Spacial Extrapolation-Based Blind DOA Estimation Approach for Closely Spaced Sources

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This paper presents a new blind direction of arrival (DOA) estimation approach for closely-spaced sources. The new method first estimates the autoregressive (AR) coefficients via an initial DOA estimation and then uses the AR coefficients for the linear extrapolation of the correlation matrix to implement a fine DOA estimation. Both initial and fine DOA estimations are performed using the estimation of signal parameters via rotational invariance techniques (ESPRIT) algorithm. Unlike a conventional AR coefficient estimation method which estimates the AR coefficients on the snapshot basis, our AR coefficients are estimated in the correlation domain once for a block of snapshots, thus significantly reducing the computational complexity of the antenna array. Moreover, the proposed spatial extrapolation-based DOA estimation approach is analyzed using perturbation theory. Both the theoretical analysis and computer simulations show that the proposed method outperforms the conventional techniques in terms of the mean square error (MSE) of the DOA estimation when the angle of separation of the signal sources is very small.

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I. INTRODUCTION

Blind direction of arrival (DOA) estimation is very important in applications involving antenna array processing. By exploiting the properties of the array response matrix, several subspace-based high-resolution DOA estimation algorithms, such as multiple signal classification (MUSIC) and estimation of signal parameter via rotational invariance techniques (ESPRIT), have been proposed to estimate DOA in a blind fashion [1, 2]. It should be noted that the DOA estimation problem is similar to the blind bearing estimation and time delay estimation problems [3, 4], whose statistical property has been well investigated [5, 6]. It has been shown in [7]–[9] that when the signal sources are very close, the performance of the subspace-based blind algorithms degrades drastically, and the variance of the estimation error increases much faster than the corresponding Cramer-Rao bound (CRB) [10, 11], implying that the angle resolution of the subspace-based methods is poor. Although the angle resolution can be increased by employing more antennas, it is a costly option and may not be feasible in wireless communication systems.

To enhance the angle resolution, one may resort to some signal preprocessing techniques. One of them is the interleaving (down-sampling) technique, which was first proposed to improve the performance of frequency estimation [7] and was then extended for array processing [12, 13]. However, this method is only effective when the signal-to-noise ratio (SNR) is low and the array size is small. In this paper, we investigate another technique—the linear extrapolation. This method has been employed in radar and acoustic signal processing to boost the resolution [14–19], and has already been extended to wireless communications [8, 20]. In this technique, a linear prediction model (autoregressive (AR) model) is used to fit the data and extrapolate the signal spectrum [17, 19]. Since linear interpolation is similar in principle to extrapolation and has been successfully applied in the field of antenna array [15, 21–23], it is expected that some linear interpolation results can be applied to or extended from linear extrapolation.

In this paper, we propose a blind DOA estimation method for closely-spaced sources by employing the spatial domain linear extrapolation. Some initial ideas about this method have been presented in [24]. The spatial extrapolation is usually implemented using AR coefficients, which could be estimated by means of one of the existing techniques such as linear prediction, eigenvalue decomposition (EVD), or the least mean square (LMS) algorithm [8, 15, 20]. In these methods, the AR coefficients are first estimated based on one snapshot. This process is repeated independently for different snapshots. The one-snapshot estimation is suitable for fast-changing sources where the signal directions are varying very rapidly. But it suffers from a heavy computational burden as the AR coefficient estimation and the corresponding extrapolation are carried out for each new snapshot acquired. On the other hand, the signal sources in wireless communications, most of the time, vary slowly or even may not vary at all, i.e., their DOAs are kept unchanged for a long period of time. As such, we propose a new AR coefficient estimation scheme that is implemented on a block-by-block basis in the correlation domain. The new scheme can not only improve the accuracy of the AR coefficient estimation, but also significantly reduce the computational load.

The second part of this paper deals with the error analysis of the proposed method via the perturbation theory [25–30]. First, we extend the analysis of the data-domain ESPRIT in [28] to the correlation-domain ESPRIT. Then, the relationship between the MSE of the DOA estimation and the number of antennas is investigated for a simple case. We also derive a closed-form expression for the mean square error (MSE) of the DOA estimation of the ESPRIT under the assumption of ideal spatial extrapolation.

