way to analyze mathematically the performance of an underwater MAC protocol. We particularly analyze a propagation delay tolerant ALOHA (PDT-ALOHA) protocol proposed recently [1]. In this scheme, guard bands are introduced at each time slot to reduce collisions between senders with different distances to the receiver, which have a great impact on acoustic transmissions. We prove several interesting properties concerning the performance of this protocol. We also show that the maximum throughput decreases as the maximum propagation delay increases, identifying which protocol parameter values realize the maximum throughput approximately. Although it turns out that exact expression for the maximum throughput of PDT-ALOHA can be quite complicated, we propose a useful simple expression which is shown numerically to be a very good approximation. Our result can be interpreted to mean that the throughput of PDT-ALOHA protocol can be 17-100% higher than the conventional slotted ALOHA, with proper protocol parameter values

1 Introduction

Underwater wireless networks have recently gained a great deal of attention as a topic of research, with a wide range of sensing and security applications just starting to be explored [2–4]. Since radio-frequency (RF) electromagnetic waves, except those of very low frequencies, decay very rapidly in water, underwater networks need to adopt acoustic transmissions instead in most cases. The long propagation delays associated with acoustic transmission, however, can significantly affect the performance of traditional network protocols, requiring the design and analysis of new approaches for different layers, including medium access.

The problem of designing a simple medium access protocol appropriate for underwater networks was addressed recently in [1]. The authors of this work show that the performance of classical slotted ALOHA deteriorates in an underwater setting where transmissions from one slot can overlap with future slots. They propose the introduction of guard bands in each time slot to address this problem; we refer to their scheme as the propagation delay tolerant ALOHA (PDT-ALOHA) protocol. Although they have demonstrated that the throughput can be increased using guard bands through simulations, [1] have failed to investigate rigorously the performance of the protocol, which can be done through the theoretical analysis.

We analyze mathematically the performance of the PDT-ALOHA protocol in this work. We investigate different metrics of performance – expected number of successful packet receptions in a time slot, throughput, and maximum throughput. We obtain exact expressions for the number of receptions and throughput. Although the exact expressions are not closed-form, they can be easily calculated numerically with given parameters. Further, we obtain very simple expressions for the maximum throughput and its maximizers which are shown to be very good approximations.

We also prove a number of interesting and useful properties concerning the performance of the PDT-ALOHA protocol. We prove that the expected number of successful packet receptions is independent of the propagation speed; and that its maximum is non-decreasing as the size of guard band increases; and derive a bound on the network load that offers the maximum throughput. We also show that the maximum throughput decreases as the maximum propagation delay increases.

As per the original study where this protocol is proposed [1], we focus the analysis on two-dimensional underwater wireless networks. Although underwater applications can naturally allow for 3D placements, it is also possible to envision practical applications involving the deployment of sensor nodes in a two-dimensional plane just below the water surface, or close to the bottom of the water body. In any case, we should note that the methodology for analyzing performance in three-dimensions would be essentially similar to the analysis that we present in this work.

Related Works

There are several literatures which have studied on the throughput of ALOHA protocols in the underwater networks. [5] has investigated the impact of the large propagation delay on the throughput of selected classical MAC protocols and their variants through simulations. [6] has compared the performance of ALOHA and CSMA with RTS/CTS protocols in underwater wireless networks. And [1] has studied on the throughput of PDT-ALOHA through simulations producing rough idea of the performance. While these works are mainly based on simulations we approach the problem from the theoretical view point.

There have been other works which take the theoretical approach as we do. [7] has analyzed slotted ALOHA without the guard band and concluded that slotted ALOHA degrades to unslotted ALOHA under high propagation delay. [8] have analyzed the performance of ALOHA in a linear multi-hop topology. However, these works do not consider the guard band to relieve the negative effect of the large propagation delay. [9, 10] have taken consideration of the guard band for the slotted ALOHA protocols in their analysis. However, they have assumed satellite networks where the imperfection or sloppiness of each node's implementation causes variable propagation delay, and they have focused on how to deal with the sloppiness using the guard bands. The main difference from their problem is that nodes are located on the ground approximately same distance away from the satellite in their problem so that the propagation delay is more or less same for each node. But, the distance to the receiver can vary greatly from node to node in underwater wireless sensor networks.

2 PDT-ALOHA Protocol and Assumptions

The PDT-ALOHA is essentially a simple enhancement of the slotted ALOHA protocol; the difference is that it has a guard band at the end of each time slot. Specifically, all transmitters in the network maintain synchronized time slots for communication in the PDT-ALOHA. A transmitter sends a packet at the start of a time slot when it has one to send, then waits until the start of the next time slot although it has another packet in its queue (See the time diagram of transmission of sender A in Fig. 1). We refer to the time duration between the end of transmission and the start of the next time slot as a guard band.

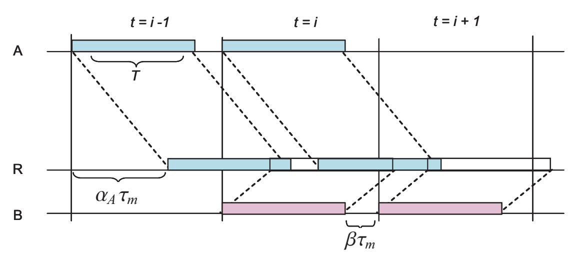


Fig. 1. Time diagram of packet transmission; A and B are transmitters and R is the receiver. B locates closer to the receiver than A.

The guard band in the PDT-ALOHA is designed in order to relieve collisions between packets of different senders transmitted in consecutive time slots. For example, if the guard band is at least as much as the maximum propagation time among all pairs of a transmitter and a receiver in the network, there would be no collision between packets transmitted in different time slots. Likewise, increasing the guard band reduces collisions, but it also increases the length of each time slot potentially reducing the throughput. Thus the selection of the appropriate guard-band length to get the maximum throughput can be formulated as an optimization problem.

