EE 585 HOMEWORK #10 SOLUTION

① (Chen 7.12)

$\dot{x}_1 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} x_1 + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_1$

$y_1 = [2 \quad 2] x_1$

eigenvalues : $\{2, 1\}$

$G_1(s) = C_1(sI - A_1)^{-1}B_1 + D_1$

$= \dfrac{2s+2}{s^2-s-2}$

$\dot{x}_2 = \begin{bmatrix} 2 & 0 \\ -1 & -1 \end{bmatrix} x_2 + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u_2$

$y_2 = [2 \quad 0] x_2$

eigenvalues $\{2, -1\}$

$G_2(s) = C_2(sI - A_2)^{-1}B_2 + D_2$

$= \dfrac{2s+2}{s^2-s-2}$

a) Are they minimal realizations?

No. $G_1(s) = \dfrac{2\cancel{(s+1)}}{(s-2)\cancel{(s+1)}} = \dfrac{2}{s-2}$

The system can be reduced to $\dot{\hat{x}}_1 = 2\hat{x}_1 + u_1$

$\hat{y}_1 = 2\hat{x}_1$

$G_2(s) = \dfrac{2\cancel{(s+1)}}{(s-2)\cancel{(s+1)}} = \dfrac{2}{s-2}$

The system can be reduced to

$\dot{\hat{x}}_2 = 2\hat{x}_2 + u_2$

$\hat{y}_2 = 2\hat{x}_2$

b) Are they algebraically equivalent?

No. Eigenvalues are not the same.

② $\dot{x} = Ax + bu$

$y = Cx + du$

realizes $H(s) = \dfrac{N(s)}{D(s)} = \dfrac{s+2}{s^2+2s+1}$ proper rational

N(s) and D(s) are coprime.

$D(s) = s^2+2s+1$ (the degree is 2)

$\det(sI-A) = s^2+2s+1$ also has ~~degree~~ order of 2.

$\Rightarrow$ It is minimal.

From Thm 7.2 (A,b) must be controllable

(A,c) must be observable.

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(3) $H(s) = \dfrac{s+1}{s^2+2}$

a) <u>Uncontrollable realization:</u>

$$H(s) = \frac{s+1}{s^2+2}\,\frac{(s-1)}{(s-1)} = \frac{s^2-1}{s^3-s^2+2s-2}$$

Use observable canonical form. (Observable but not controllable)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & -2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad C = [-1\ 0\ 1]$$

b) <u>Unobservable realization</u>

Use controllable canonical form.

$$A = \begin{bmatrix} 0 & 0 & 2 \\ 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad C = [0\ 0\ 1]$$

d) <u>Minimal realization</u>

$$A = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = [1\ 1]$$

is a minimal realization in controller form.

c) <u>Neither controllable nor observable</u>

Add to this any state equation $\dot{z} = \hat{A} z$

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & \hat{A} \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u$$

$$y = [C\ 0] \begin{bmatrix} x \\ z \end{bmatrix}$$

This realization is neither controllable nor observable.

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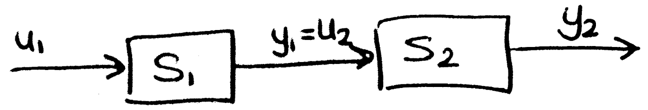
(4)



$$H_1(s) = \frac{-1}{s+2} + 1 \qquad H_2(s) = \frac{1}{s+1}$$

a) For S_1
$$\dot{x}_1 = -2x_1 + u_1 = A_1 x_1 + B_1 u_1$$
$$y_1 = -x_1 + u_1 = C_1 x_1 + D_1 u_1$$

For S_2
$$\dot{x}_2 = -x_2 + u_2 = A_2 x_2 + B_2 u_2$$
$$y_2 = x_2 = C_2 x_2 + D_2 u_2$$

$u_2 = y$

b)
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ B_2 C_1 & A_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 D_1 \end{bmatrix} u_1$$

$$y_2 = \begin{bmatrix} D_2 C_1 & C_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + D_2 D_1 u_1$$

or
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

c) It is uncontrollable but observable.

$$H(s) = \begin{bmatrix} 0 & 1 \end{bmatrix} \frac{1}{(s+1)(s+2)} \begin{bmatrix} s+1 & 0 \\ -1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{s+2}$$

or
$$H(s) = H_{21}(s) H_1(s) = \frac{s+1}{s+2} \cdot \frac{1}{s+1} = \frac{1}{s+2}$$