The rest of the paper is organized as follows. Section II describes the problem formulation of blind DOA estimation for closely-spaced sources. Section III presents a new blind DOA estimation approach, which is based on the linear extrapolation of the correlation matrix using AR coefficients that are first estimated through an initial DOA estimation. Section IV conducts a perturbation analysis of the extrapolated ESPRIT algorithm, leading to a closed-form expression for the MSE of the DOA estimation of the ESPRIT under ideal spatial extrapolation. Section V comprises a number of experimentations validating the proposed method, showing the superiority of the proposed method over the conventional DOA estimation techniques without employing the extrapolated correlation matrix. Finally, Section VI highlights some of the distinct features of the proposed algorithm.

In this paper, we adopt the following notations:

1) $^{\mathrm{T}}$ Matrix transpose;
2) $^{\mathrm{H}}$ Matrix complex conjugate transpose;
3) † Matrix pseudoinverse (Moore-Penrose inverse);
4) $\|\ \|_{\mathrm{F}}^2$ the matrix Frobenius norm;
5) * complex conjugate;
6) $\mathbf{e}_m^{\mathrm{H}}$ $[0,\ldots,0,1,0,\ldots,0]$ with 1 in the mth position;
7) $\mathbf{M}\uparrow$ A matrix by deleting the first row of matrix $\mathbf{M}$, with the property

$$(\mathbf{M}_1\mathbf{M}_2)\uparrow = (\mathbf{M}_1\uparrow)\mathbf{M}_2;$$

8) $\mathbf{M}\downarrow$ A matrix by deleting the last row of matrix $\mathbf{M}$, with the property

$$(\mathbf{M}_1\mathbf{M}_2)\downarrow = (\mathbf{M}_1\downarrow)\mathbf{M}_2.$$

II. PROBLEM STATEMENT

Consider M signal sources and an N-antenna uniform linear array (ULA), whose elements are omnidirectional. By ignoring the mutual coupling between antenna elements, the narrowband beamforming model can be described as [31]

$$\mathbf{x}(k) = \mathbf{u}(k) + \mathbf{v}(k) \tag{1}$$

$$\mathbf{u}(k) = \mathbf{A}\mathbf{s}(k) \tag{2}$$

where $\mathbf{x}(k)$ is the $N\times 1$ signal vector received by the N antennas at the kth snapshot, $\mathbf{s}(k)$ the $M\times 1$ signal vector transmitted by the M sources, $\mathbf{u}(k)$ the $N\times 1$ desired noise-free signal vector, $\mathbf{v}(k)$ the $N\times 1$ noise vector, and $\mathbf{A}$ the array response matrix given by

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ \varphi_1 & \varphi_2 & \cdots & \varphi_M \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_1^{N-1} & \varphi_2^{N-1} & \cdots & \varphi_M^{N-1} \end{pmatrix} \tag{3}$$

with

$$\varphi_m = e^{j2\pi(d/\lambda)\sin(\theta_m)} \tag{4}$$

and λ, d, θ_m being the wavelength, the ULA inter-element spacing, and the DOA of the mth source, respectively. According to the narrowband model, it has been shown in [32] that if the transmitted signal $\mathbf{s}(k)$ is known at the receiver, the optimum Wiener beamformer can be obtained in the MSE sense. The Wiener beamformer is known to maximize the output signal-to-interference-plus-noise ratio (SINR). In practice, however, the transmitted signal is normally not available unless a training sequence is employed. In this situation, another optimum beamformer, called minimum variance distortionless response (MVDR) beamformer [33, 34], can be used provided the DOAs of the signal sources are known. The MVDR beamformer guarantees that the signal propagating along the desired direction is passed without distortion and that the output noise power is minimized. As the DOA is, in general, not known, one needs to carry out the DOA estimation prior to the use of the MVDR beamformer.

