Radiation and acceleration of a relativistic electron by a focused single-cycle pulse

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ABSTRACT

We examined exact classical solutions for the radiation from an electron that is initially at rest on the axis and near the focus of a focused single-cycle TM electromagnetic pulse. We used a simple exact solution of Maxwell's equations that has zero magnetic field on the propagation z-axis where the electric vector is always along the z-axis . We examin ed the dependence on angle and frequency of the scattered radiation on the wavelength, Rayleigh length and energy of the incident pulse. We have carried numerical calculations for pulse energies from 0.01mJ to 20 J. The behavior of the scattered light is distinctly different for the case when the electron is initially at rest at the focal point of the pulse (z=0) or is initially removed by the "optimized" distance z_{opt} which makes the scattered light energy a maximum. In the latter case, we see only a very slight nonlinearity in the ratio of scattered-to-incident energies. However, when the electron is initially at pulse focus, there is always a nonlinearity that results is completely saturation of the scattered energy for pulses whose energy is beyond few millijoules. Radiation spectrum is obtained not only by the analytical approximation, but also by the fast Fourier transform.

INTRODUCTION

After suggesting new, three-dimensional, packet-like solutions to the free-space homogeneous Maxwell's equations by Brittingham, Ziolkowski has discovered EDEPT solutions. Hellwarth and Nouchi have explored a different subset of these EDEPT solutions that are called "focused doughnut" pulses. In this research, we studied a scattering of a TM one-cycle focused dough nut pulse with a relativistic electron. Since magnetic field of an focused single-cycle pulse is zero along z-axis, we only consider the electric field of the pulse,

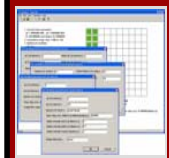
$$E_{TM,z} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{8 f_0 (q_1 q_2 + z^2 - c^2 t^2)\{(q_1(z+ct) - q_2(z-ct))\}}{[(q_1^2 + (z-ct)^2)(q_2^2 + (z+ct)^2)]^2} \qquad (1)$$

where q_1 is a nominal wave length, q_2 is the Rayleigh length or the depth of focal region, and f_0 is an electric field coefficient of a single-cycle pulse. Equation of motion for a relativistic electron with this electric field simply is

$$\frac{dp}{dt} = F_{ext} \qquad (2)$$

From the numerical solutions to the equation, total power spectrum was obtained and compared with an analytical result. We also observed that the radiation energy was not linearly proportional to a pulse intensity at high intensity region. When electrons were scattered by a focused pulse, they did not collide each other but showed a compressed state.

COMPUTATIONAL METHOD



Computer language:
- Microsoft Visual C++ version 6.0

Mathematical tools:
- Matlab R2006a
- Mathematica 5.2

CURRENT WORK

For various pulse parameters and an electron's initial conditions, we are calculating vast data sets (more than 10,000 numerical experiments) to analyze physical behaviors of a single-cycle pulse scattering with an relativistic electron.

RESULTS

Scattering of a relativistic electron by a TM focused single-cycle pulse

$(q_1 = 1\,micron,\ q_2 = 10\,micron$ and $U_{pulse} \sim 0.65\,Joule)$

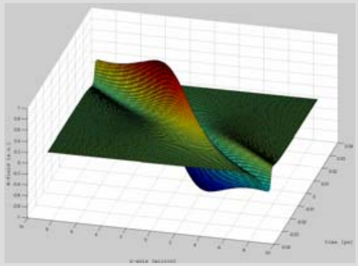


Fig 1. Spatiotemporal evolution of a focused TM single-cycle pulse

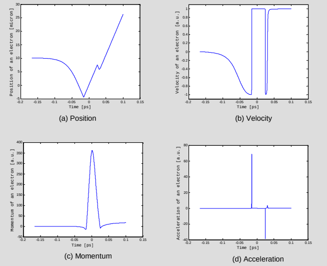


Fig 2. Numerical solutions to the relativistic equation of motion

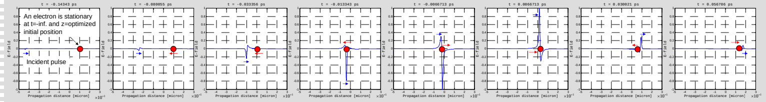


Fig 3. A time sequential plots for the scattering of a relativistic electron

Radiation Energy

The energy of a single-cycle pulse energy is give by

$$U_{pulse} = \frac{\mu_0 f_0^2 \pi^2 (q_1 + q_1)}{4 q_1^3 q_2^3} \quad [Joule] \qquad (3)$$

