

A Signal-Perturbation-Free Transmit Scheme for MIMO-OFDM Channel Estimation

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Abstract—In this paper, a novel signal-perturbation-free (SPF) approach is presented for frequency-selective MIMO-OFDM channel estimation. First, an efficient transmit scheme, which bears partial information of the correlation matrix of the transmitted signal called SPF data, is proposed for the cancellation of signal perturbation error at the receiver. A detailed transmit structure is designed to implement the SPF scheme, which is then employed along with linear prediction (LP) to derive a new semi-blind channel estimation algorithm. It is shown that the new transmit scheme can completely cancel the signal perturbation error in the noise-free case, while being able to sufficiently suppress the perturbation error in noisy conditions. It is also shown that the SPF data needs only to be transmitted over a small number of subcarriers and moreover, its overhead to the overall transmission is negligible as compared to regular pilot signals. Computer simulations show that the proposed SPF solution significantly outperforms the LP semi-blind method without using the proposed transmit scheme as well as the least-square (LS) method in terms of the mean square error (MSE) of the channel estimate.

Index Terms—MIMO-OFDM systems, channel estimation, linear prediction, signal perturbation.

I. INTRODUCTION

Driven by wide-band multimedia and integrated services, future wireless networks, called beyond 3rd generation (B3G) or 4G, are anticipated to provide reliable transmission of very high data rates ranging from 100Mb/s to 1Gb/s for high to low-mobility applications. Two key enabling technologies have been identified to meet the goals of B3G networks: multi-input multi-output (MIMO) and orthogonal frequency division multiplexing (OFDM) [1]. With multiple transmit and multiple receive antennas, MIMO systems have been known for achieving either a diversity gain to combat signal fading (e.g. space-time block codes) or to obtain a capacity gain called

spatial multiplexing (e.g. V-BLAST). It means that both high data rate and superior system performance can be achieved without increasing the total transmission power or bandwidth, by employing MIMO transmissions. Meanwhile, the common frequency-selective problem of wireless channels caused by the inter symbol interference (ISI) can be solved or alleviated by the OFDM technique without the need for complex equalization. As such, MIMO-OFDM has been considered as a strong candidate for future wireless communication systems. It is well known that the advantages promised by MIMO-OFDM systems rely on the precise knowledge of the channel state information (CSI). It has been proved [2] that when the channel is Rayleigh fading and perfectly known to the receiver, the capacity of MIMO-OFDM systems grows linearly with the lower number of the transmit and receive antennas. Therefore, channel estimation is of crucial importance in MIMO-OFDM systems.

Broadly speaking, MIMO-OFDM channel estimation approaches can be categorized into three classes, training-based, blind and semi-blind approaches. Training-based methods, such as the least-square (LS), the maximum likelihood (ML) and the minimum mean square error (MMSE) methods, employ known training signals to render an accurate channel estimation [3]. Blind MIMO-OFDM channel estimation algorithms, such as those proposed in [4]–[8], exploit the second-order stationary statistics, correlative coding and other properties, and normally have a better spectral efficiency. With a small number of training symbols, semi-blind channel estimation algorithms have been proposed to estimate the channel ambiguity matrix for space-time coded, non-redundant precoded and multiuser MIMO-OFDM systems [4], [9], [10].

It is worth pointing out that most of the existing blind and semi-blind methods, such as [4], [5], [9], [10] as mentioned above, for MIMO-OFDM channel estimation are based on the second-order statistics of a long vector, whose size is equal to or larger than the number of subcarriers. To estimate the correlation matrix reliably, these techniques need a large number of OFDM symbols, and is not suitable for fast time-varying channels. In addition, since the matrices involved in these algorithms are of huge size, their computational complexity is extremely high. Therefore, these methods impose a very high complexity on the system implementation. In contrast, linear prediction-based blind algorithms such as those proposed in [11]–[15], which are based on the second-order statistics of a short vector with a size only slightly larger than the channel length, have been well investigated for the estimation of frequency-selective MIMO channels. Moreover, the linear prediction (LP)-based semi-blind algorithm proposed in [16] has

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been shown to be much more efficient than the conventional LS methods for MIMO systems. This method was extended for MIMO-OFDM systems in [17]–[19], in which the MIMO channel is estimated with a very high accuracy by employing only a few OFDM symbols while the full or partial information of the channel correlation is not needed.

By using the perturbation theory [20]–[22], our previous study on the LP-based blind channel estimation in [18] has shown that some conventional LP-based blind algorithms such as those in [12], [15], [23] are subject to a signal perturbation error due to the finite data length effect in the calculation of the correlation matrix of the received signal. It means that these algorithms would suffer from a poor performance in the MIMO-OFDM channel estimation if the number of the OFDM symbols is not large enough. In contrast, the semi-blind algorithm proposed in [17], [18] imposes an ideal nulling constraint on the channel matrix in the absence of the noise and therefore, gives a better channel estimation performance. In [24], we have proposed for the MIMO systems a signal-perturbation-free (SPF) transmit scheme, based on which the signal perturbation error can be cancelled at the receiver, leading to a signal-perturbation-free semi-blind channel estimation solution for frequency-flat MIMO channels. Since the MIMO-OFDM channel is always frequency-selective, the estimation algorithm for frequency-flat channels in [24] cannot be applied or easily extended to frequency-selective MIMO-OFDM channels due to different system models as well as different transmission schemes. In the present paper, we will develop a new semi-blind MIMO-OFDM channel estimation technique that is devoid of signal perturbation errors. The new signal perturbation free algorithm has a significant impact on the implementation of MIMO-OFDM systems. First, as mentioned earlier, most of the existing blind estimation methods need a large number (hundreds or more) OFDM symbols to render an accurate channel estimation, whereas our semi-blind method uses a small number (tens) of OFDM symbols which reduces greatly the system implementation complexity. Second, our method offers a much better estimation performance than the conventional LS method if the same number of pilots are used or requires only one-quarter to one-third pilots to achieve a similar performance as the LS method, leading to a high spectral efficiency of the communication system.

The rest of the paper is organized as follows. Section II presents some preliminary material needed to develop the new channel estimation approach, including a formulation of the data model for MIMO-OFDM systems and a brief review of the LP-based blind channel estimation. Section III presents a novel idea of signal perturbation error cancellation for MIMO-OFDM systems. Based on a brief discussion on the signal perturbation error in the LP-based blind estimation, a signal perturbation error cancellation principle is established. In Section IV, the new signal-perturbation-free (SPF) transmit scheme is developed for MIMO-OFDM channel estimation. First, we develop a detailed transmit structure that can be used in the receiver to cancel the signal perturbation error in the estimated correlation matrix of the received signal. Based on the proposed transmit scheme, a signal-perturbation-free semi-blind channel estimation algorithm is proposed. Section V

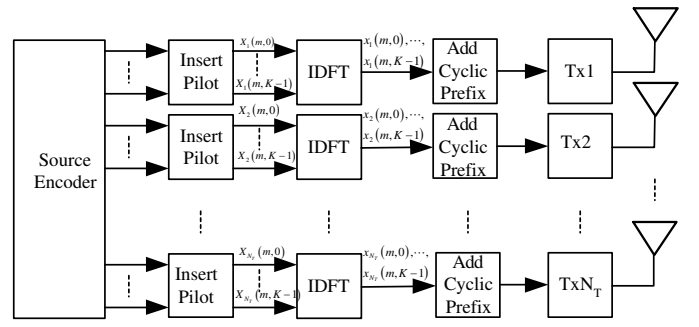


Fig. 1. Schematic representation of a MIMO-OFDM transmitter

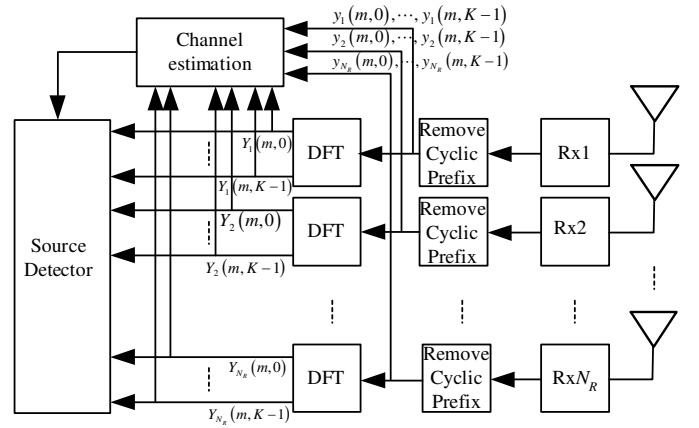


Fig. 2. Schematic representation of a MIMO-OFDM receiver

conducts a simulation study comprising a number of computer simulation based experiments to validate the proposed signal-perturbation-free semi-blind approach and show its significant advantage over some of the existing channel estimation techniques. Finally, Section VI concludes the paper by highlighting some of the contributions presented.