We made following assumptions to analyze the performance of the PDT-ALOHA protocol unless stated otherwise.

- The network has one receiver and n transmitters, which are deployed in the two-dimensional disk area.
- The receiver locates at the center of the disk area, and the transmitters are deployed uniformly at random in the area.
- The propagation speed of communication is positive finite constant regardless of the location in the network, so that the maximum propagation time from the receiver to the farthest transmitter is a positive finite constant τ.

– The transmission rate is constant for every transmitter.
– The packet size is constant so that, along with the constant transmission rate, the transmission time for a packet is constant, which we assume is one; this doesn't incur loss of generality in our analysis. Only a proper scaling is needed for some parameters, particularly τ, in order to cope with the general transmission time.
– We assume packet arrivals to the network follow the Poisson distribution. Since the Poisson arrival in a slotted time system can be well approximated through the Binomial distribution, we assume the packet departure per node at a given time slot is I.I.D. Bernoulli. Specifically, a transmitter sends a packet to the receiver with probability p in each time slot.
– If the receiver receives more than one packet simultaneously at any time in a time slot, all the packets involved fail to get delivered successfully causing a collision.
– The links over which transmissions take place are lossless (e.g., using blacklisting).
– A transmitter always transmits a packet at the start of the time slot if the transmitter wants to send the packet.
– All the nodes have the globally synchronized time slots.
– The transmission time is no less than the maximum propagation time so that $\tau \leq 1$.

The assumption that $\tau \leq 1$ is to make sure that the collision between a time slot and another is confined to the consecutive time slots. So, with this assumption, there is no possibility that a packet sent in i-th time slot collides with another in j-th time slot, where $j \notin \{i-1, i, i+1\}$.

3 Throughput Analysis

In this section we analyze the throughput of the PDT-ALOHA protocol. Let us first look at the time slot. Each time slot consists of a transmission time and a guard band following the former. Since the guard band of the size of maximum propagation time τ would eliminate all the collision between different time slots, it does not make sense to have the guard band whose size is more than τ only decreasing the throughput without any further gain. Hence, we use the normalized factor $\beta \; s.t. \; 0 \leq \beta \leq 1$ in expressing the size of guard band so that the time slot size is $1 + \beta \cdot \tau$.

3.1 Expected Number of Successful Packet Receptions

In order to analyze the throughput we first derive the expected number of successful packet receptions in a time slot. We use the linearity of expectations and conditional probabilities to calculate the expected number. Let the indicator variable I_i denote whether or not the receiver receives the packet from i-th transmitter successfully in the time slot.

$$I_i = \begin{cases} 1, & \textit{if successful reception} \\ 0, & \textit{otherwise} \end{cases} \tag{1}$$

Let N denote the random variable of the number of successful reception. Then, $N = \sum_i I_i$. Hence, the expected number is, by the linearity of expectations and conditional probability, as follows;

$$\begin{aligned} E[N] &= \sum_{i=1}^{n} E[I_i] \;=\; \sum_{i=1}^{n} \Pr\{I_i = 1\} \\ &= \sum_{i=1}^{n} \Pr\{\textit{no collision} \mid i\textit{-th sender sends}\} \cdot \Pr\{i\textit{-th sender sends}\} \\ &= n \cdot p \cdot \Pr\{NC|n_i\} \end{aligned} \tag{2}$$

where $NC|n_i$ denotes the event that no collision occurs given that i-th sender transmits. The last equality of the above equations holds since the collision probability is symmetrical among all the senders.

Therefore, in order to calculate the expected number, we need to find out the probability of no collision for the transmitted packet from the i-th sender whose location is uniform at random over the network area.

3.2 Probability of No Collision

Let us consider some situations where collision can occur in order to get some intuition. Suppose a simple network with two senders, A and B, and one receiver R. B locates right next to R while A is very far from R, and the size of guard band is small enough. Then, if A transmits in the i-th time slot, R would receive last part of the packet in the beginning of the $(i + 1)$-th slot, which would produce collision with the packet transmitted in the $(i + 1)$-th slot by B although the two packets are sent in different time slots. The time diagram in Fig. 1 visually shows this situation, where α_i is the normalized propagation time distance of $i \in \{A, B\}$ from R defined by Definition 1, the normalized guard band size β is less than 1, and $\alpha_A > \alpha_B + \beta$. However, if $\alpha_A = \alpha_B$ there is no collision between packets in different time slots. Therefore, we can see that the collision depends on nodes' locations and two packets transmitted in different time slots can experience collision between each other.

Definition 1. *The* ***normalized (propagation) time distance*** *of sender X from the receiver is the propagation time from the receiver to X divided by the maximum propagation time τ in the network.*

After all, it is not hard to identify collision regions $R_p(\alpha), R_c(\alpha), R_n(\alpha)$ for the interested transmitter I which has the normalized time distance of α, where $R_p(\alpha)$ denotes the region such that a packet sent from I collides with a packet sent in the previous consecutive time slot by a node in $R_p(\alpha)$; $R_c(\alpha)$ in the same time slot; and $R_n(\alpha)$ in the next consecutive time slot. Equation (3), (4), (5) specify the regions in terms of the normalized time distance and Fig. 2 visually presents the regions.

$$R_p(\alpha) = \{\tau_p | \alpha + \beta \le \tau_p \le 1\} \tag{3}$$

$$R_c(\alpha) = \{\tau_c | 0 \le \tau_c \le 1\} \tag{4}$$

$$R_n(\alpha) = \{\tau_n | 0 \le \tau_n \le \alpha - \beta\} \tag{5}$$

The probability of no collision given a packet sent by an arbitrary i-th sender n_i is then as follows conditioning on the n_i's normalized time distance α;

$$\Pr\{NC|n_i\} = \int_0^1 \Pr\{NC|n_i, n_i \text{ at } \alpha \text{ away}\} \cdot \textit{pdf}\{\alpha \text{ away} \mid n_i \text{ trans.}\} d\alpha$$
$$= \int_0^1 2\alpha \Pr\{NC|\alpha\} d\alpha \quad (6)$$

The last equation holds because the location of a node is independent of the packet transmission in our assumption.

