HW #8 EE 585 HOMEWORK #8

(4.9) Define $z = [z_1 \cdots z_r]$ where z_i is $q \times q$ and z is $q \times rq$

$$z = C(sI - A)^{-1}$$
$$z(sI - A) = C$$
$$sZ = zA + C$$

Using the forms of A and C we have.

$$sZ_1 = -\alpha_1 Z_1 - \alpha_2 Z_2 - \ldots - \alpha_r Z_r + I_q$$
$$sZ_2 = Z_1$$
$$\vdots$$
$$sZ_r = Z_{r-1}$$

From these we get

$$z_2 = \frac{1}{s} z_1$$
$$z_3 = \frac{1}{s} z_2 = \frac{1}{s^2} z_1 \qquad \text{and} \qquad \left(s^r + \alpha_1 s^{r-1} + \ldots + \alpha_r\right) z_1 = s^{r-1} I_q$$
$$\vdots$$
$$z_r = \frac{1}{s^{r-1}} z_1$$

Thus we have

$$z_1 = \frac{s^{r-1}}{d(s)} I_q$$
$$z_2 = \frac{s^{r-2}}{d(s)} I_q$$
$$\vdots$$
$$z_r = \frac{I_q}{d(s)}$$

where $d(s) = s^r + \alpha_1 s^{r-1} + \ldots + \alpha_r$

and the transfer function is

$$G(s) = C(sI - A)^{-1}B = zB = [z_1 \ \ldots \ z_r] \begin{bmatrix} N_1 \\ \vdots \\ N_r \end{bmatrix}$$

$$= \frac{1}{d(s)} \left[N_1 s^{r-1} + N_2 s^{r-2} + \ldots + N_{r-1} s + N_r \right]$$

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4.14
$$\hat{G}(s) = \begin{bmatrix} \dfrac{-(12s+6)}{3s+34} & \dfrac{22s+23}{3s+34} \end{bmatrix}$$

$$= \begin{bmatrix} -4 & \dfrac{22}{3} \end{bmatrix} + \begin{bmatrix} \dfrac{130}{3s+34} & \dfrac{-679/3}{3s+34} \end{bmatrix}$$

$$= \begin{bmatrix} -4 & \dfrac{22}{3} \end{bmatrix} + \dfrac{1}{s+\frac{34}{3}} \begin{bmatrix} \dfrac{130}{3} & \dfrac{-679}{9} \end{bmatrix}$$

So
$$\dot{x} = \frac{-34}{3} x + \begin{bmatrix} \dfrac{130}{3} & -\dfrac{679}{9} \end{bmatrix} u$$
$$y = x + \begin{bmatrix} -4 & \frac{22}{3} \end{bmatrix} u$$
(observable form)

OR

$$\dot{x} = \begin{bmatrix} -34/3 & 0 \\ 0 & -34/3 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} \frac{130}{3} & -\frac{679}{9} \end{bmatrix} x + \begin{bmatrix} -4 & \frac{22}{3} \end{bmatrix} u$$
(Controllable form)

6.1
$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 3 \end{bmatrix} \qquad \rho(C) = 3 \Rightarrow \text{controllable}$$

$$O = \begin{bmatrix} 1 & 2 & 1 \\ -1 & -2 & -1 \\ 1 & 2 & 1 \end{bmatrix} \qquad \rho(O) = 1 \Rightarrow \text{not observable}$$

6.18 state equation:
$$\dot{x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x$$

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \qquad \rho(C) = 3 \Rightarrow \text{controllable}$$

$$O = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \qquad \rho(O) = 2 \Rightarrow \text{not observable}$$

The RC loop is in series with the current source, therefore the response due to x will not affect the rest of the network. Thus the network is not observable.

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Additional problem

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3\omega^2 & 0 & 0 & 2\omega \\ 0 & 0 & 0 & 1 \\ 0 & -2\omega & 0 & 1 \end{bmatrix} \qquad B = [b_1 \mid b_2] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$$

a) $\mathcal{C} = [\, B \ \ AB \ \ A^2B \ \ A^3B \,]$ $\quad \rho(\mathcal{C}) = 4$ system is controllable from u.

$\Theta = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix}$ $\quad \rho(\Theta) = 4$ system is observable

b) (i) if radial thruster fails $\longrightarrow u_1 = 0$

Let $b_2 = [0 \ 0 \ 0 \ 1]^T$

$\mathcal{C}_2 = [\, b_2 \mid Ab_2 \mid A^2b_2 \mid A_3 b_2 \,]$

$\rho(\mathcal{C}_2) = 4 \longrightarrow$ system is controllable from u_2

(ii) if tangential thruster fails $\longrightarrow u_2 = 0$

Let $b_1 = [0 \ 1 \ 0 \ 0]^T$

$\mathcal{C}_1 = [\, b_1 \mid Ab_1 \mid A^2b_1 \mid A^3b_1 \,]$ $\longrightarrow$ system is not controllable from u_1.

$\rho(\mathcal{C}_1) < 4$

c) (i) $y_1 = x_1$

$C_1 = [1 \ 0 \ 0 \ 0]$

$\Theta_1 = \begin{bmatrix} C_1 \\ C_1 A \\ C_1 A^2 \\ C_1 A^3 \end{bmatrix}$

$\rho(\Theta_1) < 4$

system is not observable from y_1

(ii) $y_2 = x_3$

$C_2 = [0 \ 0 \ 1 \ 0]$

$\Theta = \begin{bmatrix} C_2 \\ C_2 A \\ C_2 A^2 \\ C_2 A^3 \end{bmatrix}$ $\quad \rho(\Theta_2) = 4$

system is observable from y_2