Throughout the paper, we adopt the following notations:

\dagger Pseudo-inverse, \otimes Kronecker product,

T Transpose, H Complex conjugate transpose,

\circledast Circular convolution, $\| \cdot \|_F$ Frobenius norm,

$\phi(k) = e^{-j2\pi \frac{k}{K}}$, and $\text{vec}(\cdot)$ a stacking of the columns of the involved matrix into a vector.

II. PRELIMINARIES

A. Data Model

Fig. 1 shows a block diagram of a typical transmitter in a MIMO-OFDM system with the V-BLAST structure [1], [25]–[27], in which there are N_T independent links, each connected to a transmit antenna and containing both pilot and information data. The m -th OFDM symbol in the i_T -th link can be written as a vector of the frequency-domain signals, namely, $[X_{i_T}(m,0), X_{i_T}(m,1), \dots, X_{i_T}(m,K-1)]^T$ where K denotes the number of subcarriers. The IDFT processing gives the time-domain OFDM signal, denoted as $[x_{i_T}(m,0), x_{i_T}(m,1), \dots, x_{i_T}(m,K-1)]^T$. After adding a cyclic prefix, each OFDM symbol is then sent out by the corresponding transmit antenna.

Fig. 2 shows the block diagram of the MIMO-OFDM receiver including N_R receive antennas as well as a channel estimation unit. After removing the cyclic prefix in each link, the signal received at the i_R -th antenna can be expressed as $[y_{i_R}(m, 0), y_{i_R}(m, 1), \dots, y_{i_R}(m, K-1)]^T$. Then, the received frequency domain signal after the DFT processing is given by $[Y_{i_R}(m, 0), Y_{i_R}(m, 1), \dots, Y_{i_R}(m, K-1)]^T$.

Most of the existing channel estimation methods work on a discrete-time MIMO-FIR channel, whose element can be characterized as an L -tap FIR filter. Assuming that the channel is constant during a number of consecutive OFDM symbols, the channel matrix for the l -th tap can be written as

$$\mathbf{H}(l) \triangleq \begin{bmatrix} h_{1,1}(l) & h_{1,2}(l) & \dots & h_{1,N_T}(l) \\ h_{2,1}(l) & h_{2,2}(l) & \dots & h_{2,N_T}(l) \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R,1}(l) & h_{N_R,2}(l) & \dots & h_{N_R,N_T}(l) \end{bmatrix} \quad (1)$$

where $h_{i_R, i_T}(l)$, ($0 \leq l \leq L-1$) represents the composite channel response between the i_R -th receive antenna and the i_T -th transmit antenna for the l -th tap. If the length of the cyclic prefix is not less than the channel length L , the time-domain signal can be written as

$$y_{i_R}(m, n) = \sum_{i_T=1}^{N_T} h_{i_R, i_T}(n) \otimes x_{i_T}(m, n) + v_{i_R}(m, n), \quad m \in \{0, \dots, g-1\} \quad (2)$$

where g is the OFDM block size, i.e., the number of OFDM symbols within which the channel is considered unchanged, and $v_{i_R}(m, n) \in \mathcal{C}$ is a spatio-temporally uncorrelated noise with zero-mean and variance σ_v^2 .

B. Brief Review of Linear Prediction based Blind MIMO Channel Estimation

We now briefly review the MIMO linear prediction and the related channel estimation methods [11], [18]. In the following, the index m is omitted for the sake of notational convenience. Define

$$\mathbf{y}(n) \triangleq [y_1(n), \dots, y_{N_R}(n)]^T, \quad (3)$$

$$\mathbf{y}_P(n-1) \triangleq [\mathbf{y}^T(n-1), \dots, \mathbf{y}^T(n-P)]^T, \quad (4)$$

$$\tilde{\mathbf{R}}_{n-1} \triangleq \mathbb{E} \{ \mathbf{y}_P(n-1) \mathbf{y}_P^H(n-1) \}, \quad (5)$$

$$\ddot{\mathbf{R}}_n \triangleq \mathbb{E} \{ \mathbf{y}(n) \mathbf{y}_P^H(n-1) \}, \quad (6)$$

where P is the length of the prediction filter. The MIMO linear predictor can be written as

$$\mathbf{P}_P \triangleq [\mathbf{P}_P(1), \mathbf{P}_P(2), \dots, \mathbf{P}_P(P)] = \ddot{\mathbf{R}}_n \tilde{\mathbf{R}}_{n-1}^{-1} \quad (7)$$

where $\mathbf{P}_P(n)$, ($n = 1, \dots, P$), is an $N_R \times N_R$ matrix representing the n -th tap of the prediction filter. The covariance matrix of the prediction error is then given by

$$\delta_{\mathbf{y}, P}^2 = \mathbf{R}_y(0) - \mathbf{P}_P \ddot{\mathbf{R}}_n^H \quad (8)$$

where

$$\mathbf{R}_y(0) \triangleq \mathbb{E} [\mathbf{y}(n) \mathbf{y}^H(n)]. \quad (9)$$

Further, define

$$\mathbf{P}_P(z) = \mathbf{I}_{N_R} - \sum_{i=1}^P \mathbf{P}_P(i) z^{-i},$$

$$\mathbf{H}(z) = \sum_{i=0}^{L-1} \mathbf{H}(i) z^{-i}.$$

It can be shown [11], [28] that if the transmitted signals are uncorrelated and moreover, $PN_R \geq (L+P-1)N_T$, we have

$$\mathbf{P}_P(z) \mathbf{H}(z) = \mathbf{H}(0), \quad (10)$$

$$\delta_{\mathbf{y}, P}^2 = \mathbf{H}(0) \mathbf{H}^H(0). \quad (11)$$

Based on (10) and (11), some blind algorithms have been proposed for MIMO channel estimation [11], [12], [15], [23]. The basic idea is to first acquire an estimate of $\mathbf{H}(0)$ from that of $\delta_{\mathbf{y}, P}^2$ according to (11), and then use (10) to obtain an estimate of the channel matrix $\mathbf{H}(z)$.

III. SIGNAL PERTURBATION CANCELLATION SCHEME

In this section, the signal perturbation in the LP-based blind channel estimation is first analyzed, showing that its performance is affected by signal perturbation error. Then, a signal perturbation cancellation scheme is developed to eliminate the effect of the signal perturbation error in the noise-free case.

A. Signal Perturbation in Linear-Prediction based Blind Channel Estimation

As shown in our previous work [18], the LP-based blind channel estimation is subject to a signal perturbation error due to the finite data length in the computation of second-order statistics. In this subsection, we disclose a perturbation form of the correlation matrix of the received signal in terms of the perturbation error of the correlation matrix of the transmitted signal, justifying that the LP-based blind technique has a poor channel estimation performance even in the noise-free case.