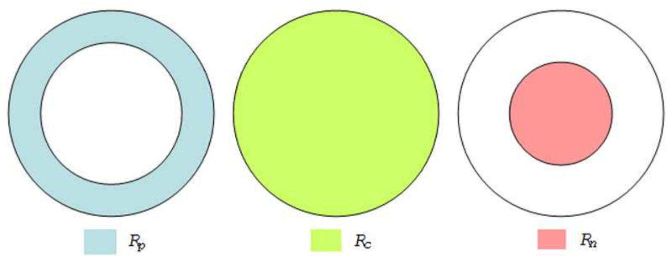


Fig. 2. Collision regions

Meanwhile, the probability of (no) collision of a specific packet does depend on the location of its sender because it defines the three collision regions, $R_n(\alpha)$, $R_c(\alpha)$, and $R_p(\alpha)$; and the regions' areas affect the probability. Hence, further conditioning on the numbers of transmitters in those three collision regions, we can get the following equation:

$$\Pr\{NC|n_i, n_i \text{ at } \alpha \text{ away}\} = \Pr\{NC| \text{ sent at } \alpha \text{ away}\}$$
$$= \sum_{x+y+z=n-1} \begin{array}{c} \Pr\{NC \mid \alpha, N_n = x, N_c = y, N_p = z\} \\ \times P\{N_n = x, N_c = y, N_p = z|\alpha\} \end{array} \quad (7)$$

where $N_n, N_c,$ and N_p denote the number of other transmitters in $R_n, R_c,$ and R_p respectively. Note that there are $(n-1)$ other transmitters (or interferers) because we focus on one specific transmitter's success.

Note also that the event of $N_n = x, N_c = y, N_p = z|\alpha$ has the multinomial distribution with parameters $n-1$, $p_n(\alpha)$, $p_c(\alpha)$, and $p_p(\alpha)$, where $p_i(\alpha),\ i \in \{n, c, p\}$ denotes the probability that a transmitter lies in $R_i(\alpha)$. And each of these probabilities is the ratio of its area to the entire area of the network;

$$p_n(\alpha) = \begin{cases} (\alpha - \beta)^2 & , \textit{if } 0 < \alpha - \beta < 1 \\ 0 & , \textit{otherwise} \end{cases}$$
$$p_c(\alpha) = 1$$
$$p_p(\alpha) = \begin{cases} 1 - (\alpha + \beta)^2 & , \textit{if } 0 < \alpha + \beta < 1 \\ 0 & , \textit{otherwise} \end{cases}$$

Now we have two cases, each of which has three sub-cases. In the first case A $(0 \leq \beta \leq 0.5)$, we have three sub-cases; (i) $0 \leq \alpha \leq \beta$, where $R_n(\alpha) = \emptyset$ so that

$p_n(\alpha) = 0$; (ii) $\beta \leq \alpha \leq 1-\beta$, where all three collision regions can have areas larger than zero; and (iii) $1-\beta \leq \alpha \leq 1$, where $R_p(\alpha) = \emptyset$ so that $p_p(\alpha) = 0$. In the other case B ($0.5 \leq \beta \leq 1$), we have another three sub-cases; (i) $0 \leq \alpha \leq 1-\beta$, where $R_n(\alpha) = \emptyset$; (ii) $1-\beta \leq \alpha \leq \beta$, where $R_p(\alpha) = R_n(\alpha) = \emptyset$; and (iii) $\beta \leq \alpha \leq 1$, where $R_p(\alpha) = \emptyset$.

Let us consider Case A.(i) first. In this case, the conditional probability of no collision turns out to involve the binomial series as follows;

$$\begin{aligned}\Pr\{NC|\alpha\} &= \sum_{z=0}^{n-1} \Pr\{NC|\alpha, N_p = z, N_c = n-1, N_n = 0\} \cdot \Pr\{N_p = z|\alpha\} \\ &= \sum_{z=0}^{n-1} \binom{n-1}{z} (1-p)^z (1-p)^{n-1} \cdot (1-(\alpha+\beta)^2)^z ((\alpha+\beta)^2)^{n-1-z} \\ &= (1-p)^{n-1}(1-p+p(\alpha+\beta)^2)^{n-1} \end{aligned} \tag{8}$$

Equation (9) and (10) are the summary after calculating the other cases in the similar way; most of them involve the binomial series although Case A.(ii) involves the multinomial series.

In Case A,

$$\Pr\{NC|\alpha\} = \begin{cases} (1-p)^{n-1}(1-p+p(\alpha+\beta)^2)^{n-1}, & \textit{if } 0 \leq \alpha \leq \beta \\ (1-p)^{n-1}(1-p+4p\alpha\beta)^{n-1}, & \textit{if } \beta \leq \alpha \leq 1-\beta \\ (1-p)^{n-1}(1-p(\alpha-\beta)^2)^{n-1}, & \textit{if } 1-\beta \leq \alpha \leq 1 \end{cases} \tag{9}$$

In Case B,

$$\Pr\{NC|\alpha\} = \begin{cases} (1-p)^{n-1}(1-p+p(\alpha+\beta)^2)^{n-1}, & \textit{if } 0 \leq \alpha \leq 1-\beta \\ (1-p)^{n-1}, & \textit{if } 1-\beta \leq \alpha \leq \beta \\ (1-p)^{n-1}(1-p(\alpha-\beta)^2)^{n-1}, & \textit{if } \beta \leq \alpha \leq 1 \end{cases} \tag{10}$$

Substituting (9) or (10) into (6) we can obtain the expression for the probability of no collision which can be evaluated easily with the numerical method.