It is well known [33] that if $N \geq M$, a subspace-based blind DOA estimation technique that exploits the autocorrelation matrix of the received signal can be obtained

$$\begin{aligned}\mathbf{R}_x &= \mathrm{E}\{\mathbf{x}(k)\mathbf{x}^{\mathrm{H}}(k)\} - \mathbf{R}_v \\ &= \mathbf{A}\mathbf{R}_s\mathbf{A}^{\mathrm{H}}\end{aligned} \tag{5}$$

where

$$\mathbf{R}_s = \begin{bmatrix} \delta_{s1}^2 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \delta_{sM}^2 \end{bmatrix} \quad (6)$$

and

$$\mathbf{R}_v = \delta_v^2 \mathbf{I}_{N\times N}. \quad (7)$$

The above expressions have been obtained under the assumption that both the signals and the noises are uncorrelated with each other. In this paper, the number of sources M is assumed to be known, a common practice in the literature [35, 36]. In practice, the value of this number can be determined by different kinds of approaches. For example, in wireless cellular communications, a ranging process can be used by the base station to detect the number of active users. The number of users can also be estimated by comparing the singular values of the correlation matrix of the received signal with a predetermined threshold [37].

Based on (5), the DOAs can be estimated by using some subspace-based blind DOA estimation algorithms, e.g., MUSIC or ESPRIT. Note that, since the number of antennas is, in general, small due to the system complexity and economic reasons, the angle resolution of the subspace-based blind DOA estimation algorithms is not sufficient for wireless communications, especially when the angle of separation of DOA is very small. Although the angle resolution can be increased by employing more antennas, this is a costly option and may not be preferred in wireless communication systems. Instead, to enhance the angle resolution, one can resort to some signal preprocessing techniques [7, 8, 12–20], such as the interleaving and linear extrapolation. Since the interleaving method is efficient only when the SNR is low and the size of the array is small, we investigate the linear extrapolation technique to enhance the resolution of the DOA estimation.

Linear extrapolation is based on the estimation of the AR model coefficients. In general, the estimation can be implemented by linear prediction [15], EVD [20] or the LMS algorithms [8]. These methods update the AR coefficients on a snapshot-by-snapshot basis and thus are suitable for signal sources with fast-varying DOAs. In wireless communications, however, the DOAs of the signal sources may vary slowly or even may not vary at all. In such a scenario, the snapshot-based estimation is less efficient since the AR coefficients do not change rapidly, and a high estimation accuracy can be achieved if a block of snapshots are used. In this paper, we propose a subspace-based AR coefficient estimation scheme in the correlation domain, which can not only improve the accuracy of the AR coefficient estimation but also significantly reduce the computational load. By employing the subspace-based AR coefficient estimation scheme, we derive an efficient blind DOA estimation method for closely-spaced sources.

III. THE PROPOSED DOA ESTIMATION WITH SPATIAL EXTRAPOLATION

In this section, we propose a new blind DOA estimation algorithm for closely-spaced sources based on the extrapolated correlation matrix. An initial DOA estimation is carried out using a high-resolution subspace algorithm, resulting in AR coefficients, which are then used for the spatial extrapolation of the correlation matrix.

Assuming that the M sources are independent and $M < N$, the matrix $\mathbf{A}$ given by (3) has a full column rank M. Then, there exists a set of coefficients $\alpha_1,\ldots,\alpha_M$ such that the $(M+1)$th row of $\mathbf{A}$ can be written as [21]

$$(\varphi_1^M, \varphi_2^M, \ldots, \varphi_M^M) = (\alpha_M^*, \ldots, \alpha_1^*)\mathbf{\Phi}_{MM} \quad (8)$$

where

$$\mathbf{\Phi}_{MM} \triangleq \begin{pmatrix} 1 & 1 & \cdots & 1 \\ \varphi_1 & \varphi_2 & \cdots & \varphi_M \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_1^{M-1} & \varphi_2^{M-1} & \cdots & \varphi_M^{M-1} \end{pmatrix}.$$