From (4)-(6), one can get

$$\tilde{\mathbf{R}}_{n-1} = \begin{bmatrix} \mathbf{R}_y(0) & \mathbf{R}_y(1) & \dots & \mathbf{R}_y(P-1) \\ \mathbf{R}_y(-1) & \mathbf{R}_y(0) & \dots & \mathbf{R}_y(P-2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_y(1-P) & \mathbf{R}_y(2-P) & \dots & \mathbf{R}_y(0) \end{bmatrix}, \quad (12)$$

$$\ddot{\mathbf{R}}_n = [\mathbf{R}_y(1) \quad \mathbf{R}_y(2) \quad \dots \quad \mathbf{R}_y(P)] \quad (13)$$

where

$$\mathbf{R}_y(l) \triangleq \mathbb{E} \{ \mathbf{y}(n) \mathbf{y}^H(n-l) \} \quad (l = 0, 1, \dots, P). \quad (14)$$

Obviously, the estimation of $\mathbf{R}_y(l)$, ($l = 0, 1, \dots, P$) plays a key role in the linear prediction-based blind method. For the sake of simplicity, let us consider first one OFDM symbol with K subcarriers. Letting $\mathbf{H}_A \triangleq [\mathbf{H}(0), \mathbf{H}(1), \dots, \mathbf{H}(L-1)]$, $\mathbf{v}(n) \triangleq [v_1(n), \dots, v_{N_T}(n)]^T$, $\mathbf{x}(n) \triangleq [x_1(n), \dots, x_{N_T}(n)]^T$ and $\mathbf{x}_L(n) \triangleq [\mathbf{x}^T(n) \dots \mathbf{x}^T(n-L+1)]^T$, ($n = 0, 1, \dots, K-1$), where

$\mathbf{x}(n) = \mathbf{x}(K+n)$ for $n < 0$, the circular convolution (2) can be rewritten in the matrix form as

$$\mathbf{y}(n) = \mathbf{H}_A \mathbf{x}_L(n) + \mathbf{v}(n). \quad (15)$$

Using (15), the estimate of $\mathbf{R}_y(l)$ given in (14) can be expressed as

$$\begin{aligned} \hat{\mathbf{R}}_y(l) &= \frac{1}{K} \sum_{n=0}^{K-1} [\mathbf{y}(n) \mathbf{y}^H(n-l)] \\ &= \mathbf{H}_A \hat{\mathbf{R}}_{x,L}(l) \mathbf{H}_A^H + \mathbf{R}_v(l) \end{aligned} \quad (16)$$

where

$$\hat{\mathbf{R}}_{x,L}(l) \triangleq \frac{1}{K} \sum_{n=0}^{K-1} \mathbf{x}_L(n) \mathbf{x}_L^H(n-l) \quad (17)$$

and $\mathbf{R}_v(l)$ is the correlation term introduced by the noise. Clearly, $\hat{\mathbf{R}}_{x,L}(l)$ can be rewritten as

$$\begin{aligned} \hat{\mathbf{R}}_{x,L}(l) &= \\ &\begin{bmatrix} \hat{\mathbf{R}}_x(l) & \hat{\mathbf{R}}_x(l+1) & \cdots & \hat{\mathbf{R}}_x(l+L-1) \\ \hat{\mathbf{R}}_x(l-1) & \hat{\mathbf{R}}_x(l) & \cdots & \hat{\mathbf{R}}_x(l+L-2) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\mathbf{R}}_x(l-L+1) & \hat{\mathbf{R}}_x(l-L+2) & \cdots & \hat{\mathbf{R}}_x(l) \end{bmatrix} \end{aligned} \quad (18)$$

where

$$\hat{\mathbf{R}}_x(l) \triangleq \frac{1}{K} \sum_{n=0}^{K-1} \mathbf{x}(n) \mathbf{x}^H(n-l) \quad (19)$$

is the estimate of the correlation matrices of the transmitted signal $\mathbf{x}(n)$. It has been proved in [18] that, when the transmitted frequency-domain signal can be considered as an i.i.d. Gaussian process with zero mean and unit variance, the transmitted time-domain MIMO-OFDM signal is uncorrelated, i.e., $\mathbf{R}_x(l) \triangleq \mathbb{E}\{\mathbf{x}(n) \mathbf{x}^H(n-l)\} = \delta(l) \mathbf{I}_{N_T}$. Using this property, (19) can be rewritten as

$$\hat{\mathbf{R}}_x(l) = \mathbf{R}_x(l) + \Delta \mathbf{R}_x(l) \quad (20)$$

where $\Delta \mathbf{R}_x(l)$ is the perturbation term of $\mathbf{R}_x(l)$ as given by

$$\Delta \mathbf{R}_x(l) \triangleq \frac{1}{K} \sum_{n=0}^{K-1} \mathbf{x}(n) \mathbf{x}^H(n-l) - \delta(l) \mathbf{I}_{N_T}. \quad (21)$$

Note that the above expressions are derived using a single OFDM symbol. When multiple OFDM symbols are used, $\hat{\mathbf{R}}_x(l)$ as well as $\hat{\mathbf{R}}_{x,L}(l)$ can be easily calculated by averaging the results obtained from each OFDM symbol.

It is obvious from (16), (18) and (20) that the perturbation error of the received signal, $\Delta \mathbf{R}_y(l) = \hat{\mathbf{R}}_y(l) - \mathbf{R}_y(l)$, can be expressed in terms of the perturbation error $\Delta \mathbf{R}_x(l)$ of the transmitted signal. It is also clear that even in the absence of the noise, the estimated correlation matrix of the received signal suffers from perturbation error due to the existence of $\Delta \mathbf{R}_x(l)$. Therefore, the existence of the signal perturbation error would in general degrade the performance of channel estimation methods utilizing the signal subspace. It has been shown in our previous work [18] that, even in the noise-free case, the conventional linear prediction-based blind algorithms such as those proposed in [11], [15], [23] are subject to a signal

perturbation error, while the semi-blind algorithm using a blind constraint derived from the linear prediction is free of signal perturbation error since it gives an ideal nulling constraint on the channel matrix. It should also be pointed out that in the presence of the noise, although both the semi-blind and the blind methods are subject to the noise perturbation terms, the semi-blind method still outperforms the blind one, since the perturbation introduced by the noise is in general significantly smaller than the signal perturbation. In order to improve the performance of the conventional blind approach, in the next subsection, we will propose a transmit scheme to cancel the signal perturbation error. The key idea is to send information of the signal perturbation matrix $\Delta \mathbf{R}_x(l)$ to the receiver. The received version of this information will then be exploited to cancel the signal perturbation error in the correlation matrix of the received signal.

B. The Principle of Signal Perturbation Cancellation

Consider the computation of the signal perturbation matrix $\Delta \mathbf{R}_x(l)$ at the transmitter using the frequency-domain data. Define the transmitted frequency-domain signal vector at the k -th subcarrier as

$$\mathbf{X}(k) \triangleq [X_1(k), \dots, X_{N_T}(k)]^T. \quad (22)$$

In [29], we developed a frequency-domain algorithm for the estimation of time-domain correlation matrices of the received signal in the nulling-based semi-blind channel estimation so as to simplify the implementation of the receiver. Inspired by this idea, we would like to express the correlation matrices $\hat{\mathbf{R}}_x(l)$ of the transmitted time-domain signal in terms of its frequency-domain version. It is proved in Appendix A that the estimated correlation matrix of the time-domain signal $\mathbf{x}(n)$, $\hat{\mathbf{R}}_x(l)$, can be calculated as

$$\hat{\mathbf{R}}_x(l) = \frac{1}{K} \sum_{k=0}^{K-1} \mathbf{X}(k) \mathbf{X}^H(k) \phi^{-l}(k) \quad (23)$$

which, by using (20), leads to

$$\Delta \mathbf{R}_x(l) = \frac{1}{K} \sum_{k=0}^{K-1} \mathbf{X}(k) \mathbf{X}^H(k) \phi^{-l}(k) - \delta(l) \mathbf{I}_{N_T}. \quad (24)$$

We have now obtained the perturbation term $\Delta \mathbf{R}_x(l)$ in terms of the transmitted frequency-domain signal $\mathbf{X}(k)$, based on which a novel SPF transmit scheme is developed. Interestingly, $\hat{\mathbf{R}}_x(l)$ given by (23) can be regarded as the IDFT of $\mathbf{X}(k) \mathbf{X}^H(k)$. Accordingly, the estimate of the correlation matrix of the received signal $\mathbf{y}(n)$, $\hat{\mathbf{R}}_y(l)$, can be represented as an IDFT of $\mathbf{Y}(k) \mathbf{Y}^H(k)$, where $\mathbf{Y}(k)$ is the frequency-domain version of $\mathbf{y}(n)$. In order to develop a new transmit scheme to cancel the perturbation error $\Delta \mathbf{R}_y(l) = \hat{\mathbf{R}}_y(l) - \mathbf{R}_y(l)$ due to $\Delta \mathbf{R}_x(l)$, we would like to further express $\Delta \mathbf{R}_x(l)$ as an IDFT of a set of data $\mathbf{T}(k)$. From (14), (16) and (18), one can see that the range of l for $\Delta \mathbf{R}_x(l)$ is given by

$$-L_1 \leq l \leq L_2, \quad (L_1 = L-1, L_2 = P+L-1). \quad (25)$$

As such, the size of the IDFT should be at least $K_T = L_1 + L_2 + 1$. Considering that the total number of subcarriers

usually satisfies $K \gg L_1 + L_2 + 1$, it would be sufficient and convenient to choose $K_T = \frac{K}{M} \geq L_1 + L_2 + 1$ where M is the largest possible integer such that K_T is the smallest possible power of two. By using the periodicity property of IDFT, we have

$$\Delta \mathbf{R}_x(l) = \begin{cases} \sum_{k=0}^{K_T-1} \mathbf{T}(k) e^{j2\pi kl/K_T}, & (l = 0, 1, \dots, L_2) \\ \sum_{k=0}^{K_T-1} \mathbf{T}(k) e^{j2\pi k(K_T+l)/K_T}, & (l = -L_1, 1 - L_1, \dots, -1) \end{cases}. \quad (26)$$