Note that the expression for probability of no collision does not involve the maximum propagation delay τ implying the probability is independent of τ so that the expected number of successful reception is also independent of τ. It turns out from Theorem 1 that the expected number is independent of τ even after relaxing the assumption of 2D unit disk of the network and the identical distribution of packet transmission for each node.

Theorem 1. *Given a network of nodes with fixed spatial locations of nodes, a fixed transmission probability p_i in a time slot for each node i, and a transmission time T for a packet, the expected number of successful packet reception f in a time slot is independent of the maximum propagation time τ in the network as long as $0 < \tau \leq T$. In other words, it is independent of the propagation speed v_p.*

Proof. Since the spatial locations of nodes are fixed, the spatial distance r_m from the receiver to the farthest node is constant; $r_m = \tau \cdot v_p = \textit{const.}$

The spatial distance r_i of an arbitrary i-th transmitter is also fixed, and so the normalized propagation time delay α_i of the node is constant regardless of r_m as long as $r_m > 0$ or $0 < v_p < \infty$ because of the following:

$$r_i = \alpha_i \cdot \tau \cdot v_p = \alpha_i \cdot r_m \quad \Rightarrow \quad \alpha_i = \frac{r_i}{r_m} = \textit{const.}$$

Let $r(R_i)$ denote the spatial region associate with the collision region R_i. Then, the spatial region of R_n, R_c, and R_p are all fixed regardless of τ because

$$\begin{aligned} r(R_n) &= \{r : 0 \le r \le (\alpha_i - \beta)\tau v_p = (\alpha_i - \beta) r_m\} \\ r(R_c) &= \{r : 0 \le r \le \tau v_p = r_m\} \\ r(R_p) &= \{r : (\alpha_i + \beta) r_m \le r \le r_m\} \end{aligned}$$

and α_i, β, and r_m are all constants.

Hence, the number of nodes in each of R_n, R_c, and R_p is constant regardless of the speed of propagation, and so the probability of no collision of the i-th transmitter is constant. Therefore,

$$f = \sum_i p_i \Pr\{NC|n_i\} = \textit{const. with respect to } \tau$$

□

3.3 Throughput for Finite Number of Nodes

In this paper we consider the throughput S in packets per transmission time. Because the expected number of successful packet receptions $f(n, \beta, p)$ in a time slot is independent of the propagation time as long as it is positive finite (Theorem 1), S can be expressed as follows:

$$S(n, \beta, p, \tau) = \frac{f(n, \beta, p)}{1 + \beta\tau} = \frac{np \Pr\{NC|n_i\}}{1 + \beta\tau} \tag{11}$$

where we know the probability of no collision from the previous section.

Using the numerical evaluation of (11), Fig. 3 shows the characteristics of the throughput depending on the size of guard band β; in (a) the maximum propagation delay τ is fixed, but the number of nodes n is varying. In (b) n is fixed but τ is varying. These plots show how the throughput responds to the variables; the optimizer β values are similar for one case, but different in the other case. And the throughput converges rapidly as n increases.

3.4 Throughput for Infinite Number of Nodes

In this section, we investigate the throughput of PDT-ALOHA protocol with an infinite number of nodes with the traffic load λ over the network, i.e, $n \to \infty$ while $p = \lambda/n$. Hence, the throughput in this case is given by

$$S' = \lim_{n\to\infty} S|_{p=\frac{\lambda}{n}} = \frac{\lambda}{1 + \beta\tau} \lim_{n\to\infty} \Pr\{NC|n_i\} \tag{12}$$

Because the integrand of (6) converges uniformly over $[0, 1]$ (see Appendix of [11]), we can exchange integral and limitation by Theorem 7.16 of [12]. Hence, with the

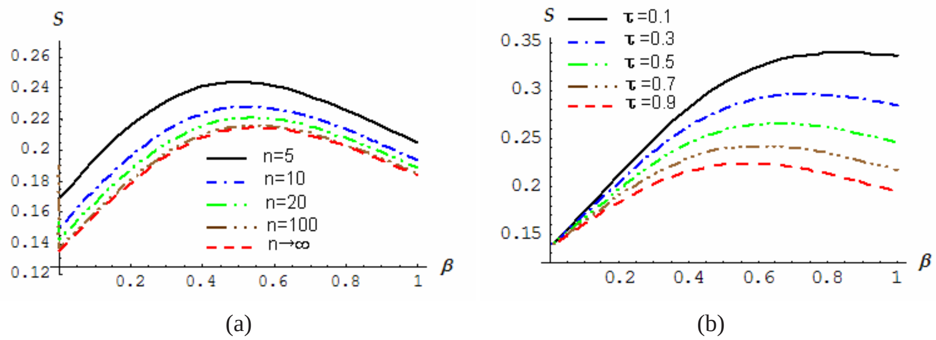


Fig. 3. Throughput of PDT-ALOHA vs. β; (a) when $\tau = 1$ and n is variable, (b) when $n = 100$ and τ is variable.

equalities in Table 1, we can achieve the conditional probability of no collision in this limiting case as follows:

If $0 \leq \beta \leq 0.5$;

$$\Pr\{NC|\alpha\} = \begin{cases} e^{-2\lambda+\lambda(\alpha+\beta)^2}, & 0 \leq \alpha \leq \beta \\ e^{-2\lambda+4\lambda\alpha\beta}, & \beta \leq \alpha \leq 1-\beta \\ e^{-\lambda-\lambda(\alpha-\beta)^2}, & 1-\beta \leq \alpha \leq 1 \end{cases} \tag{13}$$