Radiation energy by a scattered relativistic electron was obtained by

$$U_{rad} = \frac{2 q_{el}^2}{3 m_{el}^2 c^3} \int dt \left(\frac{dp}{dt}\right)^2 \quad [ergs] \qquad (4)$$

We define a dimensionless parameter, Q($\equiv q_2 / q_1$). When pulse energy is constant, high Q means a weakly focused pulse and low Q means a highly focused pulse.

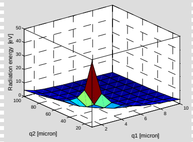


Fig 4. Radiation energy varying q1 and q2 under optimized initial condition and fixed pulse energy.
Pulse energy: 0.6578 Joule, Q: 1 ~ 100.
Radiation energy is maximized when q2 is high and q1 is small

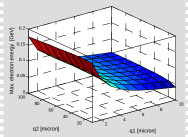


Fig 5. Maximum electron energy varying q1 and q2 under optimized initial condition and fixed pulse energy.
Pulse energy: 0.6578 Joule, Q: 1 ~ 100.
Maximum electron energy only depends on the wavelength, not Rayleigh length.

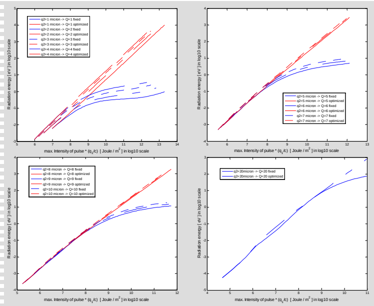


Fig 6. Radiation energy vs. maximum intensity of pulse on z-axis.
Pulse energy: 0.00001 ~ 20 Joule, Q: 1~10, 30 and q_1: 1 micron. "optimized" means the electron is initially located at optimized position for the maximum radiation at given pulse energy. "fixed" means the electron is initially at focus (z=0). For optimized initial conditions, radiation energies shows linearity like classical Thomson scattering. However, when the electron is located at focus, the radiation energies saturate at the incident pulse energies which are between 0.3 ~ 10 millijoules.

Power spectrum

The amplitude of radiation electric field for given radiation angle, θ about z-axis is obtained by

$$\mathrm{Re}\{g(\omega)\} = \frac{\omega m c^2}{A} K_1\left(x = \frac{\omega m c}{A}\sqrt{1 - \cos^2\theta}\right) \quad [cm\ \sec^{-1}] \qquad (5)$$

where K_1() is the modified Bessel function of the second kind.

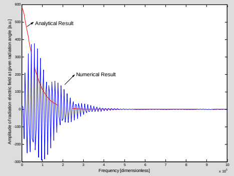


Fig 7. The amplitude of radiation electric field vs. frequency
Detection angle is 0.1038° which makes the maximum radiation.

Total power spectra (pulse energy: 0.6478 Joule)

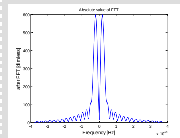


Fig 8. Radiation power vs. frequency.
Q: 10, q2/q1: 10 micron / 1 micron,
Radiation energy: $4.8734 \times 10^{-8}\ GeV$
Maximum electron energy: 0.1086 GeV

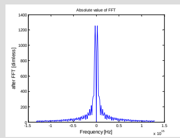


Fig 9. Radiation power vs. frequency.
Q: 10, q2/q1: 10 micron / 0.1 micron,
Radiation energy: $3.1775 \times 10^{-7}\ GeV$
Maximum electron energy: 0.3803 GeV

