The Kinematics of Musical Expression

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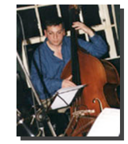
(Received 30 April 1992; revised 21 July 1994; accepted 25 October 1994)

Presented by Eric Cheng
2.1.06

Background Information



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  - Lecturer, University of Manchester, School of Psychological Sciences (since 1994)
  - Research interests:
    - Temporal Coding in the Auditory Cortex
    - Sensorimotor Processing
    - Acoustic Sensitivity of the Vestibular System
  - B.S. in Theoretical Physics, University of Exeter

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Outline

Introduction

- Musical motion <=> Physical motion
  - Connection dates back to antiquity
- Aristoxenus (c. 320 B.C.):
  - Melodic Motion
    - Notes = geometrical points in space
    - Intervals = distances between notes
  - Rhythmic Motion
    - Notes = points on a metrical grid
    - Tempo = velocity of motion with respect to metrical grid

Music and Motion

- General points on musical motion:
  1. 2 degrees of freedom:
     - Tonal movement
     - Rhythmic movement
  2. Musical movement imitates motion in physical space.
  3. Musical movement alludes to physical motion of a body or limb.

Music and Motion

- Kronman and Sundberg (1987):
  - Used regression analysis to model final ritardando in musical performances as a motion under constant negative acceleration.
  - Analagous to a runner slowing down with constant acceleration.
- Longuet-Higgins and Lisle (1989):
  - Constant accleration most natural sounding

Objectives

Goals of this paper:

1. To formulate problem in a more precise mathematical form.
2. To extend studies of Sundberg *et al*. to include the *accelerandi* as well as *ritardandi* in performances of complete pieces.
3. To show the metrical grid has its origin in the way the auditory system processes rhythm.

I. Mathematical Preliminaries

A. The Kinematic Variables

- Relate musical motion to trajectory planning
- Can think of the score as a trajectory in 2-D space.
- Vertical axis = pitch space
- Horizontal axis = metrical space
- For simplicity, examine only horizontal dimension

A. The Kinematic Variables

$$a = a(t), \quad a = a(x),$$

$$v = v(t) = \int a(t)\,dt, \quad v = v(x),$$

$$x = x(t) = \int v(t)\,dt, \quad t = t(x) = \int \frac{1}{v(x)}\,dx,$$

Physics vs. Music

	Physics	Music
$x(t)$	Position	Metrical Position
$v(t)$	Velocity	Tempo

B. Metrical Position vs. Onset Time

- Cannot measure tempo directly
- Instead, measure onset times
- Plot Metrical Position vs. Onset Time

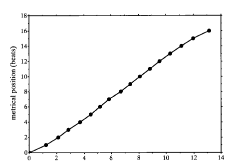


FIG. 1. Metrical position versus onset times for the first 16 beats from a performance of the *Chopin* Prelude in F# minor Op. 28, No. 8.

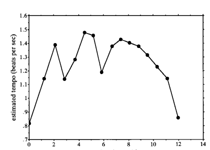
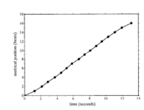


FIG. 3. Estimated tempo for the first 16 beats from a performance of the *Chopin* Prelude.

C. Estimation of tempo and accleration

- Tempo may be approximated as:

$$v_x \doteq \frac{\Delta x}{\Delta t_x} = \frac{x_2 - x_1}{t_{x_2} - t_{x_1}}$$

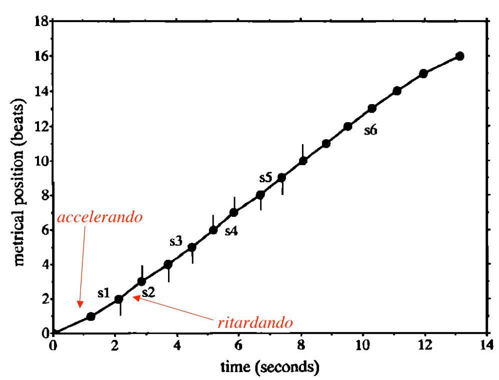


- This introduces a discretization error
- Cannot determine accurately the tempo
- Reject use of tempo for regression analysis
- Instead, use onset times

II. Method

A. Regression analysis of timing segments

- Performance timing characterized by series of connected *accelerandi* and *ritardandi*
- Onset plots therefore characterized by series of convex and concave segments of varying length
- Fit a curve to each segment



A. Regression analysis of timing segments

- In general, we will have:
  - *n* segments of length l_j
  - Regression equation for each segment
  - $\{b_{kj}\}$ coefficients and $\{R^2_j\}$ variances accounted for
- For simplicity, use same form for each segment, regardless of length
- Use 2 parameters since shortest segments are 2 onsets in length

