

A Signal Perturbation Free Transmit Scheme for MIMO Channel Estimation

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Abstract—In this paper, a novel signal perturbation free transmit scheme is proposed for MIMO channel estimation. A perturbation analysis of the WR-based method is first conducted, showing that the method is subject to a signal perturbation error and therefore, its performance is very poor under the moderate to high signal-to-noise ratios (SNRs). A new transmit structure is then proposed to cancel the signal perturbation error at the receiver in order to improve the performance of the WR-based method in the high SNR case. Computer simulations show that the WR-based method with the proposed signal perturbation free transmit scheme significantly outperforms the original WR-based method as well as the training-based LS method in terms of the MSE of the channel estimate.

I. INTRODUCTION

The MIMO (Multiple input multiple output) technique has been considered as one of the key technologies for the development of the next-generation wireless communication systems. With multiple transmit and multiple receive antennas, MIMO systems can provide either a diversity gain to combat signal fading or a capacity gain, called spatial multiplexing, to make an efficient use of channel resources [1]. It means that, with MIMO techniques, higher data rate and better performance can be achieved without increasing the total transmission power or bandwidth. On the other hand, the performance of MIMO systems depends largely upon the availability of the knowledge of the channel. Thus, an accurate estimation of the wireless channel is of crucial importance to MIMO systems [2].

Based on known pilots, the MIMO channel can be estimated by employing different kinds of training-based algorithms such as the least square (LS), the maximum likelihood (ML), the maximum a posteriori (MAP) and the minimum mean square error (MMSE) algorithms [2]. In contrast to training-based methods, blind channel estimation algorithms like those proposed in [3]–[5], can achieve a better spectral efficiency by use of second-order statistics, correlative coding or other properties. With the idea of combining the training-based and the blind algorithms, semi-blind channel estimation techniques can potentially enhance the quality of MIMO channel estimation [6]–[8]. With a small number of training symbols, problems such as ambiguities and mis-convergence of the blind methods can be solved. On the other hand, the use of the available data in semi-blind techniques can improve the accuracy of channel estimation.

More recently, a whitening-rotation (WR)-based semi-blind algorithm has been proposed for frequency-flat MIMO channel estimation [9], [10]. This algorithm consists of two phases: (1) estimation of a whitening matrix utilizing information data; and (2) estimation of a unitary rotation matrix using pilots. The Cramer-Rao bound (CRB) of this algorithm shows that it can achieve a much better channel estimation performance than the conventional LS method, when the number of receive antennas is greater than or equal to the number of transmit antennas. However, this method is found to be efficient only in the case of low SNRs.

In this paper, we first employ the perturbation theory [11]–[13] to the analysis of the WR-based semi-blind MIMO channel estimation algorithm, showing that in the noise-free case the blind part of the WR-based method is subject to a signal perturbation error. This explains why the performance of the WR-based method is poor when the SNR is moderately large. To make that the WR-based method becomes also efficient for the moderate to high SNR case, we propose a novel signal perturbation free scheme. By utilizing the singular value decomposition (SVD) of the transmit signal perturbation matrix, a very efficient transmit scheme is designed for the elimination of the signal perturbation error in the receiver, leading to a signal perturbation free WR-based semi-blind algorithm. It is shown that the new algorithm provides a much better performance than the original WR-based method as well as training-based methods for all SNR cases.

Throughout the paper, we adopt the following notations:

T Transpose,

H Complex conjugate transpose,

$\lfloor \cdot \rfloor$ the floor of a number.

II. ANALYSIS OF WR-BASED CHANNEL ESTIMATION ALGORITHM

Consider a spatial-multiplexed MIMO system with N_T transmit and N_R ($\geq N_T$) receive antennas. Suppose that the frequency-flat fading MIMO channel is characterized by an $N_R \times N_T$ matrix \mathbf{H} whose (i_R, i_T) -th element h_{i_T, i_R} represents the channel response from the i_T -th transmit antenna to the i_R -th receive antenna. Given the transmitted signal vector $\mathbf{x}(n) \triangleq [x_1(n), \dots, x_{N_T}(n)]^T$ whose elements are independent identically distributed (i.i.d.) random variables

with zero mean and unit variance $\delta_x^2 = 1$, the received signal vector $\mathbf{y}(n) \triangleq [y_1(n), \dots, y_{N_R}(n)]^T$ can be written as

$$\mathbf{y}(n) = \mathbf{H}\mathbf{x}(n) + \mathbf{v}(n) \quad (1)$$

where the noise vector $\mathbf{v}(n) \triangleq [v_1(n), \dots, v_{N_R}(n)]^T$ is spatio-temporally uncorrelated with variance δ_v^2 . Note that, in each block, the first K of the N slots are used for training purpose.