If $0.5 < \beta \leq 1$;

$$\Pr\{NC|\alpha\} = \begin{cases} e^{-2\lambda+\lambda(\alpha+\beta)^2}, & 0 \leq \alpha \leq 1-\beta \\ e^{-\lambda}, & 1-\beta \leq \alpha \leq \beta \\ e^{-\lambda-\lambda(\alpha-\beta)^2}, & \beta \leq \alpha \leq 1 \end{cases} \tag{14}$$

$$\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n-1} = e^{-\lambda}$$

$$\lim_{n\to\infty}\left(1-\frac{\lambda}{n}+\frac{\lambda}{n}(\alpha+\beta)^2\right)^{n-1} = e^{-\lambda+\lambda(\alpha+\beta)^2}$$

$$\lim_{n\to\infty}\left(1-\frac{\lambda}{n}+4\frac{\lambda}{n}\alpha\beta\right)^{n-1} = e^{-\lambda+4\lambda\alpha\beta}$$

$$\lim_{n\to\infty}\left(1-\frac{\lambda}{n}(\alpha-\beta)^2\right)^{n-1} = e^{-\lambda(\alpha-\beta)^2}$$

Table 1. Equalities to use as building blocks to calculate the throughput in the limiting case

a	0.2464
b	−2.9312
c	−0.9887
p	0.0784
q	0.2638
r	0.9173
p_1	0.1805
q_1	0.6543
r_1	0.8898
p_2	0.2257
q_2	0.6959
r_2	0.9049

Table 2. Constants for Approximation Models

Therefore, we can obtain the expression for the probability of no collision for a packet of a transmitter substituting (13) or (14) into (6) which can be evaluated easily with the numerical method. We shall use the numerical evaluations as one of tools to investigate the properties of the maximum throughput and its approximation, which turns out to have a very simple expression in Sect. 4.2.

4 Optimization

In this section we investigate the *maximum* number of successful packet receptions in a time slot and the *maximum* throughput of PDT-ALOHA protocol. We also have an interest in the protocol parameters, particularly the size of guard band and the traffic load, which realize the maximum throughput.

We start with special cases, i.e. $\beta = 0$, or $\beta = 1$, which can be analyzed analytically. Then, we examine general cases given a network size in terms of the maximum propagation delay. Because it is very hard to obtain the closed form expression (if possible), we resort to use the numerical method to analyze the optimum behavior of the system. Based on the result of the numerical analysis, we propose simple approximations for the optimum behavior and its protocol parameters.

In this section we also consider the traffic load per transmission time λ_r because it is useful to compare traffic loads between systems of different size of time slot and it turns out it gives simpler approximation for the optimum values. The traffic load per transmission time has the relationship as $\lambda_r = \lambda/(1+\beta\tau)$ with the traffic load per time slot which we have dealt with so far.

Although we assume in this section the limiting case where the number of nodes in the network is infinite, it is fairly straightforward to adapt the method we used here for the finite number of nodes.

4.1 Special Cases

When there is no guard band (i.e $\beta = 0$) or the guard band is full so that there is no collision between packets from different time slots (i.e. $\beta = 1$) we have a closed-form expression for throughput which is simple enough to analyze analytically the maximum throughput. When $\beta = 0$ it is easy to see from (6), (12), and (13) that the throughput is as follows:

$$S_0(\lambda) \doteq S(\beta = 0, \lambda, \tau) = \lambda e^{-2\lambda} \tag{15}$$

There is no guard band in this case which makes the maximum propagation time irrelevant to the throughput, which (15) confirms. Note that the throughput in this case become the throughput of the classical unslotted ALOHA protocol [13, 14] as pointed by [7, 1].

The maximum throughput can be obtained simply using the derivative since S_0 is convex. The maximum is achieved at $\lambda = 0.5$ (i.e. $\lambda_r = 0.5$) as follows:

$$S_0^* = e^{-1}/2 \tag{16}$$

When $\beta = 1$ the throughput is as follows from (6), (12), and (14):

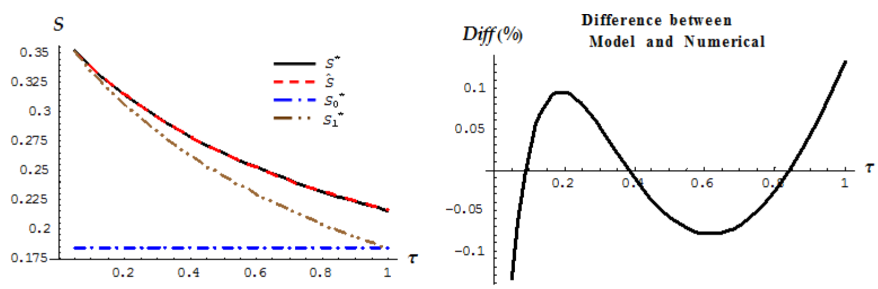
$$S_1(\lambda, \tau) \doteq S(\beta = 1, \lambda, \tau) = \frac{\lambda e^{-\lambda}}{1+\tau} \tag{17}$$

Because S_1 is convex regarding λ at any $\tau \in (0, 1]$, we can obtain its maximum given τ using the partial derivative as follows:

$$S_1^*(\tau) = \frac{e^{-1}}{1 + \tau} \tag{18}$$

where the maximizer is $\lambda = 1$ while the corresponding traffic load per transmission time is $\lambda_r = 1/(1 + \tau)$.

Figure 4.(a) shows S_0^* and S_1^* as well as the maximum throughput S^* which is discussed in Sect. 4.2; it can be seen that both of them are suboptimal although S_1^* approaches the maximum as τ goes to 0. When $\tau = 1$ for which the maximum propagation delay is equal to the transmission time of a packet, the throughput becomes same whether PDT-ALOHA has the full guard band or no guard band at all.