Clearly, (8) can be rewritten as

$$\varphi_m^M = \alpha_1^* \varphi_m^{M-1} + \cdots + \alpha_M^*, \qquad (m = 1,\ldots,M). \quad (9)$$

By multiplying both sides of (9) by φ_i^{n-M}, $(n > M)$, we have

$$\varphi_m^n = \alpha_1^* \varphi_m^{n-1} + \cdots + \alpha_M^* \varphi_m^{n-M}, \qquad (m = 1,\ldots,M). \quad (10)$$

Obviously, (10) can be expressed in the matrix form

$$(\varphi_1^n, \varphi_2^n, \ldots, \varphi_M^n) = (\alpha_M^*, \ldots, \alpha_1^*) \begin{pmatrix} \varphi_1^{n-M} & \varphi_2^{n-M} & \cdots & \varphi_M^{n-M} \\ \varphi_1^{n-M+1} & \varphi_2^{n-M+1} & \cdots & \varphi_M^{n-M+1} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_1^{n-1} & \varphi_2^{n-1} & \cdots & \varphi_M^{n-1} \end{pmatrix}. \quad (11)$$

By postmultiplying both sides of (11) by $[s_1(k),\ldots,s_M(k)]^{\mathrm{T}}$ and using (2), one can obtain the desired noise-free signal at the nth $(n > M)$ receive antenna for the kth snapshot,

$$u_n(k) = \sum_{m=1}^{M} \alpha_m^* u_{n-m}(k). \quad (12)$$

It is clear that (12) corresponds to the formulation of spatial extrapolation for the noise-free case, since the signal received at any virtual antenna can be obtained from the last M received signals $u_{n-1},\ldots,u_{n-m}$

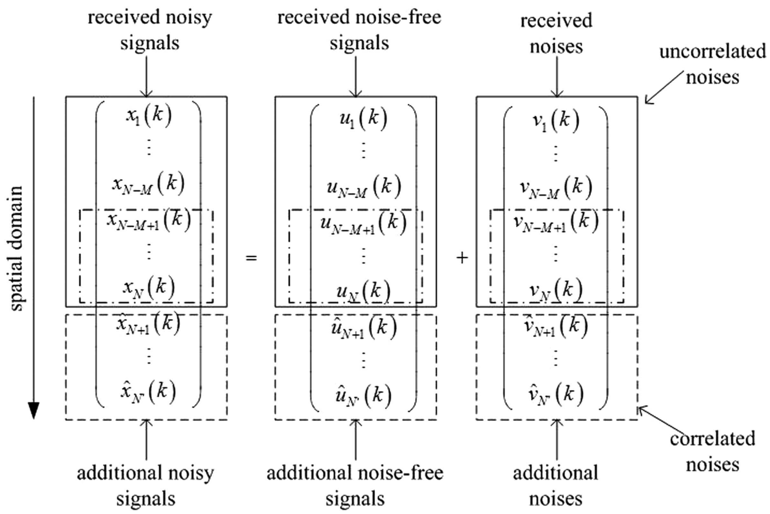

Fig. 1. Extrapolated data model.

once the coefficients $\alpha_1,\ldots,\alpha_M$ are known. Thus, $\alpha_1,\ldots,\alpha_M$ represent the optimal coefficients for spatial extrapolation in the noise-free case. However, like AR modeling for linear predictive coding (LPC), spectral estimation, and time series forecasting, the signal to be modeled here is corrupted by the white Gaussian noise. Substituting (12) into (1) gives the corresponding noisy measurement of the AR process

$$x_n = \sum_{m=1}^{M} \alpha_m^* u_{n-m} + v_n \tag{13}$$

where the index k has been omitted for notational simplicity. Some of the existing techniques such as linear prediction, EVD, and the LMS algorithm [8, 15, 21, 38, 39] have been employed to estimate the AR coefficients $\alpha_1,\ldots,\alpha_M$ from the observed noisy signal.