We now briefly review a whitening-rotation (WR) based semi-blind MIMO channel estimation algorithm [9], [10]. Its idea originates from a decomposition of the channel matrix,

$$\mathbf{H} = \mathbf{W}\mathbf{Q}^H \quad (2)$$

where \mathbf{W} is a whitening matrix and \mathbf{Q} is a unitary rotation matrix. Performing the singular value decomposition (SVD) of \mathbf{H} gives

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H. \quad (3)$$

Obviously, one possible choice of \mathbf{W} and \mathbf{Q} can be $\mathbf{U}\mathbf{\Sigma}$ and \mathbf{V} . Thus, the WR-based channel estimation method can be implemented with two steps:

- (i) Estimate the whitening matrix \mathbf{W} in a blind fashion using the autocorrelation matrix of the received signal and a subspace-based method;
- (ii) Estimate the unitary rotation matrix \mathbf{Q} by utilizing training pilots and a constrained maximum-likelihood (ML) method.

It is known that the solution of subspace based methods is always perturbed by various sources, such as the finite data length, the measurement noise etc [11], [12]. The perturbation theory has been employed for the analysis of subspace based methods [12], [13]. In the following, we would like to show that, by considering only the perturbation due to the finite data length in the computation of correlation matrices, the whitening matrix would be perturbed even in the absence of noise.

Using (1), the autocorrelation matrix of the received signal, $\hat{\mathbf{R}}_{\mathbf{Y}}$, with such a perturbation can be written as

$$\hat{\mathbf{R}}_{\mathbf{Y}} = \hat{\mathbf{E}}[\mathbf{y}(n)\mathbf{y}^H(n)] - \delta_v^2\mathbf{I} = \mathbf{H}[\mathbf{I} + \Delta\mathbf{R}_x]\mathbf{H}^H + \Delta\mathbf{R}_v \quad (4)$$

where $\Delta\mathbf{R}_x$ denotes the signal perturbation matrix,

$$\Delta\mathbf{R}_x \triangleq \frac{1}{N} \sum_{n=1}^N \mathbf{x}(n)\mathbf{x}^H(n) - \delta_x^2\mathbf{I}, \quad (5)$$

and $\Delta\mathbf{R}_v$ the perturbation matrix introduced by the noise. It should be mentioned that in the noise-free case, the perturbation term introduced by the noise would disappear. Then, (4) reduces to

$$\hat{\mathbf{R}}_{\mathbf{Y}} = \mathbf{H}[\mathbf{I} + \Delta\mathbf{R}_x]\mathbf{H}^H. \quad (6)$$

Obviously, from (6), one can find that even in the noise-free case $\hat{\mathbf{W}}$ is subject to a signal perturbation error, which is dictated by the signal perturbation matrix $\Delta\mathbf{R}_x$. This could explain why the performance of the WR-based method is very poor in the moderate to high SNR cases.

III. PROPOSED TRANSMIT SCHEME

In this section, we propose a novel transmit scheme to improve the estimation performance of the whitening matrix in the WR-based method. Our idea is to send some information of the autocorrelation matrix of the transmitted signal to the receiver. The received version of this information will be then exploited to cancel the signal perturbation error.