(a) (b)

Fig. 4. The maximum throughput: (a) numerically calculated values and its approximation, (b) the difference between numerical values and its approximation

4.2 Maximum Throughput

Now we investigate the maximum throughput S^* over all possible non-negative guard band β and network load per time slot λ given the network size in terms of the maximum propagation delay. Note that it is sufficient to look into only $\beta \in [0, 1]$ and $\lambda \in [0, 1]$ because $S(\beta, \lambda, \tau) \le S(1, 1, \tau)$, $\forall \beta \ge 1$, $\forall \lambda \ge 1$ due to Theorem 2 (for a finite number of nodes), Theorem 3 (for an infinite number of nodes), and Theorem 4.

Theorem 2. *Suppose a network of n number of nodes is assumed as that of Sect. 3.3 with $p = \lambda/n$. Then, the throughput S_n of the PDT-ALOHA protocol with $\lambda \ge 1$ for the network is no higher than when $\lambda = 1$. That is,*

$$\lambda \ge 1 \;\Rightarrow\; S_n(\beta, \lambda, \tau) \le S_n(\beta, 1, \tau),\; \forall \beta \in [0, 1]$$

Proof. The expected number of successful receptions $f_n(\beta, \lambda)$ in a time slot can be expressed as follows using (2), (6), (9), and (10):

$$f_n(\beta, \lambda) = \lambda \int_0^1 2\alpha \Pr\{NC|\alpha\} d\alpha = \lambda \left(1 - \frac{\lambda}{n}\right)^{n-1} \int_0^1 g_n(\beta, \lambda) d\alpha$$

where $g_n(\beta, \lambda)$ is a proper function after extracting $\left(1 - \frac{\lambda}{n}\right)^{n-1}$.

Suppose $\lambda \geq 1$. Since $0 < \lambda_1 \leq \lambda_2 < n$ implies $g_n(\beta, \lambda_1) \geq g_n(\beta, \lambda_2)$ for all $\beta \in [0, 1]$,

$$f_n(\beta, \lambda) = \lambda \left(1 - \frac{\lambda}{n}\right)^{n-1} \int_0^1 g_n(\beta, \lambda) d\alpha$$
$$\leq \lambda \left(1 - \frac{\lambda}{n}\right)^{n-1} \int_0^1 g_n(\beta, 1) d\alpha \;\leq\; \left(1 - \frac{1}{n}\right)^{n-1} \int_0^1 g_n(\beta, 1) d\alpha \;=\; f_n(\beta, 1)$$

where the last inequality holds since $x(1 - x/n)^{n-1} \leq (1 - 1/n)^{n-1}$ for $\forall x \geq 1$ and $\forall n \geq 2$.

Therefore,

$$S_n(\beta, \lambda, \tau) = \frac{f_n(\beta, \lambda)}{1 + \beta\tau} \leq \frac{f_n(\beta, 1)}{1 + \beta\tau} = S_n(\beta, 1, \tau)$$ □

Theorem 3. *Theorem 2 holds for the infinite number of nodes as long as the throughput limit exists.*

Proof. Since $S_n(\beta, \lambda, \tau) \leq S_n(\beta, 1, \tau)$ for $\forall \lambda \geq 1$ and $\forall n \geq 2$ from Theorem 2,

$$S(\beta, \lambda, \tau) = \lim_{n \to \infty} S_n(\beta, \lambda, \tau) \leq \lim_{n \to \infty} S_n(\beta, 1, \tau) = S(\beta, 1, \tau)$$

as long as the limits exist. □

Theorem 4. *The throughput S with the normalized guard band size $\beta \geq 1$ of an arbitrary network is no higher than that of $\beta = 1$. That is,*

$$\beta \geq 1 \;\Rightarrow\; S(\beta, \boldsymbol{p}, \tau) \leq S(1, \boldsymbol{p}, \tau)$$

Proof. If $\beta \geq 1$, there is no longer collision of packets between different time slots and β does not have any effect on packets sent in the same time slot. Hence, the expected number of successful packet receptions in a time slot is same for $\beta \geq 1$ as that of $\beta = 1$. However, increasing β makes the size of time slot increases. Therefore, the claim follows. □

We evaluate the maximum throughput for 20 values of τ starting from 0.05 to 1 incrementing 0.05 using the numerical method. After examining the behavior of S^*, we propose the following simple expression as an approximation of S^*:

$$\hat{S}(\tau) = p + \frac{q}{\tau + r} \tag{19}$$

where p, q, and r are constants.

The black solid line of Fig. 4.(a) shows the interpolation of 20 data points of the maximum throughput found in aforementioned way. The red dash line is of our approximation $\hat{S}$ with constants achieved through numerical curve fitting (Table 2). And Fig. 4.(b) visually shows the accuracy of $\hat{S}$. As can be seen, our approximation has reasonably good accuracy. From the figure we can also see that the PDT-ALOHA with proper parameter values can achieve about 17% (when $\tau = 1$) to 100% (when $\tau \to 0$) improvement on throughput over the traditional ALOHA protocol.

The optimum values of protocol parameters which realize the optimum throughput are also of interest. In particular, we are interested in the optimum size of the guard band β^* and the optimum traffic load given the network size in terms of τ.

Through the numerical analysis we are able to propose a simple approximation model for the optimizer β^* as follows:

$$\hat{\beta}(\tau) = p_1 + \frac{q_1}{\tau + r_1} \tag{20}$$

where p_1, q_1, and r_1 are constants; their proper values are given in Table 2 through curve fitting.

As for the optimum traffic load per transmission time, we propose the following approximation model:

$$\hat{\lambda_r}(\tau) = p_2 + \frac{q_2}{\tau + r_2} \tag{21}$$

where p_2, q_2, and r_2 are constants (Table 2).