Multi-electron Acceleration

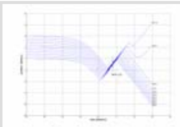


Fig 10. Electron position vs. time.
positive f0,
Q=10, Pulse energy: 0.6478 Joule,
compression ratio: ~92%

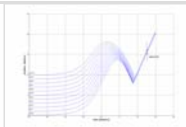


Fig 11. Electron position vs. time.
negative f0,
Q=10, Pulse energy: 0.6478 Joule,
compression ratio: ~99%

CONCLUSION

In this poster, we presented the ability of a TM focused single-cycle pulse to accelerate a relativistic electron. We have solved a equation of motion for a stationary relativistic electron with a focused TM single-cycle pulse with pulse parameters: wavelength, Rayleigh length and pulse energy. Since the radiative reaction force is negligible and magnetic fields on z-axis is zero , the electron only feels the z-directional electric field along propagation z-axis. The electric field of the pulse rapidly change its sign near focal plane. This makes an abrupt variation of an electron's momentum and acceleration which yield radiation of a relativistic electron. The analytical calculation of radiation energy agreed with the result of Jackson [6]. It also coincided with the numerical value of energy from the total power spectrum by fast Fourier transform. Varying the pulse energy from 0.01 mJ to 20 J, there was a distinct difference between the case of an electron's fixed initial position at z=0 and the case of an electron' s optimized initial position at each given pulse energy. In the latter, scattered radiation energy from an electron increased linearly like the classical Thomson scattering. However, the radiation energy from the electron which is initially at focus increased in a saturation curve. Also, we examined the multi-electron scattering. When the pulse injected into 11 electrons on z-axis which are distributed with equal relative distances about the focus, the scattered electrons did not collide each other. Especially, at negative f_0, the scattered electrons finally moved together with 99% compression ratio.

FUTURE WORK

In the nonlinearity of scattered radiation, we have not found a fundamental reason for this behavior. But, from the fact that maximum energy of electron also saturated, we learned there is the electron's optimal trajectory which has always three turning points and results in the maximum radiation of the scattering. We need to know why this optimal trajectory makes the radiation maximized. If we change our concern from the maximum radiation to the maximum electron' s energy, we found that the pulse scattering with negative f_0 has more advantages than one with positive f_0. We are also curious about how much fast an electron can be accelerated by the scattering with a focused single-cycle pulse. In order to apply our results to laboratory experiments, we are considering extending our research to other kinds of the single-cycle pulses or 1½ cycle pulses.

REFERENCES

[1] R. W. Hellwarth Focused one-cycle electromagnetic pulses, Phys. Rev. E **54**, 889 (1996).
[2] J. N. Brittingham Foucus waves modes in homogeneous Maxwell's equation: Transverse electric mode, J. Appl. Phys. **54**, 1159. (1983)
[3] R. W. Ziolkowski Localized transmission of electromagnetic energy, Phys. Rev.A **39**, 2005 (1989)
[4] S. Feng, H. G. Winful and R. W. Hellwarth Gouy shift and temporal reshaping of focused single-cycle electromagnetic pulses, Opt. Lett, **23**, 385. (1998)
[5] S. Feng, H. G. Winful and R. W. Hellwarth Spatio-temporal evolution of focused single-cycle pulses, Phys. Rev. E **59**, 4630. (1999)
[6] J. D. Jackson Classical electrodynamics, 3rd ed. (JohnWiley & Sons, 1998)
[7] W. K Panofsky and M. Phillips Classical electricity and magnetism, 2nd ed. (Addison-Wesley, 1962)
[8] L. D. Landau and E. M. Lifshitz The classical theory of fields, 4th ed. (Pergamon press, 1975)
[9] W. H. Press et al. Numerical recipes in C : The art of scientific computing, 2nd ed. (The press syndicate, 1992)
[10] A. Carati et al. Nonuniqueness properties of the physical solutions of the Lorentz-Dirac equation, Nonlinearity **8**, 65-79(1995)

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