The new idea begins with the decomposition of the signal perturbation matrix, which is re-defined as the scaled version of (5) for notational convenience,

$$\Delta\mathbf{R}_A = \sum_{n=1}^N \mathbf{x}(n)\mathbf{x}^H(n) - N\delta_x^2\mathbf{I}. \quad (7)$$

Performing the SVD on $\Delta\mathbf{R}_A$ gives

$$\Delta\mathbf{R}_A = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_{N_T}] \begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_{N_T} \end{bmatrix} \begin{bmatrix} \mathbf{v}_1^H \\ \mathbf{v}_2^H \\ \vdots \\ \mathbf{v}_{N_T}^H \end{bmatrix} \quad (8)$$

where the singular values σ_i , ($i = 1, 2, \dots, N_T$) are nonnegative real numbers. Moreover, as $\Delta\mathbf{R}_A$ is Hermitian matrix, if any two of the singular values are different, one can verify that

$$\mathbf{u}_i = \pm\mathbf{v}_i, (i = 1, 2, \dots, N_T). \quad (9)$$

In the case where some of the singular values of $\Delta\mathbf{R}_A$ are identical, one can modify $\Delta\mathbf{R}_A$ slightly by adding a random matrix with a very small norm, say 10^{-6} , such that the singular values of the modified $\Delta\mathbf{R}_A$ are different. Note that the revised matrix is almost the same as the original one, and the deviation due to the addition of the random matrix can be neglected. Thus, the equality in (9) is almost guaranteed. It should also be mentioned that for zero singular values, since they have no contribution to $\Delta\mathbf{R}_A$, their effect can be ignored regardless of the associated \mathbf{u}_i and \mathbf{v}_i . Using (9), one can separate $\Delta\mathbf{R}_A$ into two parts as

$$\Delta\mathbf{R}_A = \Delta\mathbf{R}_{\text{pos}} - \Delta\mathbf{R}_{\text{neg}} \quad (10)$$

where

$$\Delta\mathbf{R}_{\text{pos}} = \sum_{i=1}^{L_{\text{pos}}} \sigma_{\text{pos},i} \mathbf{u}_{\text{pos},i} \mathbf{u}_{\text{pos},i}^H, \quad (11)$$

$$\Delta\mathbf{R}_{\text{neg}} = \sum_{i=1}^{L_{\text{neg}}} \sigma_{\text{neg},i} \mathbf{u}_{\text{neg},i} \mathbf{u}_{\text{neg},i}^H. \quad (12)$$

Here, L_{pos} is the number of eigenvalues with respect to $\mathbf{u}_i = \mathbf{v}_i$, and $\sigma_{\text{pos},i}$ and $\mathbf{u}_{\text{pos},i}$ represent the associated singular value and the left singular vector, respectively. Likewise, L_{neg} , $\sigma_{\text{neg},i}$ and $\mathbf{u}_{\text{neg},i}$ are the similar terms with respect to $\mathbf{u}_i = -\mathbf{v}_i$. In what follows, we will derive a further decomposed form of $\Delta\mathbf{R}_A$ based on (10) such that the information of $\Delta\mathbf{R}_{\text{pos}}$ and $\Delta\mathbf{R}_{\text{neg}}$ can easily be transmitted to the receiver, which will be employed to cancel the signal perturbation error.

By letting

$$\Delta\mathbf{R}_{\text{pos}} = \eta \mathbf{X}_{\text{pos}} \mathbf{X}_{\text{pos}}^H, \quad (13)$$

$$\Delta \mathbf{R}_{\text{neg}} = \eta \mathbf{X}_{\text{neg}} \mathbf{X}_{\text{neg}}^H, \quad (14)$$

(10) can be rewritten as

$$\Delta \mathbf{R}_A = \eta (\mathbf{X}_{\text{pos}} \mathbf{X}_{\text{pos}}^H - \mathbf{X}_{\text{neg}} \mathbf{X}_{\text{neg}}^H) \quad (15)$$

where η is a scaling factor, and \mathbf{X}_{pos} and \mathbf{X}_{neg} are two matrices containing the information of $\Delta \mathbf{R}_{\text{pos}}$ and that of $\Delta \mathbf{R}_{\text{neg}}$, respectively. We now first show that as long as \mathbf{X}_{pos} and \mathbf{X}_{neg} are transmitted to the receiver, the signal perturbation error can completely be cancelled. We will then show that \mathbf{X}_{pos} and \mathbf{X}_{neg} can easily be constructed using the singular values and the singular vectors of $\Delta \mathbf{R}_A$. As shown later, the size of \mathbf{X}_{pos} and \mathbf{X}_{neg} can be made comparable to the dimension of $\Delta \mathbf{R}_A$, namely, the number of the transmit antennas N_T , the spectral resources used for transmitting \mathbf{X}_{pos} and \mathbf{X}_{neg} is negligible as compared to that of the user data.