Clearly, (26) gives a K_T -size IDFT of $\mathbf{T}(k)$ with a gain of K_T , which means $\mathbf{T}(k)$ can easily be obtained from a K_T -size DFT of $\Delta \mathbf{R}_x(l)$ with a gain of $1/K_T$, namely,

$$\mathbf{T}(k) = \frac{1}{K_T} \left[\sum_{l=0}^{L_2} \Delta \mathbf{R}_x(l) e^{-j2\pi kl/K_T} + \sum_{l=K_T-L_1}^{K_T-1} \Delta \mathbf{R}_x(l - K_T) e^{-j2\pi kl/K_T} \right] (k = 0, 1, \dots, K_T - 1). \quad (27)$$

In what follows, we show that as long as the $N_T \times N_T$ matrix $\mathbf{T}(k)$ can be factorized into

$$\mathbf{T}(k) = \mathbf{T}_L(k) \mathbf{T}_R^H(k) \quad (28)$$

and $\mathbf{T}_L(k)$ and $\mathbf{T}_R^H(k)$ are transmitted to the receiver, the signal perturbation error in $\hat{\mathbf{R}}_y(l)$ can be eliminated in the absence of the noise.

First of all, we reveal that only a small number of subcarriers are needed to transmit $\mathbf{T}(k)$. Noting that $e^{j2\pi k(l+K_T)/K_T} = e^{j2\pi kl/K_T} = e^{j2\pi kMl/K}$, (26) can be rewritten as

$$\Delta \mathbf{R}_x(l) = \sum_{k=0}^{K_T-1} \mathbf{T}(k) \phi^{-Ml}(k), (l = -L_1, -L_1 + 1, \dots, L_2). \quad (29)$$

Interestingly, (29) corresponds to an M -rate decimated version of the K -size DFT of $\mathbf{T}'(k)$, where $\mathbf{T}'(k)$ is an up-sampled version of $\mathbf{T}(k)$ by a factor of M , i.e., it can easily be obtained by inserting $M - 1$ zero matrices following each $\mathbf{T}(k)$. This observation gives us an idea that $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$ should be transmitted over the kM -th subcarriers only, ($k = 0, 1, \dots, K_T - 1$), which is particularly in favor of practical applications since not all the subcarriers are required for the transmission of $\mathbf{T}(k)$.

In order to exploit the received version of $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$ for the cancellation of signal perturbation error $\Delta \mathbf{R}_y(l)$, let us first express the estimated correlation matrix of the received signal in terms of the channel matrix and the estimate of the correlation matrix of the transmitted signal in the absence of the noise. Noting that the received frequency-domain signal at the k -th subcarrier $\mathbf{Y}(k)$ can be written as

$$\mathbf{Y}(k) = \mathbf{H}_F(k) \mathbf{X}(k) \quad (30)$$

where $\mathbf{H}_F(k)$, ($k = 0, 1, \dots, K - 1$) is the frequency-domain channel matrix defined by

$$\mathbf{H}_F(k) \triangleq \sum_{l=0}^{L-1} \mathbf{H}(l) \phi^l(k), (k = 0, 1, \dots, K - 1). \quad (31)$$

In a manner similar to the derivation of (23) as shown in Appendix A, one obtains

$$\hat{\mathbf{R}}_y(l) = \frac{1}{K} \sum_{k=0}^{K-1} \mathbf{Y}(k) \mathbf{Y}^H(k) \phi^{-l}(k), \quad (32)$$

which, by using (30) and (31), leads to

$$\hat{\mathbf{R}}_y(l) = \sum_{l_1=0}^{L-1} \sum_{l_2=0}^{L-1} \mathbf{H}(l_1) \frac{1}{K} \sum_{k=0}^{K-1} [\mathbf{X}(k) \mathbf{X}^H(k) \phi^{-(l-l_1+l_2)}(k)] \mathbf{H}^H(l_2). \quad (33)$$

Further by using (23) into (33), we have

$$\hat{\mathbf{R}}_y(l) = \sum_{l_1=0}^{L-1} \sum_{l_2=0}^{L-1} \mathbf{H}(l_1) \hat{\mathbf{R}}_x(l - l_1 + l_2) \mathbf{H}^H(l_2). \quad (34)$$

On the other hand, by using the received noise-free version of the user specific data $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$, denoted as

$$\mathbf{Y}_{TL}(k) = \mathbf{H}_F(kM) \mathbf{T}_L(k), \quad (35)$$

$$\mathbf{Y}_{TR}(k) = \mathbf{H}_F(kM) \mathbf{T}_R(k), \quad (36)$$

we can construct matrices $\mathbf{R}_{YT}(l)$, ($l = 0, 1, \dots, L - 1$) as

$$\mathbf{R}_{YT}(l) = \sum_{k=0}^{K_T-1} \mathbf{Y}_{TL}(k) \mathbf{Y}_{TR}^H(k) \phi^{-Ml}(k). \quad (37)$$

Substituting (35) and (36) into (37) and using the definition of $\mathbf{H}_F(k)$ in (31), we have

$$\mathbf{R}_{YT}(l) = \sum_{l_1=0}^{L-1} \sum_{l_2=0}^{L-1} \mathbf{H}(l_1) \sum_{k=0}^{K_T-1} [\mathbf{T}_L(k) \mathbf{T}_R^H(k) \phi^{-M(l-l_1+l_2)}(k)] \mathbf{H}^H(l_2), \quad (38)$$

which, by using (28) and (29), can be further rewritten as

$$\mathbf{R}_{YT}(l) = \sum_{l_1=0}^{L-1} \sum_{l_2=0}^{L-1} \mathbf{H}(l_1) \Delta \mathbf{R}_x(l - l_1 + l_2) \mathbf{H}^H(l_2). \quad (39)$$

From (34) and (39), and noting that $\Delta \mathbf{R}_x(l) = \hat{\mathbf{R}}_x(l) - \mathbf{R}_x(l)$ and $\Delta \mathbf{R}_y(l) = \hat{\mathbf{R}}_y(l) - \mathbf{R}_y(l)$, it is now clear that $\mathbf{R}_{YT}(l)$ gives exactly the signal perturbation error $\Delta \mathbf{R}_y(l)$. Therefore, the correlation matrix of the received signal without the signal perturbation error can be calculated by

$$\hat{\mathbf{R}}'_y(l) = \hat{\mathbf{R}}_y(l) - \mathbf{R}_{YT}(l) = \sum_{l_1=0}^{L-1} \sum_{l_2=0}^{L-1} \mathbf{H}(l_1) \mathbf{R}_x(l - l_1 + l_2) \mathbf{H}^H(l_2) = \mathbf{R}_y(l) \quad (41)$$

The above discussion shows that via the transmission of $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$, the signal perturbation error in the receiver has been completely eliminated in the noise free case. From (41), it is interesting to note that, under the assumption of

$\mathbf{R}_x(l) = \delta(l)\mathbf{I}_{N_T}$, the ideal correlation matrix of the received signal, $\mathbf{R}_y(l)$, can be expressed in terms of the channel matrices $\mathbf{H}(l)$, i.e.,

$$\mathbf{R}_y(l) = \sum_{i=l}^{L-1} \mathbf{H}(i) \mathbf{H}^H(i-l), \quad (l = 0, 1, \dots, L-1). \quad (42)$$

It should also be mentioned that, in the noisy case, $\hat{\mathbf{R}}'_y(l)$ calculated using (40) would be, in general, different from $\mathbf{R}_y(l)$ obtained from (41) or (42) due to the existence of both signal and noise perturbation errors in $\hat{\mathbf{R}}'_y(l)$. However, it is consistently found through a computer simulation study that $\|\hat{\mathbf{R}}'_y(l) - \mathbf{R}_y(l)\|_F^2 \ll \|\hat{\mathbf{R}}_y(l) - \mathbf{R}_y(l)\|_F^2$ as seen from Fig. 4 in the simulation section. This is because the noise perturbation is much smaller than the signal perturbation whereas the latter has been completely cancelled in $\hat{\mathbf{R}}'_y(l)$ by using (40). This means that the estimation accuracy of the correlation matrix of the received noisy signal has been significantly improved using $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$. In the following section, we will propose a detailed structure for $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$ to facilitate the implementation of the new SPF transmit scheme.

