SOLUTIONS TO HOMEWORK #11

8.1 $\dot{x} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$

$u = r - [k_1 \ k_2] x$

$\dot{x} = \begin{bmatrix} 2-k_1 & 1-k_2 \\ -1-2k_1 & 1-2k_2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} r$

$\det \begin{bmatrix} s-2+k_1 & -1+k_2 \\ 1+2k_1 & s-1+2k_2 \end{bmatrix} = s^2 + (k_1+2k_2-3)s + k_1 - 5k_2 + 3$

$\Delta_f(s) = (s+1)(s+2) = s^2 + 3s + 2$

$\therefore \ k_1 + 2k_2 - 3 = 3 \Rightarrow k_1 + 2k_2 = 6$

$k_1 - 5k_2 + 3 = 2 \Rightarrow k_1 - 5k_2 = -1$

Solving these yields $k_2 = 1$, $k_1 = 4$.

8.2 $\Delta(s) = \det \begin{bmatrix} s-2 & -1 \\ 1 & s-1 \end{bmatrix} = (s-1)(s-2) + 1$

$= s^2 - 3s + 3$

$\bar{k} = [3+3 \ \ 2-3] = [6 \ \ -1]$

$\bar{C}^{-1} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$C = [b \ \ Ab] = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}, \quad C^{-1} = \frac{1}{-7}\begin{bmatrix} 1 & -4 \\ -2 & 1 \end{bmatrix}$

$k = \bar{k}\bar{C}C^{-1} = [6 \ \ -1] \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{-1}{7} & \frac{4}{7} \\ \frac{2}{7} & \frac{-1}{7} \end{bmatrix}$

$= [6 \ \ 17] \begin{bmatrix} \frac{-1}{7} & \frac{4}{7} \\ \frac{2}{7} & \frac{-1}{7} \end{bmatrix} = [\frac{28}{7} \ \ \frac{7}{7}] = [4 \ \ 1].$

8.4 $\dot{x} = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u$

We use (8.13) to compute feedback gain k. We compute

$\Delta(s) = (s-1)^3 = s^3 - 3s^2 + 3s - 1$

$\Delta_f(s) = (s+2)(s+1+j)(s+1-j)$

$= s^3 + 4s^2 + 6s + 4$

$\bar{k} = [4-(-3) \ \ 6-3 \ \ 4-(-1)] = [7 \ \ 3 \ \ 5]$

$\bar{C}^{-1} = \begin{bmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

$C = \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}, \quad C^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ -2 & -3 & 2 \\ 1 & 2 & -1 \end{bmatrix}$

$k = \bar{k}\,\bar{C}C^{-1} = [15 \ \ 47 \ \ -8].$