Letting \mathbf{Y}_{pos} and \mathbf{Y}_{neg} be the received signals corresponding to \mathbf{X}_{pos} and \mathbf{X}_{neg} , respectively, namely,

$$\mathbf{Y}_{\text{pos}} = \mathbf{H} \mathbf{X}_{\text{pos}} + \mathbf{V}_{\text{pos}}, \quad (16)$$

$$\mathbf{Y}_{\text{neg}} = \mathbf{H} \mathbf{X}_{\text{neg}} + \mathbf{V}_{\text{neg}} \quad (17)$$

where \mathbf{V}_{pos} and \mathbf{V}_{neg} are the corresponding noise matrices, the received version of the signal perturbation matrix can be defined as

$$\Delta \hat{\mathbf{R}}_Y = \frac{\eta}{N} [(\mathbf{Y}_{\text{pos}} \mathbf{Y}_{\text{pos}}^H - \mathbf{Y}_{\text{neg}} \mathbf{Y}_{\text{neg}}^H) - (N_{\text{pos}} - N_{\text{neg}}) \delta_v^2 \mathbf{I}] \quad (18)$$

where N_{pos} and N_{neg} denote the number of the columns of \mathbf{Y}_{pos} and that of \mathbf{Y}_{neg} , respectively. Using (16) and (17) into (18) and noting that $\Delta \mathbf{R}_A = N \Delta \mathbf{R}_x$, we obtain

$$\Delta \hat{\mathbf{R}}_Y = \mathbf{H} \Delta \mathbf{R}_x \mathbf{H}^H + \Delta \mathbf{R}_{vp} \quad (19)$$

where $\Delta \mathbf{R}_{vp}$ represents a perturbation term introduced by the noise. By utilizing (4) and (19), the received correlation matrix without the signal perturbation error can be obtained from

$$\hat{\mathbf{R}}_Y = \hat{\mathbf{R}}_Y - \Delta \hat{\mathbf{R}}_Y = \mathbf{H} \mathbf{H}^H + \Delta \mathbf{R}'_v \quad (20)$$

where

$$\Delta \mathbf{R}'_v = \Delta \mathbf{R}_v - \Delta \mathbf{R}_{vp}. \quad (21)$$

As a result, the signal perturbation error has been completely eliminated through the transmission of \mathbf{X}_{pos} and \mathbf{X}_{neg} . What remains to be solved in the proposed scheme is to determine the matrices \mathbf{X}_{pos} and \mathbf{X}_{neg} from the singular values σ_i and the singular vectors \mathbf{u}_i .

Note that the total power of N_T transmit antennas in each time slot can be written as $\delta_{\text{int}} \triangleq N_T \delta_x^2$. It is found from a large number of simulation experiments that the value of $\sigma_{\text{pos},i}$ is much larger than δ_{int} . To transmit $\sigma_{\text{pos},i}$ with a small number of slots, it is first divided by the scaling factor η and then is split into $N_{\text{pos},i}$ terms of δ_{int} and one fractional term as

$$\frac{\sigma_{\text{pos},i}}{\eta} = (N_{\text{pos},i} \delta_{\text{int}} + \delta_{\text{pos-frac},i}) \quad (22)$$

where

$$N_{\text{pos},i} = \lfloor \frac{\sigma_{\text{pos},i}}{\eta \delta_{\text{int}}} \rfloor, \quad (23)$$

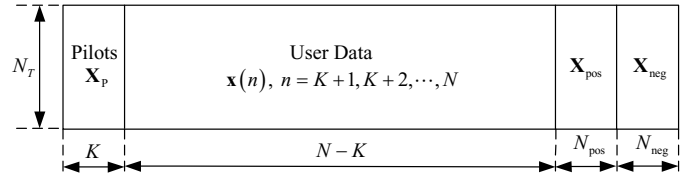


Fig. 1. New transmit structure

$$\delta_{\text{pos-frac},i} = \frac{\sigma_{\text{pos},i}}{\eta} - N_{\text{pos},i} \delta_{\text{int}}. \quad (24)$$