IV. IMPLEMENTATION OF THE SPF TRANSMIT SCHEME FOR CHANNEL ESTIMATION

A. The Signal-Perturbation-Free Data Structure

We now propose an idea to obtain $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$ by using the singular value decomposition (SVD) technique. Performing the SVD on $\mathbf{T}(k)$ gives

$$\mathbf{T}(k) = \mathbf{U}_T(k) \mathbf{\Sigma}_T(k) \mathbf{V}_T^H(k) \quad (43)$$

where

$$\mathbf{U}_T(k) = [\mathbf{u}_{T,1}(k), \mathbf{u}_{T,2}(k), \dots, \mathbf{u}_{T,N_T}(k)],$$

$$\mathbf{V}_T(k) = [\mathbf{v}_{T,1}(k), \mathbf{v}_{T,2}(k), \dots, \mathbf{v}_{T,N_T}(k)],$$

and $\mathbf{\Sigma}_T(k)$ is a diagonal matrix composed of the singular values $\sigma_{T,i}(k)$, ($i = 1, 2, \dots, N_T$) of $\mathbf{T}(k)$. We can construct the matrices $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$ using the singular values $\sigma_{T,i}(k)$ and the singular vectors $\mathbf{u}_{T,i}(k)$ and $\mathbf{v}_{T,i}(k)$.

Note that the total power of N_T transmit antennas at one subcarrier in each OFDM symbol can be written as $\delta_{\text{int}} \triangleq N_T$. Clearly, the power required to transmit $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$ depends on $\sigma_{T,i}(k)$. It is found from extensive computer simulations that the value of $\sigma_{T,i}(k)$ is much smaller than δ_{int} . In order to ensure a reliable transmission of the SPF data in noisy conditions, we should allow the use of one or multiple OFDM symbols at the kM -th subcarrier to carry an amplified version of $\sigma_{T,i}(k)$. To this end, we use a scaling factor η to split $\sigma_{T,i}(k)$ into $M_{T,i}(k) \geq 0$ times of δ_{int} plus one fractional term $\delta_{T,i}(k)$ as

$$\frac{\sigma_{T,i}(k)}{\eta} = M_{T,i}(k) \delta_{\text{int}} + \delta_{T,i}(k) \quad (44)$$

where

$$M_{T,i}(k) = \lfloor \frac{\sigma_{T,i}(k)}{\eta \delta_{\text{int}}} \rfloor, \quad (45)$$

$$\delta_{T,i}(k) = \frac{\sigma_{T,i}(k)}{\eta} - M_{T,i}(k) \delta_{\text{int}}. \quad (46)$$

By letting

$$\mathbf{T}_{L-\text{int},i}(k) = \sqrt{\delta_{\text{int}}} \mathbf{u}_{T,i}(k),$$

$$\mathbf{T}_{L-\text{frac},i}(k) = \sqrt{\delta_{T,i}(k)} \mathbf{u}_{T,i}(k),$$

one can construct an $N_T \times [M_{T,i}(k) + 1]$ matrix $\mathbf{T}_{L,i}(k)$ for the i -th singular value by stacking $M_{T,i}(k)$ consecutive vectors $\mathbf{T}_{L-\text{int},i}(k)$ and one vector $\mathbf{T}_{L-\text{frac},i}(k)$. In a similar manner, by using the right singular vector $\mathbf{v}_{T,i}(k)$, an $N_T \times [M_{T,i}(k) + 1]$ matrix $\mathbf{T}_{R,i}(k)$ that satisfies

$$\mathbf{T}_{L,i}(k) \mathbf{T}_{R,i}^H(k) = \frac{\sigma_{T,i}(k)}{\eta} \mathbf{u}_{T,i}(k) \mathbf{v}_{T,i}^H(k)$$

can be constructed. Thus, the complete $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$ can be formed as

$$\mathbf{T}_L(k) = [\mathbf{T}_{L,1}(k), \mathbf{T}_{L,2}(k), \dots, \mathbf{T}_{L,N_T}(k)], \quad (47)$$

$$\mathbf{T}_R(k) = [\mathbf{T}_{R,1}(k), \mathbf{T}_{R,2}(k), \dots, \mathbf{T}_{R,N_T}(k)]. \quad (48)$$

Obviously, each of $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$ has a total of $N_T + \sum_{i=1}^{N_T} M_{T,i}(k)$ columns.

From the above discussion, a new data structure, which consists of user's data, pilots and the signal-perturbation-free (SPF) data $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$, can be designed as shown in Fig.3. Here the pilots allocated in the first g_p OFDM symbols are transmitted only at the subcarriers $k = 0, M, \dots, (K_T - 1)M$ as normally specified in OFDM systems. As such, the SPF data $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$ are presumably transmitted at the same subcarriers as the pilot. Note that the pilots will be utilized in this paper to estimate the ambiguity matrix of the blind channel estimation, as will be explained in the next section. The zero symbol, namely, the symbol with zero amplitude, proceeding the SPF data is used to identify $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$. It is now clear that the total column size of $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$ is inversely proportional to the scaling factor η . One can easily find a value of η such that as low as $2N_T$ symbols are required for the transmission of $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$. In general, the choice of η should depend on the number of the transmit antennas as well as the length of the user data. Our simulations show that, for a 2×4 MIMO-OFDM system, with a properly chosen value of η , on the average, up to 8 OFDM symbols per SPF subcarrier are required for the transmission of SPF data to sufficiently suppress the signal perturbation error. This overhead is negligible as compared to the pilot budget required by OFDM systems. It should be mentioned that the construction of $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$ is rather simple since they are readily obtained by stacking a scaled version of the left and right singular vectors of $N_T \times N_T$ matrix $\mathbf{T}(k)$. Moreover, $\mathbf{T}(k)$ can easily be computed by a K_T -size DFT of $\Delta \mathbf{R}_x(l)$ where K_T is a small number, for example, $K_T = 8$ in our study when 512 subcarriers are employed. It should be noted that the SPF data $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$ suffer from the channel noise during their transmission just as the pilot and user data do. However, this noise effect has been significantly reduced by using multiple OFDM symbols to bear properly scaled singular vectors of $\mathbf{T}(k)$. As will be seen from Fig.4 in Section V, the proposed signal perturbation

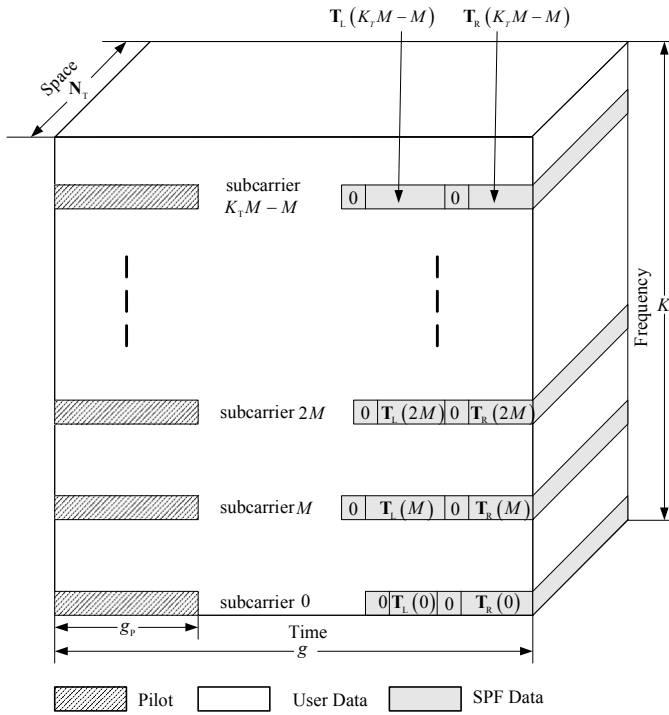


Fig. 3. Signal perturbation free transmit structure for MIMO-OFDM systems

cancellation scheme is indeed very efficient at moderate to high SNR levels.