B. The Model

- Boundary condition:
  $t = 0$ when $x = 0$ --> $b_o = 0$
- Therefore, our equations will be of the form:
  $x(t) = b_1 t + b_2 f(t) + e_i$
- Simplest plausible model of this form is that used by Kronman and Sundberg (1987):

$$a(t)=b_2, \quad a(x)=b_2 \qquad (1a)$$
$$v(t)=b_1+b_2 t, \quad v(x)=(b_1^2+2b_2 x)^{1/2}, \qquad (1b)$$
$$x(t)=b_1 t+1/2 b_2 t^2, \quad t(x)=\frac{1}{b_2}\{(b_1^2+2b_2 x)^{1/2}-b_1\}. \qquad (1c)$$

B. The Model

$$a(t)=b_2, \quad a(x)=b_2 \qquad (1a)$$
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$$x(t)=b_1 t+1/2 b_2 t^2, \quad t(x)=\frac{1}{b_2}\{(b_1^2+2b_2 x)^{1/2}-b_1\}. \qquad (1c)$$

- Constant acceleration
- Linear tempo increase
- Piecewise Linear Tempo (PLT)

B. The Model

- Simpler model:

$$a(t)=0, \quad a(x)=0, \qquad (2a)$$
$$v(t)=b_1, \quad v(x)=b_1, \qquad (2b)$$
$$x(t)=b_1 t, \quad t(x)=x/b_1. \qquad (2c)$$

- Zero acceleration
- Constant tempo
- Piecewise Constant Tempo (PCT)

C. Statistics

- Distribution of $\{R^2_j\}$ highly skewed
  - All close to 1.
  - Use median variance accounted for $\{M_R^2\}$ for each segment length to compare each model
- $\{M_R^2\}$ values will be very high
- To make a judgment between models, we require:
  - Number of segments *n* is large
  - The variance of $\{R^2_j\}$ values is small

III. Data

III. Data

- Used three performances of Chopin pieces by a single concert pianist
- Chopin Etude No. 2 in A-flat
- Chopin Prelude in F#-minor, Op. 28, No. 8

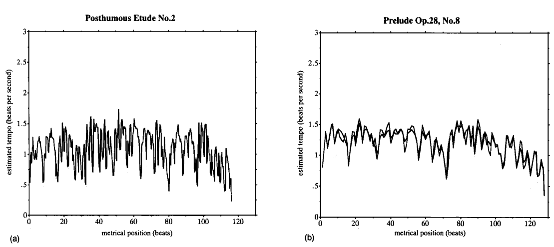


FIG. 6. Estimated tempo for (a) a performance of the *Chopin* Posthumous Etude No. 2 in A-flat major and (b) two performances of the *Chopin* Prelude F#-minor.

IV. Results

IV. Results

- PCT model performed very well
- PLT contributed about 3% $\{R_j^2\}$ in Chopin Etude
- Smallness of contribution of acceleration term suggests that any 2-parameter model which is monotonic and intrinsically linear in the parameters will fit well.

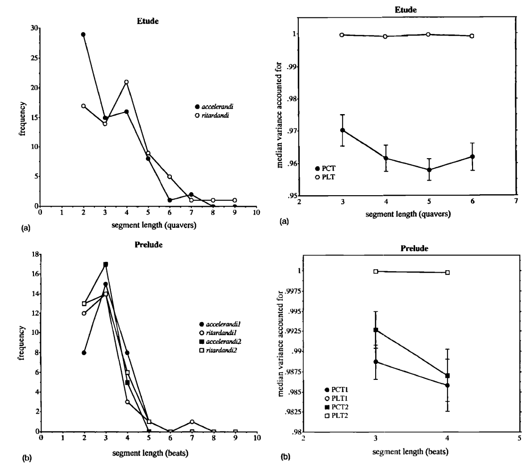


FIG. 7. Frequency distribution of segment lengths. (a) *Chopin* Etude and (b) *Chopin* Prelude (two performances).

FIG. 8. Variance accounted for by piecewise linear versus piecewise constant tempo as a function of segment length. (a) *Chopin* Etude and (b) the *Chopin* Prelude.

V. Discussion

A. Allusion of Tempo Variation to Physical Motion

- Must exercise caution in making connection between music and physical motion
- Essential differences between motion of particle and motion of musical expression:
  - For a particle, motion is determinate
  - We have a set of forces and laws to predict the trajectory
  - In music, motion varies in a structured but nondeterminate way for communication
  - We are given a trajectory, and attempt to predict intention based on trajectory

A. Allusion of Tempo Variation to Physical Motion

- Must distinguish metaphorical notions of musical motion with more psychologically concrete phenomena that correspond to the metaphorical.
- Rhythmic motion <--> perception of metrical music
- Tempo variation <--> conception metrical space

B. The auditory-motor basis of metrical space

- How do we explain the spatial-motional view of meter and tempo?
- Can construct theory of rhythm perception
- A model for rhythm perception can provide the psychological basis for metrical space and thus connect physical and musical motion

B. The auditory-motor basis of metrical space

- Components of the model:
  1. Auditory periphery
  2. Time-domain process
  3. Frequency-domain process
  4. Sensory-motor feedback filter

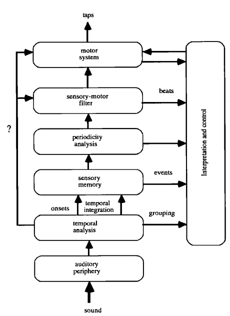


FIG. 9. An auditory-motor model of rhythm perception.