Letting

$$\mathbf{x}_{\text{pos-int},i} = \sqrt{\delta_{\text{int}}} \mathbf{u}_{\text{pos},i},$$

$$\mathbf{x}_{\text{pos-frac},i} = \sqrt{\delta_{\text{pos-frac},i}} \mathbf{u}_{\text{pos},i},$$

one can construct an $N_T \times (N_{\text{pos},i} + 1)$ matrix $\mathbf{X}_{\text{pos},i}$ for the i -th singular value by stacking $N_{\text{pos},i}$ consecutive vectors $\mathbf{x}_{\text{pos-int},i}$ and one vector $\mathbf{x}_{\text{pos-frac},i}$, which meets

$$\eta \mathbf{X}_{\text{pos},i} \mathbf{X}_{\text{pos},i}^H = \sigma_{\text{pos},i} \mathbf{u}_{\text{pos},i} \mathbf{u}_{\text{pos},i}^H.$$

Thus, the complete \mathbf{X}_{pos} can be formed as

$$\mathbf{X}_{\text{pos}} = [\mathbf{X}_{\text{pos},1}, \mathbf{X}_{\text{pos},2}, \dots, \mathbf{X}_{\text{pos},L_{\text{pos}}}] \quad (25)$$

Obviously, the number of columns of \mathbf{X}_{pos} is given by $N_{\text{pos}} = L_{\text{pos}} + \sum_{i=1}^{L_{\text{pos}}} N_{\text{pos},i}$.

In a similar manner, \mathbf{X}_{neg} can be constructed as follows

$$\mathbf{X}_{\text{neg}} = [\mathbf{X}_{\text{neg},1}, \mathbf{X}_{\text{neg},2}, \dots, \mathbf{X}_{\text{neg},L_{\text{neg}}}] \quad (26)$$

where $\mathbf{X}_{\text{neg},i}$ consists of $N_{\text{neg},i} = \lfloor \frac{\sigma_{\text{neg},i}}{\eta \delta_{\text{int}}} \rfloor$ consecutive vectors $\mathbf{x}_{\text{neg-int},i}$ and one vector $\mathbf{x}_{\text{neg-frac},i}$ as given by

$$\mathbf{x}_{\text{neg-int},i} = \sqrt{\delta_{\text{int}}} \mathbf{u}_{\text{neg},i}, \quad (27)$$

$$\mathbf{x}_{\text{neg-frac},i} = \sqrt{\delta_{\text{neg-frac},i}} \mathbf{u}_{\text{neg},i}, \quad (28)$$

$$\delta_{\text{neg-frac},i} = \frac{\sigma_{\text{neg},i}}{\eta} - N_{\text{neg},i} \delta_{\text{int}}. \quad (29)$$

Note that the number of columns of \mathbf{X}_{neg} is

$$N_{\text{neg}} = L_{\text{neg}} + \sum_{i=1}^{L_{\text{neg}}} N_{\text{neg},i}. \quad (30)$$

From the above discussion, a new transmit structure, which consists of conventional pilots, user's data and the additional data \mathbf{X}_{pos} and \mathbf{X}_{neg} , can be obtained as shown in Fig.1. It is now clear that the total column size of \mathbf{X}_{pos} and \mathbf{X}_{neg} is inversely proportional to the scaling factor η . It can be shown that when η is sufficiently large, as low as N_T slots can be used for the transmission of \mathbf{X}_{pos} and \mathbf{X}_{neg} . In general, the choice of η should depend on the number of transmit antennas as well as the length of user data. Our extensive simulations show that $\eta = 16$ is a proper choice to achieve a very good channel estimate for a 4×8 MIMO system, in which case, the transmission of \mathbf{X}_{pos} and \mathbf{X}_{neg} requires only 9 slots when the user data length is about 1000.