It is of interest to consider the implementation complexity of the proposed SPF transmit scheme. Note that the main computational effort to implement the SPF scheme is on the transmitter. First, the calculation of the correlation matrix as shown in (23) requires $K(N_T + N_T^2)$ complex multiplications. Performing a K_T size DFT cost K_T^2 multiplications. The realization of an SVD of $\mathbf{T}(k)$, ($k = 0, 1, \dots, K_T - 1$), needs $K_T N_T^3$ multiplications. Considering that the values of K_T and N_T are usually small in practical applications, the computational complexity of the proposed SPF method is very small, which means that the implementation of the SPF transmit scheme introduces only a light burden on the transmitter while providing a significant gain in the SOS estimation at the receiver.

B. Application to MIMO-OFDM Channel Estimation

By employing the proposed signal-perturbation-free transmit scheme, the correlation matrix of the received signal after the perturbation cancellation, $\hat{\mathbf{R}}_y^l(l)$, can be obtained from (40). Then, by employing (12) and (13), the estimate of $\tilde{\mathbf{R}}_{n-1}$ and $\tilde{\mathbf{R}}_n$ is obtained, which is to be used to compute the MIMO linear predictor (7). In the absence of the noise, by following the linear prediction process [18], one can easily obtain the expression for $\mathbf{H}(i)$, ($i = 1, 2, \dots, L - 1$) in terms of $\mathbf{H}(0)$ as given below

$$\mathbf{H}(i) = \mathbf{P}_R(i) \mathbf{H}(0), \quad (i = 1, 2, \dots, L - 1), \quad (49)$$

where $\mathbf{P}_R(i)$ can be iteratively calculated by

$$\mathbf{P}_R(1) = \mathbf{P}_P(1),$$

$$\begin{aligned} \mathbf{P}_R(2) &= \mathbf{P}_P(2) + \mathbf{P}_P(1) \mathbf{P}_P(1), \\ \mathbf{P}_R(3) &= \mathbf{P}_P(3) + \mathbf{P}_P(2) \mathbf{P}_P(1) + \mathbf{P}_P(1) \mathbf{P}_P(2) \\ &\quad + \mathbf{P}_P(1) \mathbf{P}_P(1) \mathbf{P}_P(1), \\ &\quad \vdots \end{aligned}$$

Eq. (49) indicates that once the channel matrix at zero tap, $\mathbf{H}(0)$, is available, the matrices at other taps $\mathbf{H}(i)$ ($i = 1, 2, \dots, L - 1$), can be obtained from the above recursion. In the following, we use the whitening rotation (WR) algorithm [30], [31] to determine $\mathbf{H}(0)$.

The idea of WR method starts from the decomposition of $\mathbf{H}(0)$ as

$$\mathbf{H}(0) = \mathbf{W}_0 \mathbf{Q}_0^H \quad (50)$$

where \mathbf{W}_0 is a whitening matrix and \mathbf{Q}_0 is a unitary rotation matrix. From the singular value decomposition (SVD) of $\mathbf{H}(0)$,

$$\mathbf{H}(0) = \mathbf{U}_0 \mathbf{\Sigma}_0 \mathbf{V}_0^H, \quad (51)$$

one can see that a possible choice of \mathbf{W}_0 and \mathbf{Q}_0 is $\mathbf{U}_0 \mathbf{\Sigma}_0$ and \mathbf{V}_0 . Substituting (51) into (11) gives

$$\delta_{\hat{\mathbf{y}}, P}^2 = \mathbf{U}_0 \mathbf{\Sigma}_0 \mathbf{\Sigma}_0^H \mathbf{U}_0^H, \quad (52)$$

implying that $\mathbf{W}_0 = \mathbf{U}_0 \mathbf{\Sigma}_0$ can easily be estimated from the SVD of the prediction error $\delta_{\hat{\mathbf{y}}, P}^2$. We now consider the estimation of \mathbf{Q}_0 by using the training pilots placed in the kM -th subcarriers ($k = 0, 1, \dots, K_T - 1$) during the first g_p OFDM symbols as shown in Fig.3. By stacking a pilot matrix from all g_p symbols at the kM -th subcarrier as

$$\mathbf{X}_P(k) = [\mathbf{X}(kM, 1), \mathbf{X}(kM, 2), \dots, \mathbf{X}(kM, g_p)], \quad (53)$$

a received version of $\mathbf{X}_P(k)$ can be expressed as

$$\mathbf{Y}_P(k) = \mathbf{H}_F(kM) \mathbf{X}_P(k). \quad (54)$$

On the other hand, using (49) in (31) gives

$$\mathbf{H}_F(k) = \mathbf{P}_{R,F}(k) \mathbf{H}(0) \quad (55)$$

where

$$\mathbf{P}_{R,F}(k) \triangleq \sum_{l=0}^{L-1} \mathbf{P}_R(i) \phi^l(k). \quad (56)$$

By defining

$$\mathbf{Y}_Q(k) \triangleq \mathbf{W}_0^H \mathbf{P}_{R,F}(k) \mathbf{Y}_P(k) \mathbf{X}_P^H(k), \quad (57)$$

$$\mathbf{Y}_Q \triangleq \sum_{k=1}^{K_T} \mathbf{Y}_Q(k), \quad (58)$$

one can prove that the rotation matrix \mathbf{Q}_0 can be calculated as

$$\mathbf{Q}_0 = \mathbf{V}_Q \mathbf{U}_Q^H \quad (59)$$

where \mathbf{U}_Q and \mathbf{V}_Q are obtained from an SVD of \mathbf{Y}_Q as given by

$$\mathbf{Y}_Q = \mathbf{U}_Q \mathbf{\Sigma}_Q \mathbf{V}_Q^H. \quad (60)$$

Having estimated \mathbf{W}_0 and \mathbf{Q}_0 , the channel matrix $\mathbf{H}(0)$ is readily given by (50) and thereby, $\mathbf{H}(l)$, ($l = 1, \dots, L - 1$) can easily be computed from (49).

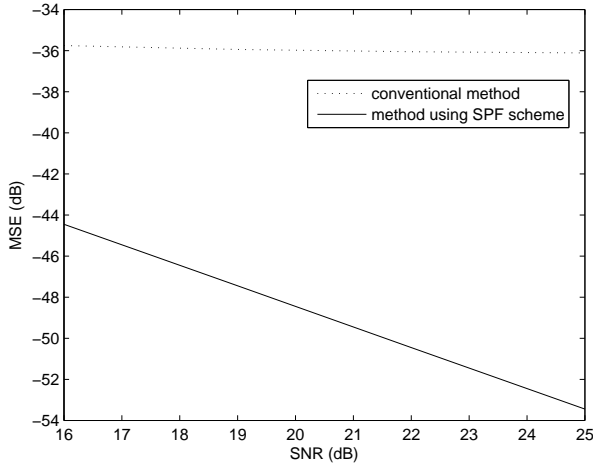


Fig. 4. MSE of the estimate of the correlation matrix of the received signal versus SNR

V. SIMULATION RESULTS

We consider a MIMO-OFDM system with 2 transmit and 4 receive antennas. The number of subcarriers is set to 512, the length of cyclic prefix is 10. In this paper, the QPSK modulation is used and an SUI-3 type MIMO channel is assumed. In particular, the channel is modelled as a 3-tap MIMO-FIR filter, in which each tap corresponds to a 2×4 random matrix whose elements are i.i.d. complex Gaussian variables with zero mean and an equal variance. Moreover, the channel has an exponentially decaying profile, giving 0 dB, -5 dB and -10 dB powers for the first, second and third taps, respectively. In *Experiment 1* to *Experiment 4*, M and K_T are set to be 64 and 8, respectively, suggesting that the pilots and SPF data are transmitted only in the subcarriers indexed by $64 \times k$, ($k = 0, 1, \dots, 7$), which, for the convenience, are referred to as the SPF subcarriers.

