Supplementary Material for: Lifelong Path Planning with Kinematic Constraints for Multi-Agent Pickup and Delivery

Proof of Theorem 1

Proof (*By Induction*). Consider a path returned by SIPP-wRT. By definition, the agent starts in the first configuration $n_{start}.cfg$ at time *current_t* without any collisions. Assume that the agent arrives at cell $n.cfg.cell$ with configuration $n.cfg$ all the way from the first configuration $n_{start}.cfg$ without any collisions. Let $n'.cfg$ be the (successor) configuration next to configuration $n.cfg$ in the path. As the agent moves from cell $l = n.cfg.cell$ to the next cell $l' = n'.cfg.cell$, we use the following arguments:

(1) Line 25 (26) (when node n is being expanded) guarantees that any dynamic obstacle, which departs from cell l at a time earlier (later) than when the agent departs from cell l and which then moves in the same direction as the agent, must arrive at the next cell l' earlier (later) than when the agent arrives at cell l'.

(2) Line 24 (when node n is being expanded) guarantees that the agent must not collide with any dynamic obstacle, which arrives at cell l in a non-opposite direction at a time later than when the agent departs from cell l, until the dynamic obstacle has arrived at cell l.

(3) Line 35 (when node n is being expanded and node n' is being generated) guarantees that the agent must not collide with any dynamic obstacle, which departs from cell l' in a non-opposite direction at a time earlier than when the agent arrives at cell l', until the agent has arrived at cell l'. The line also guarantees that, if any dynamic obstacle departs from cell l' in the opposite direction at a time earlier than when the agent arrives at cell l', it also arrives at cell l at a time earlier than when the agent departs from cell l and thus also departs from cell l at a time earlier than when the agent arrives at cell l because its corresponding reserved interval does not intersect with the safe interval $n.int$ for cell l. Therefore, in both cases, the agent does not collide with such a dynamic obstacle (that departs from cell l' at a time earlier than when the agent arrives at cell l') as it moves from cell l to cell l' due to the induction assumption.

(4) For a non-goal node n', Line 24 (when node n' is being expanded) guarantees that the agent must not collide with any dynamic obstacle, which arrives at cell l' in a non-opposite direction at a time later than when the agent departs from cell l', until the dynamic obstacle has arrived at cell l'.

From the induction assumption, it suffices to prove that the agent does not collide with any given dynamic obstacle as the agent departs from cell l until it arrives at cell l'. Assume, for a proof by contradiction, that the agent collides with the dynamic obstacle.

(i) If the dynamic obstacle arrives at cell l at a time earlier than when the agent departs from cell l, it also departs from cell l at a time earlier than when the agent departs from cell l because its corresponding reserved interval does not intersect with the safe interval $n.int$ for cell l. It can collide with the agent only if it then moves from cell l to cell l' at a fixed velocity v'_{trans} smaller than the fixed velocity v_{trans} at which the agent moves from cell l to cell l'. It must arrive at cell l' at a time earlier than when the agent arrives at cell l' due to (1) and also departs at a time earlier than when the agent arrives at cell l' because its corresponding reserved interval does not intersect with the safe interval $n'.int$ for cell l'. The collision thus remains when the agent arrives at cell l', which contradicts (3).

(ii.a) If the dynamic obstacle arrives at cell l in a non-opposite direction at a time later than when the agent departs from cell l, it does not collide with the agent until it arrives at cell l due to (2). As a consequence, the dynamic obstacle also departs from cell l at a time later than when the agent departs from cell l. It can collide with the agent only if it then moves from cell l to cell l' at a fixed velocity v'_{trans} larger than the fixed velocity v_{trans} at which the agent moves from cell l to cell l'. It must arrive at cell l' at a time later than when the agent arrives at cell l' due to (1) and also later than when the agent departs from cell l' because its corresponding reserved interval does not intersect with the safe interval $n'.int$ for cell l'. Node n' is thus a non-goal node due to Line 8 (after n' is popped from *OPEN*), and the collision thus remains when the agent departs from cell l', which contradicts (4).

(ii.b) If the dynamic obstacle arrives at cell l in the opposite direction at a time later than when the agent departs from cell l, Line 24 (when node n is being expanded) guarantees that it departs form cell l' at a time later than when the agent arrives at cell l'. The dynamic obstacle thus also arrives at cell l' at a time later than when the agent departs from cell l' because its corresponding reserved interval does not intersect with the safe interval

n'.*int* for cell l'. Node n' is thus a non-goal node due to Line 8 (after n' is popped from *OPEN*). If the dynamic obstacle arrives at cell l' from another cell l'' in the opposite direction to the one in which the agent departs from cell l' (toward cell l''), Line 24 (Case (c) of Function *Offset*) (when node n' is being expanded) guarantees that the dynamic obstacle arrives at cell l'' at a time later than when the agent departs from cell l', ensuring that the agent does not collide with the dynamic obstacle until the agent departs from cell l'. Otherwise, any collision thus remains when the agent departs from cell l', which contradicts (4).

(iii) If the dynamic obstacle does not fall into Case (i) but also arrives at cell l' at a time earlier than when the agent arrives at cell l', it must also depart from cell l' earlier than when the agent arrives at cell l' because its corresponding reserved interval does not intersect with the safe interval n'.*int* for cell l'. Any collision thus remains when the agent arrives at cell l', which contradicts (3).

(iv.a) If node n' is a non-goal node and the dynamic obstacle does not fall into Case (ii.b) but also arrives at cell l' at a time later than when the agent arrives at cell l', the dynamic obstacle must arrives at cell l' later than when the agent departs from cell l' because its corresponding reserved interval does not intersect with the safe interval n'.*int* for cell l'. We use the same argument as for Case (ii.b): If the dynamic obstacle arrives at cell l' from another cell l'' in the opposite direction to the one in which the agent departs from cell l' (toward cell l''), Line 24 (Case (c) of Function *Offset*) (when node n' is being expanded) guarantees that the dynamic obstacle arrives at cell l'' at a time later than when the agent departs from cell l', ensuring that the agent does not collide with the dynamic obstacle until the agent departs from cell l'. Otherwise, any collision thus remains when the agent departs from cell l', which contradicts (4).

(iv.b) If node n' is the goal node n_{goal}, no dynamic obstacle arrives at cell l' at a time later than when the agent arrives at cell l' due to Line 8 (after n' is popped from *OPEN*). No collision is possible until the agent has arrived at cell l'.

Therefore, the agent arrives at cell $l' = n'.cfg.cell$ with configuration $n'.cfg$ without any collisions. □