EE 585 HOMEWORK #9

(6.8) $A = \begin{bmatrix} -1 & 4 \\ 4 & -1 \end{bmatrix}$ $\qquad \mathcal{C} = \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}$ not controllable

$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\qquad$ select $P^{-1} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then $P = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

$C = [1 \;\; 1]$ $\quad \bar{A} = PAP^{-1} = \begin{bmatrix} 3 & 4 \\ 0 & -5 \end{bmatrix}$ $\quad \bar{B} = PB = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad \bar{C} = CP^{-1} = [2 \;\; 1]$

Thus $\bar{x} = Px$ will transform the equation to

$$\dot{\bar{x}} = \begin{bmatrix} 3 & 4 \\ 0 & -5 \end{bmatrix} \bar{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = [2 \;\; 1] \bar{x}$$

and the equation is reduced to $\quad \dot{\bar{x}}_1 = 3\bar{x}_1 + u$, $\quad y = 2\bar{x}_1$

This reduced equation is observable.

(6.10) From Corollary 6.8 we see that x_3 is not controllable. We arrange the equation as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_3 \end{bmatrix} = \left[\begin{array}{cccc|c} \lambda_1 & 1 & 0 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 & 1 \\ 0 & 0 & \lambda_2 & 1 & 0 \\ 0 & 0 & 0 & \lambda_2 & 0 \\ \hline 0 & 0 & 0 & 0 & \lambda_1 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_4 \\ x_5 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ \hline 0 \end{bmatrix} u$$

$$y = [0 \;\; 1 \;\; 0 \;\; 1 \,|\, 1] \bar{x}$$

Thus the equation can be reduced as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_1 & 0 & 0 \\ 0 & 0 & \lambda_2 & 1 \\ 0 & 0 & 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [0 \;\; 1 \;\; 0 \;\; 1] \tilde{x}$$

Using Corollary 6.8 we conclude that the reduced equation is controllable.

Using Corollary 6.08 we see that x_1 and x_4 are not observable. We rearrange the equation as

$$\begin{bmatrix} \dot{x}_2 \\ \dot{x}_5 \\ \dot{x}_1 \\ \dot{x}_4 \end{bmatrix} = \left[\begin{array}{cc|cc} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ \hline 1 & 0 & \lambda_1 & 0 \\ 0 & 1 & 0 & \lambda_2 \end{array}\right] \begin{bmatrix} x_2 \\ x_5 \\ x_1 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ \hline 0 \\ 0 \end{bmatrix} u$$

$$y = [1 \;\; 1 \,|\, 0 \;\; 0] \hat{x}$$

This can be reduced to $\begin{bmatrix} \dot{x}_2 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} x_2 \\ x_5 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$

$$y = [1 \;\; 1] \begin{bmatrix} x_2 \\ x_5 \end{bmatrix}$$

This is controllable and observable.

HOMEWORK #9

(6.11) Select an arbitrary Q_2 such that $[Q_1\ Q_2]$ is nonsingular. Define $\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = [Q_1\ Q_2]^{-1}$

Then $\begin{bmatrix} P_1 \\ P_2 \end{bmatrix}[Q_1\ Q_2] = \begin{bmatrix} P_1Q_1 & P_1Q_2 \\ P_2Q_1 & P_2Q_2 \end{bmatrix} = \begin{bmatrix} I_{n_1} & 0 \\ 0 & I \end{bmatrix}$

$P_2Q_1 = 0$

Because Q_1 consists of all linearly independent columns of $[B\ AB \dots A^{n-1}B] = 0$ we have

$$P_2B = 0 \qquad P_2AQ_1 = 0$$

Consider the transform $\bar{x} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} x$

Then

$$\bar{A} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}[A][Q_1\ Q_2] = \begin{bmatrix} P_1AQ_1 & P_1AQ_2 \\ P_2AQ_1 & P_2AQ_2 \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} B = \begin{bmatrix} P_1B \\ P_2B \end{bmatrix}$$

$$\bar{C} = C[Q_1\ Q_2] = [CQ_1\ CQ_2]$$

Since $P_2B = 0$ and $P_2AQ_1 = 0$ the equation is in the form of (6.40) and can be reduced to the controllable form

$$\dot{\bar{x}}_1 = P_1AQ_1\bar{x}_1 + P_1Bu$$
$$y = CQ_1\bar{x}_1 + Du$$

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① $\dot{x} = Ax + Bu$ $\qquad$ ((A,c) observable)
$y = Cx + du$

$$y = cx + du$$
$$y' = cx' + du' = c(Ax + Bu) + du' = cAx + cBu + du'$$
$$y'' = cAx' + cBu' + du'' = cA(Ax + Bu) + cBu' + du''$$
$$= cA^2x + cABu + cBu' + du''$$

$$y''' = cA^3 + cA^2Bu + cABu' + cBu'' + du'''$$
$$\vdots$$
$$y^{n-1} = cA^{n-1} + cA^{n-2}Bu + cA^{n-3}Bu' + \dots + du^{n-1}$$

$$\underbrace{\begin{bmatrix} y \\ y' \\ \vdots \\ y^{n-1} \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} c \\ cA \\ \vdots \\ cA^{n-1} \end{bmatrix}}_{\theta} x(t) + \underbrace{\begin{bmatrix} d & 0 & \cdots & 0 \\ cB & d & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ cA^{n-2}B & \cdots & cB & d \end{bmatrix}}_{M} \underbrace{\begin{bmatrix} u \\ u' \\ \vdots \\ u^{n-1} \end{bmatrix}}_{u}$$

Since (A,c) is [redacted] observable θ has inverse.

$$x(t) = \theta^{-1}(y - Mu)$$

HOMEWORK #9

(3) a) Given $\dot{x} = \underset{n\times n}{A} x + \underset{n\times m}{B} u$; $F_{m\times n}$

$y = C_{m\times n}\, x$; $K_{n\times m}$

Define

$$\mathcal{C} = [\, B \mid AB \mid \dots \mid A^{n-1}B \,]$$

$$\mathcal{C}_f = [\, B \mid (A+BF)B \mid \dots \mid (A+BF)^{n-1}B \,]$$

$$\mathcal{C}_f = \mathcal{C} \begin{bmatrix} 1 & FB & F(A+BF)B & \cdots & F(A+BF)^{n-2}B \\ 0 & 1 & FB & & F(A+BF)^{n-3}B \\ \vdots & 0 & 1 & \ddots & \vdots \\ \vdots & 0 & & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & 1 \end{bmatrix}$$

The triangular matrix is nonsingular for any B, A, F

Rank of $\mathcal{C}_f$ is equal to rank of $\mathcal{C}$

Therefore (A,B) is controllable iff $(A+BF, B)$ is controllable.

b) Using duality (A,C) is observable iff $(A+KC, C)$ is observable.