For the purpose of comparison, the channel vector \mathbf{h} is estimated by the proposed SPF LP-based semi-blind algorithm, the LP-based semi-blind algorithm without using the SPF transmit scheme and the LS method. For easy citation, we call these three methods as the SPF LP semi-blind, LP semi-blind and LS methods. The estimation performance is evaluated in terms of the MSE of the estimate of the channel matrix given by

$$\text{MSE} = \frac{1}{N_{\text{MC}}} \sum_{n=1}^{N_{\text{MC}}} \left\| \hat{\mathbf{h}}_n - \mathbf{h}_n \right\|^2 \quad (61)$$

where N_{MC} is the number of Monte Carlo iterations, and \mathbf{h}_n and $\hat{\mathbf{h}}_n$ are the true and the estimated channel vectors with respect to the n -th Monte Carlo iteration, respectively.

Experiment 1: MSE versus SNR

In the first experiment, the channel estimation performance in terms of the MSE versus the SNR is investigated. The simulation involves 2000 Monte Carlo runs of the transmission of 60 OFDM symbols with pilot length $g_p = 20$. Here, the scaling factor in the SPF scheme is set to be $\eta = 9.5 \times 10^{-4}$, which corresponds, on the average, to 5.8 OFDM symbols

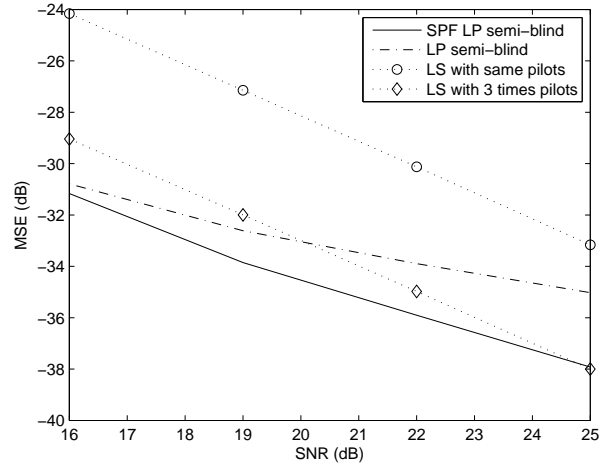


Fig. 5. MSE of the channel estimate versus SNR

TABLE I
THE AVERAGE NUMBER OF SPF SYMBOLS NEEDED FOR THE TRANSMISSION OF $\mathbf{T}_L(k)$ AND $\mathbf{T}_R(k)$ VS VALUE OF η

scaling factor	$\eta_1 = 9.5 \times 10^{-4}$	$\eta_2 = 1.9 \times 10^{-3}$
average number of SPF symbols	5.8	4.1

per SPF subcarrier used for the transmission of $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$. First of all, Fig. 4 shows the MSE of the estimate of the correlation matrices of the received signal from N_{MC} Monte Carlo iterations which is defined, in a manner similar to (61), by using the norm of the error correlation matrix. Clearly, the conventional correlation matrix estimation without using the proposed SPF cancellation scheme achieves very little gain in the MSE with increasing the SNR level. In contrast, by using the SPF cancellation scheme, the MSE of the estimated correlation matrix has been significantly improved, which is linearly proportional to the increase of the SNR. Fig. 5 shows the channel estimation results of the SPF LP semi-blind, LP semi-blind and the LS methods with 20 pilot symbols. Moreover, the result from the LS method using 60 pilot symbols, which is three times the pilot length of other methods, is also provided for comparison. It is seen that the SPF LP semi-blind algorithm consistently outperforms the LP semi-blind method and the LS method. Also, one can find that the performance gain of the SPF LP semi-blind algorithm over the LP semi-blind algorithm becomes larger with increasing the SNR level. In particular, the MSE is improved by 2.9 dB when the SNR is 25 dB.

Experiment 2: The effect of scaling factor η on the channel estimation performance

In this experiment, the channel estimation performance in terms of the MSE for different scaling factors η is investigated. Using the same condition as in *Experiment 1*, the simulation is undertaken based on 2000 Monte Carlo runs for the scaling factor η given by $\eta_1 = 9.5 \times 10^{-4}$ and $\eta_2 = 1.9 \times 10^{-3}$, respectively. Table I shows the average number of OFDM

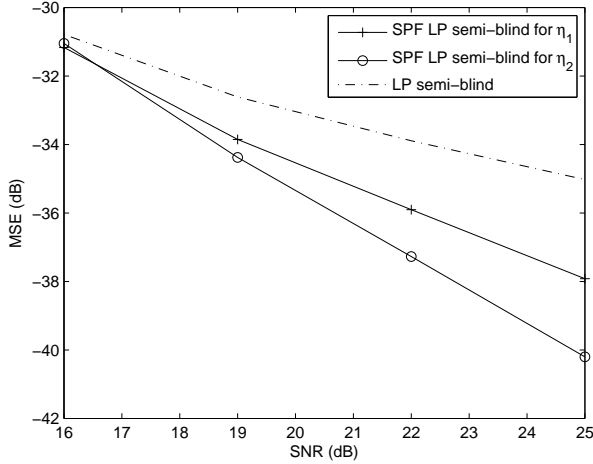


Fig. 6. The effect of the scaling factor η on the MSE of channel estimate versus SNR

TABLE II
THE SELECTED η VALUES FOR FIVE OFDM BLOCK SIZES

OFDM block size	The value of η
40	1.164×10^{-3}
60	9.5×10^{-4}
80	8.216×10^{-4}
100	7.313×10^{-4}
120	6.712×10^{-4}

symbols per SPF subcarrier used for $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$ for three values of η . Clearly, more OFDM symbols are needed for a smaller value of η . Fig. 6 shows the MSE plots of the LP semi-blind algorithm and the SPF LP semi-blind algorithm for the two η values. It is seen that the performance of the SPF LP semi-blind algorithm for both two cases is better than that of the LP semi-blind algorithm. Moreover, the performance of the SPF LP semi-blind algorithm corresponding to each value of η depends on the SNR range. For example, η_1 gives slightly better performance than η_2 when the SNR is less than 17 dB, while η_2 achieves much better performance than η_1 when the SNR is larger than 17 dB.

Experiment 3: The effect of OFDM block size on the channel estimation performance

Now, we examine the effect of the OFDM block size on channel estimation performance. In order to have a fair comparison, we have chosen a proper value of η for a different OFDM block size $g = 40, 60, 80, 100, 120$ as shown in Table II such that the same average number of OFDM symbols is used for the transmission of the SPF data. It has been verified through simulations that the average number of OFDM symbols for the SPF data is then 5.8 for all the five cases.

Fig. 7 shows the MSE of the correlation matrices as a function of the OFDM block size by the conventional and SPF estimation methods based on 5000 Monte Carlo runs for the pilot length $g_p = 20$ at an SNR of 18 dB. It is seen that the MSE of the estimated correlation matrix by using the SPF cancellation scheme consistently outperforms

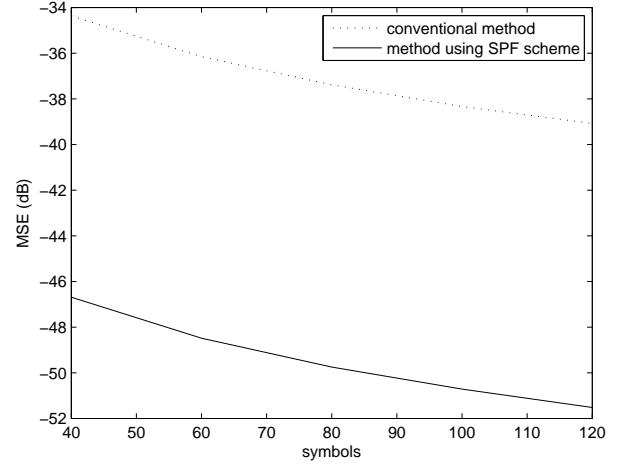


Fig. 7. MSE of the estimate of the correlation matrix of the received signal versus the OFDM block size for fixed number of pilots

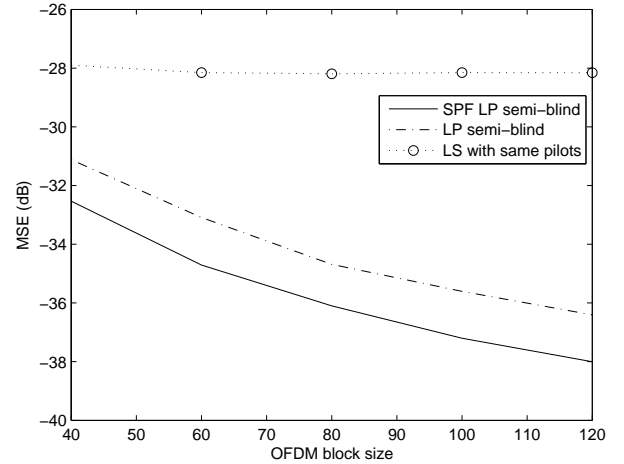


Fig. 8. MSE versus the OFDM block size for fixed number of pilots

that without using the proposed SPF cancellation scheme by about 12.4 dB. Fig. 8 shows the corresponding channel estimation results. For comparison, the performance of the LS method with the same pilot length is also included. It is seen that the performance of both the SPF LP semi-blind and the LP semi-blind algorithms gets much better with increasing OFDM block size. Also, one can find that the SPF LP semi-blind algorithm consistently achieves a gain of 1.5 dB over the LP semi-blind algorithm.