We now propose to estimate the coefficients $\alpha_1,\ldots,\alpha_M$ from an initial estimate of the DOA that is obtained through a subspace-based algorithm. Suppose the initial DOA is estimated as $\hat{\theta}_m$, $(m = 1,2,\ldots,M)$, which leads to $\hat{\varphi}_m$ as shown in (4). Then from (8), we have

$$(\hat{\alpha}_M,\ldots,\hat{\alpha}_1) = ((\hat{\varphi}_1^M, \hat{\varphi}_2^M, \ldots, \hat{\varphi}_M^M)\hat{\boldsymbol{\Phi}}_{MM}^{-1})^*. \tag{14}$$

These coefficients can be used as AR model coefficients for spatial extrapolation. The extrapolated signal samples $\hat{x}_n$ $(n > N)$ for each snapshot can be written as

$$\hat{x}_n = \begin{cases} \displaystyle\sum_{m=1}^{M} \hat{\alpha}_m^* x_{n-m}, & (n = N+1) \\ \displaystyle\sum_{m=i-N}^{M} \hat{\alpha}_m^* x_{n-m} + \sum_{m=1}^{i-N-1} \hat{\alpha}_m^* \hat{x}_{n-m}, & (n = N+2,\ldots,N+M) \\ \displaystyle\sum_{m=1}^{M} \hat{\alpha}_m^* \hat{x}_{n-m}, & (n = N+M+1,\ldots,N') \end{cases} \tag{15}$$

where N' is the total number of antennas including virtual ones.

The above extrapolation process is illustrated in Fig. 1. The spatial extrapolation performed on the received noisy signal can be interpreted as the extrapolation of the clean signal plus that of the noise. Obviously, the additional samples from extrapolation are correlated with one another, as well as with part of the received samples. In general, the estimated coefficient $\hat{\alpha}_m$ does not equal α_m, thus leading to a corrupted estimate $\hat{u}_{N+1},\ldots,\hat{u}_{N'}$ of the ideal additional noise-free signals $u_{N+1},\ldots,u_{N'}$. However, $\hat{\alpha}_m$ gets closer to α_m, with an increasing SNR. Thus, for a very high SNR, $\hat{u}_{N+1},\ldots,\hat{u}_{N'}$ can be approximated as $u_{N+1},\ldots,u_{N'}$. Although additional noise-free signals are desired, the performance of the DOA estimation based on the extrapolated data would not be as good as the performance using N' real antennas due to the addition of the correlated noise samples caused by the extrapolation. However, it will be shown that the DOA estimation performance using spatial extrapolation would be superior to that of N actual antennas when the angle of separation is very small.

Using the received data x_n along with the additional signal samples $\hat{x}_n$, the extrapolated correlation matrix can be estimated as

$$\hat{\mathbf{R}}_x' = \frac{1}{K}\sum_{k=1}^{K} \begin{bmatrix} x_1(k) \\ \vdots \\ x_N(k) \\ \hat{x}_{N+1}(k) \\ \vdots \\ \hat{x}_{N'}(k) \end{bmatrix} [x_1^*(k)\cdots x_N^*(k) \quad \hat{x}_{N+1}^*(k)\cdots \hat{x}_{N'}^*(k)]. \tag{16}$$

It is of interest to note that the calculation of the extrapolated correlation matrix $\hat{\mathbf{R}}_x' \in \mathbb{C}^{N'\times N'}$ can be simplified by applying the linear extrapolation to both the column and the row of the original correlation matrix $\hat{\mathbf{R}}_x \in \mathbb{C}^{N\times N}$. With the extrapolated correlation matrix, a fine estimation of the DOA can be obtained by employing a subspace-based algorithm again.

The proposed complete ESPRIT algorithm with spatial extrapolation can be summarized as follows:

Step 1 estimate the autocorrelation matrix $\hat{\mathbf{R}}_x$ from K-snapshot received data.

Step 2 estimate an initial DOA $\{\hat{\theta}_m, m = 1,\ldots,M\}$ using a subspace method.

Step 3 calculate $\hat{\varphi}_m$ using (4).

Step 4 compute $\hat{\alpha}_m$ using (14).