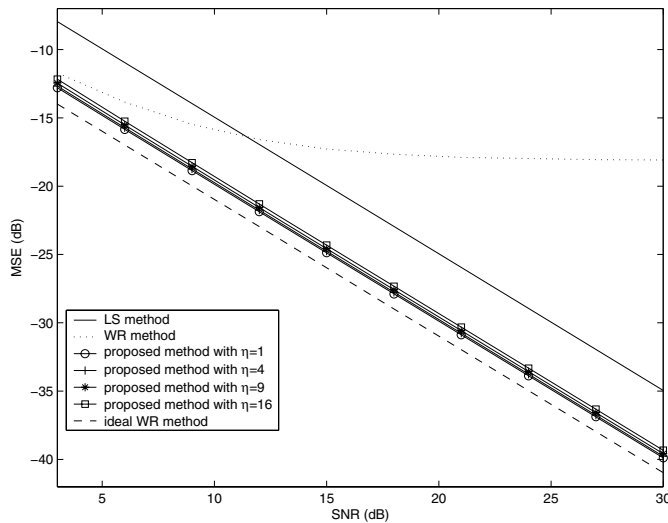


Fig. 2. MSE versus SNR

As a conclusion, the scheme developed above gives a signal perturbation free estimate of the whitening matrix in the noise-free case as seen from (20), leading to an ideal WR-based method. It should be pointed out that in the presence of noise, although the WR-based method with the proposed transmit scheme is subject to the noise perturbation, the method still outperforms the WR-based method, since the perturbation introduced by the noise is, in general, significantly smaller than the signal perturbation. It should also be mentioned that the proposed signal perturbation free scheme is also very useful for other correlation-based methods for MIMO channel estimation.

IV. SIMULATION RESULTS

We consider a MIMO system with 4 transmit and 8 receive antennas, in which the QPSK modulation is used and a Rayleigh channel, whose elements are i.i.d. complex Gaussian variables with zero mean and unit variance, is assumed. Here, the orthogonal pilots are generated by using the scheme proposed in [14]. It is noted that, a joint optimization approach was proposed in [9] to improve the estimation of the whitening matrix. Although this approach can improve the estimation accuracy of the whitening matrix slightly, the complexity of this method is extremely high since it involves many iterations in the computation of the “fminunc” function in MATLAB, making its implementation very difficult for real-world applications. Thus, this approach is not considered in our experiments. Instead, an ideal WR-based method, which obtains the whitening matrix directly from the true channel matrix, is simulated in our experiments for comparison.

Experiment 1: MSE versus SNR

In the first experiment, the channel estimation performance in terms of the plot of the MSE versus the SNR is investigated. The simulation is undertaken based on 20000 Monte Carlo

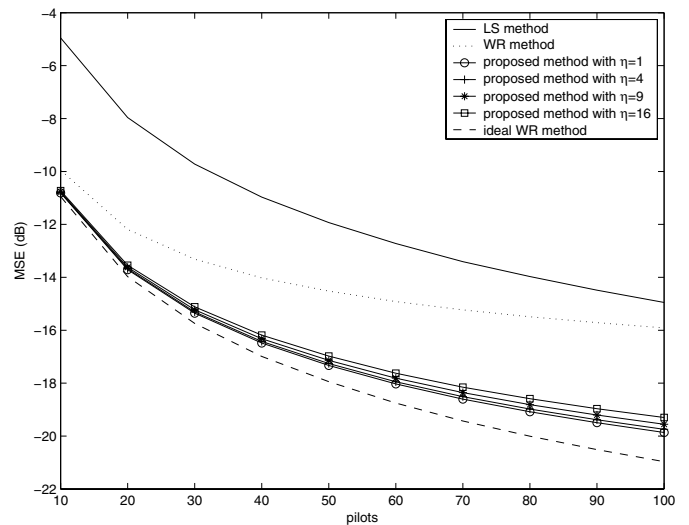


Fig. 3. MSE versus pilot length

runs of the transmission of one data frame with 1000 slots out of which 100 are used as pilots. Fig. 2 shows the MSE plots of the LS method, the ideal WR-based method, the WR-based method and the proposed method with $\eta = 1, 4, 9, 16$, respectively. It is seen that the MSE of the proposed method is closest to that of the ideal WR-based method in comparison to other methods irrespective of the choice of η . Interestingly, the different values of η only make a little difference on the MSE result. However, the number of slots for the transmission of \mathbf{X}_{pos} and \mathbf{X}_{neg} depends largely on the value of η . We have found that $\eta = 16$ is a very good choice for the proposed method, since it requires only 9 slots for \mathbf{X}_{pos} and \mathbf{X}_{neg} while significantly outperforming the WR-based and the LS methods at all SNR levels.