Experiment 4: MSE versus number of subcarriers per OFDM symbol

In this experiment, the channel estimation performance in terms of the MSE versus the number of subcarriers per OFDM symbol, K , is investigated. The simulation is undertaken based on 500 Monte Carlo runs of the transmission of 60 OFDM symbols with 8 SPF subcarriers and pilot length $g_p = 20$ at an SNR of 18dB. Here, we use four different values of K , namely, 128, 256, 512 and 1024, and set the

TABLE III
THE AVERAGE NUMBER OF SPF SYMBOLS NEEDED FOR THE TRANSMISSION OF $\mathbf{T}_L(k)$ AND $\mathbf{T}_R(k)$ VS NUMBER OF SUBCARRIERS PER OFDM SYMBOL

Number of subcarriers per OFDM symbol	128	256	512	1024
Average number of SPF symbols needed	6.9	6.2	6.4	5.7

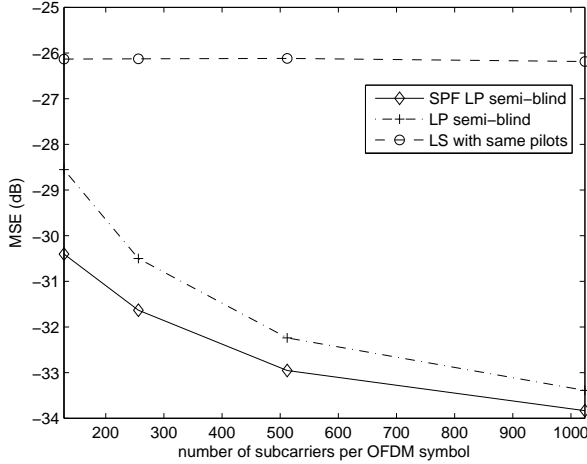


Fig. 9. MSE versus number of subcarriers per OFDM symbol

corresponding η values as $\eta = 1.5 \times 10^{-3}$, $\eta = 1.2 \times 10^{-3}$, $\eta = 8.2 \times 10^{-4}$ and $\eta = 6.8 \times 10^{-4}$, respectively. The pilots and SPF data are transmitted only in the subcarriers indexed by $M \times k$, ($k = 0, 1, \dots, 7$) for $M = 16, 32, 64$ and 128, for the four scenarios of K , respectively. Table III shows the average number of OFDM symbols per SPF subcarrier used for $\mathbf{T}_L(k)$ and $\mathbf{T}_R(k)$ for the four values of K . Fig. 9 shows the MSE values for the four different cases of the three methods. It is seen that the performance of the SPF LP semi-blind and the LP semi-blind algorithms becomes much better with increasing the number of subcarriers, which shows a significant advantage of the two methods as compared to the LS algorithm. Moreover, it is observed that, the gain of the SPF LP semi-blind algorithm over the LP semi-blind algorithm decreases slightly with increasing the number of subcarriers, indicating that the SPF transmit scheme is more useful for the case with less subcarriers (i.e. the case with respect to the SOS estimation with less samples).

VI. CONCLUSIONS

We had previously shown that the LP-based blind channel estimation method is subject to signal perturbation error and therefore gives a poor estimation performance in the moderate to high SNR levels. To improve its performance, in this paper, we have developed a new scheme that transmits user specific data bearing the information of the correlation matrix of the transmitted signal to cancel the signal perturbation error at the receiver. The new transmit scheme has then been used to develop a signal-perturbation-free semi-blind LP-based channel estimation algorithm. It has been shown

that the new semi-blind MIMO-OFDM channel estimation solution is devoid of any signal perturbation error in the noise-free case and is capable of efficiently suppressing the signal perturbation error in the noisy case. Simulation results have confirmed that, by using a small number of additional slots for the transmission of the user specific SPF data, the new semi-blind approach can achieve a significantly improved channel estimation performance as compared to the LP-based semi-blind method without using the signal-perturbation-free transmit scheme as well as the LS method.

APPENDIX A FREQUENCY-DOMAIN ESTIMATION OF THE CORRELATION MATRIX $\hat{\mathbf{R}}_x(l)$

From (19) and $\mathbf{x}(n) \triangleq [x_1(n), \dots, x_{N_T}(n)]^T$, we have

$$\hat{\mathbf{R}}_x(l) = \begin{bmatrix} \hat{R}_{1,1}(l) & \hat{R}_{1,2}(l) & \cdots & \hat{R}_{1,N_T}(l) \\ \hat{R}_{2,1}(l) & \hat{R}_{2,2}(l) & \cdots & \hat{R}_{2,N_T}(l) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{R}_{N_T,1}(l) & \hat{R}_{N_T,2}(l) & \cdots & \hat{R}_{N_T,N_T}(l) \end{bmatrix} \quad (\text{A-1})$$

where

$$\hat{R}_{i_{T1},i_{T2}}(l) = \frac{1}{K} \sum_{n=0}^{K-1} x_{i_{T1}}(n) x_{i_{T2}}^H(n-l). \quad (\text{A-2})$$

Note that $x_{i_{T2}}(n)$ for $n < l$ can be obtained using $x_{i_{T2}}(n-l) \triangleq x_{i_{T2}}(K+n-l)$ due to the circular convolution. Utilizing

$$x_{i_{T1}}(n) = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} X_{i_{T1}}(k) e^{j2\pi(kn/K)}, \quad (\text{A-3})$$

$$x_{i_{T2}}(n) = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} X_{i_{T2}}(k) e^{j2\pi(kn/K)}, \quad (\text{A-4})$$

$\hat{R}_{i_{T1},i_{T2}}$ as defined in (A-2) can be written as

$$\begin{aligned} & \hat{R}_{i_{T1},i_{T2}}(l) \\ &= \frac{1}{K} \sum_{n=0}^{K-1} x_{i_{T1}}(n) \left[\frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} X_{i_{T2}}^*(k) e^{-j2\pi k(n-l)/K} \right] \\ &= \frac{1}{K} \sum_{k=0}^{K-1} \left[\frac{1}{\sqrt{K}} \sum_{n=0}^{K-1} x_{i_{T1}}(n) e^{-j2\pi kn/K} \right] \\ & \quad X_{i_{T2}}^*(k) \phi^{-l}(k) \\ &= \frac{1}{K} \sum_{k=0}^{K-1} X_{i_{T1}}(k) X_{i_{T2}}^*(k) \phi^{-l}(k). \end{aligned} \quad (\text{A-5})$$

Substituting (A-5) into (A-1) yields

$$\begin{aligned}
 \hat{\mathbf{R}}_x(l) &= \frac{1}{K} \sum_{k=0}^{K-1} \begin{bmatrix} X_1(k) X_1^*(k) & X_1(k) X_2^*(k) & \cdots \\ X_2(k) X_1^*(k) & X_2(k) X_2^*(k) & \cdots \\ \vdots & \vdots & \ddots \\ X_{N_T}(k) X_1^*(k) & X_{N_T}(k) X_2^*(k) & \cdots \\ X_1(k) X_{N_T}^*(k) \\ X_2(k) X_{N_T}^*(k) \\ \vdots \\ X_{N_T}(k) X_{N_T}^*(k) \end{bmatrix} \phi^{-l}(k), \\
 &= \frac{1}{K} \sum_{k=0}^{K-1} \mathbf{X}(k) \mathbf{X}^H(k) \phi^{-l}(k). \quad (\text{A-6})
 \end{aligned}$$

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