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Found Mathematical Objects by Tom Johnson

In this paper, Tom Johnson described an unconventional viewpoint of making art by finding objects. Specifically, Johnson explained how to compose music by using found mathematical objects, such as Pascal's triangle and the Narayana series. He argued through several of his own compositions that artistic techniques become unnecessary in this way and instead, the esthetic foundations become a little more solid.

First Johnson explained his philosophy of composing, regarding music as symbols of numbers and notes. He believed that notes are the essential components of composition, and sounds are just the phenomena which can not be correctly measured. Based on the symbolic characteristics of music, if number existed before music, then music that comes from numbers is somehow connected to mathematical concepts. He further emphasized the automaton of music. When the music is generated by a sequence of automaton with mathematical objects, the music is self-contained and derives outside of the composer.

In order to present the relationship between music composition and mathematical objects, Johnson gave three examples of automata and three transformations of Pascal's Triangle. The objective of composition here is to use an existing mathematical object and arrange it into an artistic object without altering, decorating, or composing it in a traditional way. The direct mapping between music and mathematics does not exist, but certainly it can be created. The first example is a simple number sequence containing basic three-number component n, n+1, n for number n. He assigned a note for each number, and composed a piece with a sequence of neighboring-moving notes. The second example is about the Narayana sequence, a type of delayed Fibonacci series, produced by an interesting mathematical problem of the birth of claves. He left the third example in another article.

The transformations of the mathematic objects, Pascal's Triangle, into music are achieved by the formation of chords. The triangle was found in the first piece showing all the possible combinations of chords within one octave. A triangle could also be seen in the second piece where all the chords consist of only major second and minor third intervals. The third piece that transformed Pascal's triangle was a 7-note scale piece with infinite self-similar structure.

In the second part of the paper, Johnson focused on self-replicating (melody) loops and rhythmic canons. He constructed a melodic loop that playing the first, third, fifth, etc.; notes will be the same melody as playing all the notes. That melody is called self-replicating at the ratio of 2:1. The rhythmic canon, on the contrary, is seeking a rhythm so that every point will be filled on the loop when it is repeated. The similarity between these two cases came from the group theory that both pieces divide a cyclic group into similar sub-groups.

Johnson then showed an orchestra piece he composed by a 21-note loop with the explanation of the constructing processes. He was satisfied with the results since not only the piece was self-contained, but also the mathematics objects helped him to get rid of certain old-fashioned ways of composing works and orchestrating. More musically and mathematically interesting examples about self-replicating loops and rhythmic canons

were mentioned in the e-mail conversations between him and others, such as Thomas Noll, Guerino Mazzola, and Markus Reineke. Examples include questions about how to make rhythmic canons with voice moving at different tempi, how to find the shortest self-replicating loop given a small ratio, and how to find a loop of three or more numbers prime to each other.

This is a very interesting paper with an original perspective. In the conventional thinking, music composing is regarded as a creative process which can be achieved only by people who have special artistic senses. Johnson argues that music actually derives from existing objects, the mathematics objects in particular, and composers are the one who find those objects and realize them, or transform them into pieces. His statements become quite convincing as he presents several compositions he has done in this way. I believe that there are some common structures of beauty shared by various kinds of art, for example, symmetry and rational relations. People can model those structures in terms of mathematics, or create them from mathematics. I respect the power of mathematical objects he used in composition, but I still admire more of the esthetics that composers show in music. After all, not all compositions can be constructed by mathematical objects, and not all combinations of mathematical objects can produce a good piece of music.