The black solid lines of Fig. 5.(a) and (c) show the interpolation of 20 data points of β^* and λ_r^*, respectively, which are found numerically. The associated red dash lines are of our approximation $\hat{\beta}$ and $\hat{\lambda_r}$ with constants in Table 2. And Fig. 5.(b) and (d) visually show the accuracy of corresponding optimizers. As you can see, both of our approximations have reasonably good accuracy.

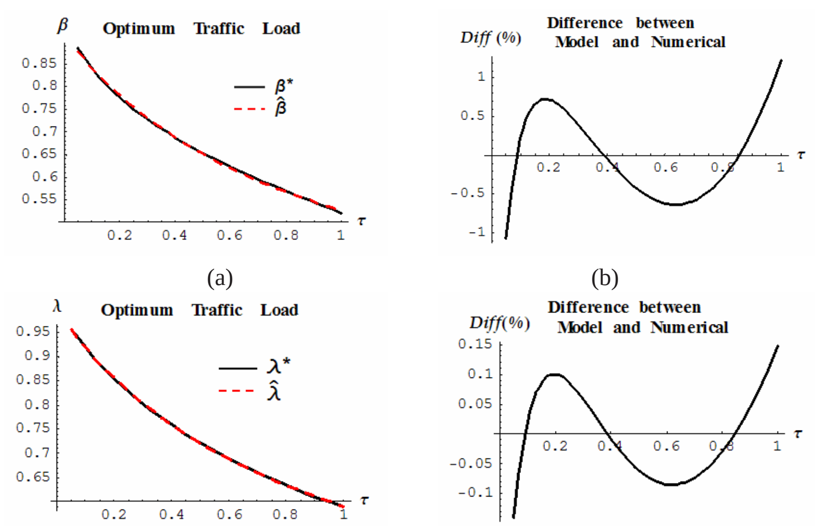


Fig. 5. The optimizers: (a) numerically calculated β^* values and its approximation, (b) the difference between numerical β^* values and its approximation, (c) λ_r^* values and its approximation, (d) the difference between numerical λ_r^* and its approximation

4.3 Maximum Expected Number of Successful Packet Receptions

In this subsection we consider the maximum expected number of successful receptions. We first present the analytic findings about the properties of the maximum successful number. The findings are more general than what we assume previously. We prove through Theorem 5 that the maximum expected number of receptions is monotonically non-decreasing with respect to the guard band β even when the network area is no longer 2D disk and the sending probability is not identical for each node as long as the maximum propagation delay is less than the transmission time of a packet.

Lemma 1. *Given the arbitrary location distribution of n transmitters and the probability p_i that i-th transmitter transmits a packet in a time slot, the expected number of successful reception in a time slot, $f(\beta, \boldsymbol{p})$, is monotonically non-decreasing as the normalized guard band β increases when the maximum propagation delay τ in the network is less than the transmission time T.*

In other words,

$$0 \leq \beta_1 \leq \beta_2 \leq 1 \quad \Rightarrow \quad f(\beta_1, \boldsymbol{p}) \leq f(\beta_2, \boldsymbol{p}),$$
$$\textit{for all } \boldsymbol{p} = (p_1, \ldots, p_n) \textit{ s.t } 0 \leq p_i \leq 1, \, \forall i \in \{1, \ldots, n\}$$

Proof. Because $\tau \leq T$, a transmission can interfere only with the transmission of immediate previous, current, and/or immediate next time slot. Hence, there are at most three collision regions given a transmitter as we investigated in Sect. 3.2. The three regions for an arbitrary i-th transmitter which has the normalized propagation time distance of α_i are in summary as follows in terms of normalized time distance;

For $R_n(\alpha_i)$ the region for possible collision with the next consecutive time slot: $0 \leq \tau_n \leq \alpha_i - \beta$. For $R_c(\alpha_i)$ the region for possible collision with the current time slot: $0 \leq \tau_c \leq 1$. And, for $R_p(\alpha_i)$ the region for possible collision with the previous consecutive time slot: $\alpha_i + \beta \leq \tau_p \leq 1$

Hence, when β increases, $R_n(\alpha_i)$ decreases monotonically up to $\emptyset$, making the corresponding collision probability monotonically non-increasing; $R_c(\alpha_i)$ stays constant, not changing the probability; and $R_p(\alpha_i)$ decreases monotonically up to $\emptyset$, making the probability monotonically non-increasing. These implies that the probability of no collision for the i-th transmitter $\Pr\{NC|n_i\}$ is monotonically non-decreasing for each i.

Therefore, the expected number of successful receptions in a time slot $f(\beta, \boldsymbol{p}) = \sum_i p_i \Pr\{NC|n_i\}$ is monotonically non-decreasing with respect to β. □

Theorem 5. *With the same assumptions of Lemma 1, the maximum expected number (over $\boldsymbol{p}$) of successful packet receptions $f^*(\beta)$ in a time slot (i.e. $f^*(\beta) = \max_{\boldsymbol{p}} f(\beta, \boldsymbol{p})$) is monotonically non-decreasing with respect to the normalized guard band size β. In other words,*

$$0 \leq \beta_1 \leq \beta_2 \leq 1 \quad \Rightarrow \quad f^*(\beta_1) \leq f^*(\beta_2)$$

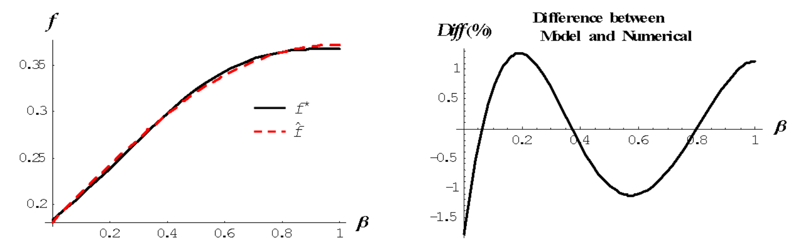
Proof. From the definition of f^* and Lemma 1,

$$f^*(\beta_2) \geq f(\beta_2, \boldsymbol{p}) \geq f(\beta_1, \boldsymbol{p}), \; \forall \boldsymbol{p}$$