Step 5 extrapolate $\hat{\mathbf{R}}_x$ to obtain $\hat{\mathbf{R}}'_x$.

Step 6 implement a fine DOA estimation using $\hat{\mathbf{R}}'_x$ in conjunction with a subspace method.

In this paper, the ESPRIT algorithm is employed for both the initial and fine DOA estimations. In this case, Step 3 above can be omitted since $\hat{\varphi}_m$ has been directly calculated in ESPRIT without requiring $\hat{\theta}_m$. In the following, we call the proposed algorithm the ESPRIT with spatial extrapolation.

IV. PERFORMANCE ANALYSIS OF ESPRIT WITH SPATIAL EXTRAPOLATION

In this section, the performance of the ESPRIT with spatial extrapolation is studied. First, an MSE expression for the conventional ESPRIT is derived in the correlation domain, based on which the effect of the number of antennas on the MSE is investigated as a case study. Then, an MSE expression for the extrapolated ESPRIT algorithm is developed.

A. MSE Derivation of the Correlation-Domain ESPRIT

It is known that the solution of subspace methods is always perturbed by various sources, such as finite data length and measurement noise [26, 29, 30]. First- or second-order perturbation analysis has been proposed for subspace decomposition in [25, 26]. In [27], a general framework for the analysis of the MSE of DOA estimation has been obtained using the first-order perturbation. A closed-form MSE for ESPRIT has been given in the data-domain in terms of the array geometry, the number of snapshots, and the signal covariance matrix in [28]. In what follows, we extend the method in [28] for the analysis of the correlation-domain ESPRIT.

Given K-snapshot data, the autocorrelation matrix of the received signal can be estimated as

$$\begin{aligned}\hat{\mathbf{R}}_x &= \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}(k)\mathbf{x}^{\mathrm{H}}(k) - \mathbf{R}_v \\ &= \mathbf{A}\hat{\mathbf{R}}_s\mathbf{A}^{\mathrm{H}} + \mathbf{A}\hat{\mathbf{R}}_{sv} + \hat{\mathbf{R}}_{vs}\mathbf{A}^{\mathrm{H}} + \hat{\mathbf{R}}_v - \mathbf{R}_v \qquad (17)\end{aligned}$$

where $\hat{\mathbf{R}}_s$ and $\hat{\mathbf{R}}_v$ are the estimates of $\mathbf{R}_s$ and $\mathbf{R}_v$, respectively, and

$$\hat{\mathbf{R}}_{sv} \triangleq \frac{1}{K}\sum_{k=1}^{K}\mathbf{s}(k)\mathbf{v}^{\mathrm{H}}(k) \qquad (18)$$

$$\hat{\mathbf{R}}_{vs} \triangleq \frac{1}{K}\sum_{k=1}^{K}\mathbf{v}(k)\mathbf{s}^{\mathrm{H}}(k). \qquad (19)$$

It should be noted that $\hat{\mathbf{R}}_{sv}$ and $\hat{\mathbf{R}}_{vs}$ are non-zero matrices. Letting $\Delta\mathbf{R}_v \triangleq \hat{\mathbf{R}}_v - \mathbf{R}_v$, we can separate $\hat{\mathbf{R}}_x$ into the ideal and the perturbation parts as

$$\hat{\mathbf{R}}_x = \bar{\mathbf{R}}_x + \Delta\bar{\mathbf{R}}_x \qquad (20)$$

where $\bar{\mathbf{R}}_x \triangleq \mathbf{A}\hat{\mathbf{R}}_s\mathbf{A}^{\mathrm{H}}$ and $\Delta\bar{\mathbf{R}}_x \triangleq \mathbf{A}\hat{\mathbf{R}}_{sv} + \hat{\mathbf{R}}_{vs}\mathbf{A}^{\mathrm{H}} + \Delta\mathbf{R}_v$. Performing the singular value decomposition (SVD) on $\bar{\mathbf{R}}_x$ gives