Experiment 2: MSE versus pilot length

Here, we investigate the channel estimation performance versus the pilot length. Fig. 3 shows the MSE plots from 20000 Monte Carlo runs of the transmission of one data frame of 1000 slots for an SNR of 10 dB, indicating a high estimation consistency of the proposed method with different values of η . Clearly, the performance improvement of the proposed method gets more prominent compared to the WR-based method with the increase of pilot length. For example, when the pilot length is increased to 100 from 30, the performance gain of the proposed method over the WR-based method is boosted to 3.4 dB from 2dB.

V. CONCLUSIONS

A novel signal perturbation free transmit scheme has been proposed for MIMO channel estimation. The perturbation analysis of the WR-based method has revealed that the signal perturbation error makes its performance very poor in the moderate to high SNR case. To improve the performance of the WR-based method in the high SNR case, a new transmit structure, which contains known data bearing the information

of the input signal perturbation matrix, has been proposed for the cancellation of the signal perturbation error at the receiver. Simulation results have confirmed that, by using a small number of additional slots for bearing the information of the autocorrelation matrix of the transmitted signal, a significant improvement in terms of the MSE of the channel estimate can be achieved over the WR-based method for all SNRs.

REFERENCES

- [1] A. J. Paulraj, D. A. Gore, R. U. Nabar, and H. Bolcskei, "An overview of MIMO communications—a key to gigabit wireless," *Proceedings of the IEEE*, vol. 92, no. 2, pp. 198–218, 2004.
- [2] M. Biguesh and A. B. Gershman, "MIMO channel estimation: optimal training and tradeoffs between estimation techniques," in *Proc. of IEEE International Conference on Communications*, vol. 5, 2004, pp. 2658–2662.
- [3] V. Barroso and J. Xavier, "Blind identification of MIMO channels: a closed form solution based on second order statistics," in *Thirty-Third Asilomar Conference on Signals, Systems, and Computers*, vol. 1, 1999, pp. 70–74.
- [4] J. K. Tugnait, "Blind estimation and equalization of MIMO channels via multidelay whitening," *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 8, pp. 1507–1519, 2001.
- [5] Z. Ding and L. Qiu, "Blind MIMO channel identification from second order statistics using rank deficient channel convolution matrix," *IEEE Trans. on Signal Processing*, vol. 51, no. 2, pp. 535–544, 2003.
- [6] V. Buchoux, O. Cappe, E. Moulines, and A. Gorokhov, "On the performance of semi-blind subspace-based channel estimation," *IEEE Trans. on Signal Processing*, vol. 48, no. 6, pp. 1750–1759, 2000.
- [7] A. Medles, D. T. M. Slock, and E. De Carvalho, "Linear prediction based semi-blind estimation of MIMO FIR channels," in *Proc. of IEEE Third Workshop on Signal Processing Advances in Wireless Communications*, 2001, pp. 58–61.
- [8] Y. H. Zeng, S. D. Ma, and T. S. Ng, "Semi-blind estimation of channels and symbols for asynchronous MIMO systems," *IEE Proceedings-Communications*, vol. 152, no. 6, pp. 883–889, 2005.
- [9] A. K. Jagannatham and B. D. Rao, "Whitening-rotation-based semi-blind MIMO channel estimation," *IEEE Trans. on Signal Processing*, vol. 54, no. 3, pp. 861–869, 2006.
- [10] —, "Cramer-Rao lower bound for constrained complex parameters," *IEEE Signal Processing Letters*, vol. 11, no. 11, pp. 875–878, 2004.
- [11] B. Friedlander and A. J. Weiss, "On the second-order statistics of the eigenvectors of sample covariance matrices," *IEEE Trans. on Signal Processing*, vol. 46, no. 11, pp. 3136–3139, 1998.
- [12] Z. Xu, "Perturbation analysis for subspace decomposition with applications in subspace-based algorithms," *IEEE Trans. on Signal Processing*, vol. 50, no. 11, pp. 2820–2830, 2002.
- [13] F. Wan, W.-P. Zhu, and M. Swamy, "A semi-blind channel estimation approach for MIMO-OFDM systems," *accepted for publication as a Regular Paper in the IEEE Trans. on Signal Processing*. http://users.encs.concordia.ca/~f_wan/paper_accepted/TSP_Feng1.pdf.
- [14] I. Barhumi, G. Leus, and M. Moonen, "Optimal training design for MIMO OFDM systems in mobile wireless channels," *IEEE Trans. on Signal Processing*, vol. 51, no. 6, pp. 1615–1624, 2003.