Therefore, $f^*(\beta_2)$ is an upper bound of $f(\beta_1, \boldsymbol{p})$ for all $\boldsymbol{p}$, which implies the following:

$$f^*(\beta_2) \geq \max_{\boldsymbol{p}} f(\beta_1, \boldsymbol{p}) = f^*(\beta_1) \qquad \square$$

As same as the previous section, we use the numerical method to evaluate f^*. The black solid line of Fig. 6.(a) shows the interpolation of 21 data points of f^* found numerically.



(a) (b)

Fig. 6. The maximum number of successful receptions in a time slot: (a) numerically calculated values and its approximation, (b) the difference between numerical values and its approximation

Although it is hard to obtain the exact expression of f^*, we know from Theorem 5 that the maximized function $f^*(\beta) = \max_\lambda f(\beta, \lambda)$ is monotonically non-decreasing. From this fact and the observation that the log-scale plot of the numerically evaluated $f^*(\beta)$ is approximately of cubic function, we are able to propose the following approximation model for $f^*(\beta)$:

$$\hat{f}(\beta) = e^{a(\beta-1)^2(\beta+b)+c} \qquad (22)$$

where a, b, and c are constants and the constraint that $b < -1$ makes sure that the function is monotonically increasing.

The red dash line of Fig. 6.(a) is plotted by this approximation with proper constants suggested in Table 2 acquired through the numerical curve fitting. And Fig. 6.(b) shows its accuracy.

5 Acknowledgement

We would like to thank Affan A. Syed, John Heidemann, and Wei Ye from USC/ISI, who developed and discussed with us the original PDT-ALOHA algorithm. Affan A. Syed also helped greatly in discussing the original problem formulation.

This work was funded in part by NSF through the CAREER grant CNS-0347621.

6 Conclusion

We have presented theoretical analysis of an underwater MAC protocol in this work. Specifically, we have analyzed mathematically the performance of the PDT-ALOHA protocol. We have investigated different metrics of performances – expected number of successful packet receptions in a time slot, throughput, and maximum throughput. We have obtained exact expressions for the number of receptions and throughput. Although it is very hard to obtain meaningful closed-form expressions from the exact expressions, it is fairly fast to numerically calculate them with given parameters. Further, we have obtained simple expressions for the maximum throughput and optimum protocol parameter values which are shown to be very good approximations. From a practical point of view, our result shows that the throughput of optimized PDT-ALOHA protocol is 17-100% better than that of conventional slotted ALOHA.

We have also proven a number of interesting and useful properties concerning the performance of the PDT-ALOHA protocol. We have proven that the expected number of successful packet receptions is independent of the propagation speed; that its maximum is non-decreasing as the size of guard band increases; and derive a bound on the network load that offers the maximum throughput.

In the future, we would like to extend these results for 3-dimensional underwater networks.

References

1. Syed, A.A., Ye, W., Heidemann, J., Krishnamachari, B.: Understanding spatio-temporal uncertainty in medium access with ALOHA protocols. In: WuWNet ’07. (2007)
2. Akyildiz, I.F., Pompili, D., Melodia, T.: Underwater Acoustic Sensor Networks: Research Challenges. In: Ad Hoc Networks Journal. (Mar. 2005) 257–279
3. Partan, J., Kurose, J., Levine, B.N.: A Survey of Practical Issues in Underwater Networks. In: WUWNet ’06. (2006)
4. Heidemann, J., Li, Y., Syed, A., Wills, J., Ye, W.: Underwater Sensor Networking: Research Challenges and Potential Applications. In: WCNC ’06. (April 2006)
5. Guo, X., Frater, M., Ryan, M.: A Propagation-delay-tolerant Collision Avoidance Protocol for Underwater Acoustic Sensor Networks in Proc. IEEE Oceans Conference (2006)
6. Xie, P., Cui, J.H.: Exploring Random Access and Handshaking Techniques in Large-Scale Underwater Wireless Acoustic Sensor Networks. In: OCEANS ’06. (September 2006)
7. Vieira, L.F.M., Kong, J., Lee, U., Gerla, M.: Analysis of ALOHA protocols for underwater acoustic sensor networks. In: Extended abstract in WUWNet’06. (2006)
8. Gibson, J.H., Xie, G.G., Xiao, Y., Chen, H.: Analyzing the Performance of Multi-Hop Underwater Acoustic Sensor Networks. In: OCEANS ’07. (June 2007)
9. Crozier, S., Webster, P.: Performance of 2-dimensional sloppy-slotted ALOHA random accesssignaling. Wireless Communications, Conference Proceedings, IEEE International Conference on Selected Topics in (1992) 383–386
10. Crozier, S.: Sloppy-slotted ALOHA. In JPL, California Inst. of Tech., Proceedings of the 2nd International Mobile Satellite Conference (1990) 357–362
11. Ahn, J., Krishnamachari, B.: Performance of Propagation Delay Tolerant ALOHA Protocol for Underwater Wireless Networks. Technical Report CENG-2007-13, USC (Nov. 2007)
12. Rudin, W.: Principles of Mathematical Analysis. 3rd edn. McGraw-Hill (1976)
13. Rosner, R.: Distributed Telecommunications Networks via Satellite and Packet Switching. Wadsworth, Inc (1982)
14. Stallings, W.: Data and Computer Communications. Macmillan Publishing Company, New York (1985)