$$\bar{\mathbf{R}}_x = [\bar{\mathbf{U}}_s, \bar{\mathbf{U}}_n]\begin{bmatrix}\bar{\boldsymbol{\Sigma}}_s & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\bar{\mathbf{U}}_s^{\mathrm{H}}\\ \bar{\mathbf{U}}_n^{\mathrm{H}}\end{bmatrix} \qquad (21)$$

where $\bar{\mathbf{U}}_s$ and $\bar{\mathbf{U}}_n$ span the signal and the noise spaces of $\bar{\mathbf{R}}_x$, respectively, the diagonal matrix $\bar{\boldsymbol{\Sigma}}_s$ contains the signal singular values. In a manner similar to the one used in [28], the first-order perturbation of $\bar{\mathbf{U}}_s$ can be obtained as

$$\Delta\bar{\mathbf{U}}_s \approx \bar{\mathbf{U}}_n\bar{\mathbf{U}}_n^{\mathrm{H}}\Delta\bar{\mathbf{R}}_x\bar{\mathbf{U}}_s\bar{\boldsymbol{\Sigma}}_s^{-1}. \qquad (22)$$

For a medium to high SNR, we have $\bar{\mathbf{U}}_n^{\mathrm{H}}\mathbf{A} = \mathbf{0}$ and $\|\hat{\mathbf{R}}_{vs}\|_{\mathrm{F}}^2 \gg \|\Delta\mathbf{R}_v\|_{\mathrm{F}}^2$, leading (22) to

$$\Delta\bar{\mathbf{U}}_s \approx \bar{\mathbf{U}}_n\bar{\mathbf{U}}_n^{\mathrm{H}}\hat{\mathbf{R}}_{vs}\mathbf{A}^{\mathrm{H}}\bar{\mathbf{U}}_s\bar{\boldsymbol{\Sigma}}_s^{-1}. \qquad (23)$$

Also, the DOA estimation error of the correlation-domain ESPRIT can be derived as

$$\Delta\hat{\theta}_m = \mathrm{Im}[\boldsymbol{\alpha}_m^{\mathrm{H}}\hat{\mathbf{R}}_{vs}\boldsymbol{\beta}_m] \qquad (24)$$

where

$$\boldsymbol{\alpha}_m^{\mathrm{H}} = c_m\mathbf{e}_m^{\mathrm{H}}(\mathbf{A}\downarrow)^{\dagger}[(\mathbf{I}\uparrow)\varphi_m^{-1} - (\mathbf{I}\downarrow)] \qquad (25)$$

$$\boldsymbol{\beta}_m = \mathbf{A}^{\mathrm{H}}\bar{\mathbf{U}}_s\bar{\boldsymbol{\Sigma}}_s^{-1}\bar{\mathbf{U}}_s^{\mathrm{H}}\mathbf{A}\mathbf{e}_m \qquad (26)$$

with $c_m = \lambda/2\pi d\cos(\theta_m)$. Noting that

$$\mathbf{A}^{\mathrm{H}}\bar{\mathbf{U}}_s\bar{\boldsymbol{\Sigma}}_s^{-1}\bar{\mathbf{U}}_s^{\mathrm{H}}\mathbf{A} = \mathbf{A}^{\mathrm{H}}(\mathbf{A}\hat{\mathbf{R}}_s\mathbf{A}^{\mathrm{H}})^{\dagger}\mathbf{A} = \hat{\mathbf{R}}_s^{-1}$$

(26) can be reduced to

$$\boldsymbol{\beta}_m = \hat{\mathbf{R}}_s^{-1}(m,m)\mathbf{e}_m. \qquad (27)$$

From (24), one can easily verify that the mean of $\Delta\hat{\theta}_m$ is zero. Thus, the MSE of the DOA estimation can be derived as the variance of $\Delta\hat{\theta}_m$. Note that the MSE expression given in [28] is no longer applicable, since the independent and identically distributed (IID) data perturbation $\Delta\mathbf{Y}$ in [28, eq. (36)] has been replaced by $\hat{\mathbf{R}}_{vs}$ in (24) above, which represents the correlation of the signal and the noise. In Appendix A, we have derived a closed-form expression for $\mathrm{Var}(\Delta\hat{\theta}_m)$ as given below

$$\mathrm{Var}(\Delta\hat{\theta}_m) = \frac{1}{2K}\|\boldsymbol{\alpha}_m\|_{\mathrm{F}}^2 \delta_v^2 \hat{\mathbf{R}}_s^{-1}(m,m). \tag{28}$$

