HOMEWORK #7 Solution

4.8

$$\dot{x}_1 = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} x_1 + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u_1 \qquad \dot{x}_2 = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix} x_2 + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u_2$$

$$y_1 = [1 \ -1 \ 0]\, x_1 \qquad y_2 = [1 \ -1 \ 0]\, x_2$$

a) System 1 and 2 are <u>not</u> <u>equivalent</u> because eigenvalues are not the same.

$\lambda_1 = 2, 2, 1$

$\lambda_2 = 2, 2, -1$

b) They are <u>zero-state equivalent</u> because

$$G_1 = G_2 = \frac{1}{(s-2)^2}$$

4.15 $\dot{x} = Ax + bu$

$y = Cx$

(i) $d = 0 \longrightarrow$ strictly proper transfer function

(ii) $\hat{g}(s) = C(sI - A)^{-1}b$ can be expanded into infinite series as

$$\hat{g}(s) = cbs^{-1} + cAbs^{-2} + \dots + (cA^k b)s^{-k-1} + \dots$$

$$+ (cA^{n-m-3}b)s^{-(n-m-2)} + (cA^{n-m-2}b)s^{-(n-m-1)}$$

$$+ (cA^{n-m-1}b)s^{-(n-m)} + (cA^{n-m}b)s^{-(n-m+1)} + \dots$$

$$= \frac{1}{s^n}\left[cbs^{n-1} + cAbs^{n-2} + \dots + cA^{n-m-2}bs^{m+1} + cA^{n-m-1}s^m + \dots\right]$$

if $CA^ib = 0$ for $i = 0, 1, \dots n-m-2$

and $CA^{n-m-1}b \neq 0$

$$\hat{g}(s) = \frac{1}{s^n}\left[\underbrace{CA^{n-m-1}b}_{\neq 0}\, s^{m} + \dots\right]$$

guaranted that the numerator has degree m