B. The auditory-motor basis of metrical space

1. Auditory periphery
   - Simulated with cochlear model in conjunction with array of hair cells
2. Time-domain Process
   - Triggered by onsets
   - Carries out temporal segmentation
   - Gives information on loudness and grouping of neural events

B. The auditory-motor basis of metrical space

3. Frequency-domain process
   - Carries out periodicity analysis
   - Determines metrical harmonics of a rhythm
4. Sensory-motor feedback filter
   - Selects tactus from metrical harmonics

B. The auditory-motor basis of metrical space

- Possible to associate each event with a number of cycles of harmonics
- Independent of tempo
- This provides the perceptual basis of metrical space

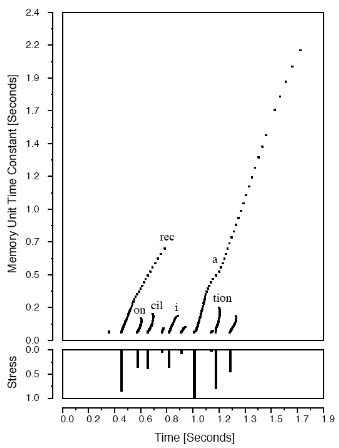


Figure 5. Rhythmogram of the utterance 'reconciliation'. Source: Todd and Brown (1994)

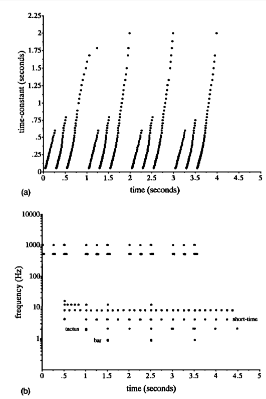
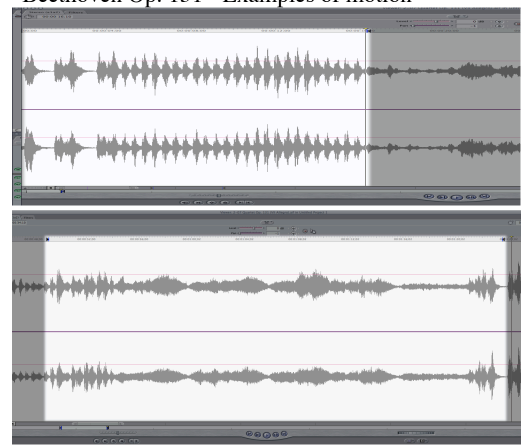


FIG. 10. Response of (a) the temporal analysis and (b) the periodicity analysis to a simple anapest rhythm. The naturalness of placing pitch and rhythm in the same map is evident. The tone bursts have two components an octave apart (512 and 1024 Hz). The main rhythmic harmonics are also an octave apart (1, 2, and 4 Hz).

C. Centers of Moment in Music and Movement

- Can talk about motion on 2 levels:
  1. Tactus
     - Associated with the natural foot-tapping period of 600ms
     - Motion inferred from a constant tempo rhythm with a strong beat
  2. Whole body motion
     - Natural body sway period of 5s
     - Gestural motion associated with *tempo rubato*

Beethoven Op. 131 - Examples of motion



Tactus

Body Sway

VI. Conclusion

- Regression analysis should be limited in three ways:
  1. Use onset times instead of tempo
  2. Use time as independent variable
  3. Use no more than 2 parameters
- PLT is simplest model that fits these limitations:
- PLT better than PCT, but PCT provides reasonable model for performances with shallow rubato

VI. Conclusion

- Explanation the origins of music-motion connection lies in the way the auditory system perceives rhythm
- Two types of motion associated with foot tapping and body sway can be accounted for by the sensory-motor process which selects the tactus

Comments

- Todd successful in achieving first two goals:
  - Formulated problem mathematically
  - Included *accelerandi* in analysis
- Regression analysis provides little compelling evidence for music-motion connection
  - PCT and PLT models not uniquely tied to physics. They are simply mathematical equations.
  - As Todd said, probably any 2-parameter model linear in the parameters would have worked

Comments

- Less successful in achieving third goal
- Presented a model to account for music-motion connection, but provided little compelling evidence for why we should believe the model other than that it accounts for the music-motion connection