It can easily be shown that the MSE expression of the correlation-domain ESPRIT is equivalent to that of the data-domain ESPRIT. Moreover, it is of interest to note that by using (25) in (28), the MSE in (28) can be rewritten as a product of two terms involving the array response matrix $\mathbf{A}$ and the power of the signal source $\hat{\mathbf{R}}_s(m,m)$, namely,

$$\mathrm{Var}(\Delta\hat{\theta}_m) = \omega_1(\mathbf{A},m)\omega_2(\hat{\mathbf{R}}_s,m) \tag{29}$$

where

$$\omega_1(\mathbf{A},m) \stackrel{\Delta}{=} \|\mathbf{e}_m^{\mathrm{H}}(\mathbf{A}\downarrow)^{\dagger}[(\mathbf{I}\uparrow)\varphi_m^{-1} - (\mathbf{I}\downarrow)]\|_{\mathrm{F}}^2 \tag{30}$$

$$\omega_2(\hat{\mathbf{R}}_s,m) \stackrel{\Delta}{=} \frac{c_m}{2K}\delta_v^2\hat{\mathbf{R}}_s^{-1}(m,m). \tag{31}$$

B. Case Study for Close DOAs

Here we analyze the effect of the number of antennas on the MSE of the ESPRIT algorithm when DOAs are very close. Note that only $\omega_1(\mathbf{A},m)$ depends on the number of antennas. Given the SVD of $\mathbf{A}$, $\mathbf{A} = \mathbf{U}_A\boldsymbol{\Sigma}_A\mathbf{V}_A^{\mathrm{H}}$, we have

$$\omega_1(\mathbf{A},m) = \|\mathbf{e}_m^{\mathrm{H}}\mathbf{V}_A\boldsymbol{\Sigma}_A^{-1}(\mathbf{U}_A\downarrow)^{\mathrm{H}}[(\mathbf{I}\uparrow)\varphi_m^{-1} - (\mathbf{I}\downarrow)]\|_{\mathrm{F}}^2. \tag{32}$$

When the angle of separation of DOAs is very small, one can find that the minimum singular value of $\mathbf{A}$ would become very small. For illustration, we consider the simplest case of only two sources with a very small angle of separation $\Delta\theta_{\mathrm{sep}} \stackrel{\Delta}{=} \theta_2 - \theta_1$. For the antenna array with N elements and $d = \frac{1}{2}\lambda$, the two singular values of $\mathbf{A}$ can be derived (see Appendix B for details)

$$\delta_1 \approx \sqrt{2N} \tag{33}$$

$$\delta_2 \approx \begin{cases} \left(\sqrt{\displaystyle\sum_{n=1}^{(N-1)/2} 4\pi^2 n^2}\right)\cos\theta_1\Delta\theta_{\mathrm{sep}}; & N \text{ odd} \\ \left(\sqrt{\displaystyle\sum_{n=0}^{(N/2)-1} 4\pi^2 (n+1/2)^2}\right)\cos\theta_1\Delta\theta_{\mathrm{sep}}; & N \text{ even} \end{cases}. \tag{34}$$

It is seen from (34) that δ_2 is very small when N is small, leading to $\delta_2^{-1} \gg \delta_1^{-1}$. Since $\omega_1(\mathbf{A},m)$ is mainly dominated by the minimum singular value of $\mathbf{A}$ as seen from (32), the MSE is determined by δ_2. It is also seen that when N is very large, the value of δ_2 dramatically increases, giving a much smaller MSE. This explains why the angle resolution of ESPRIT algorithm can be improved by increasing the number of the antenna elements.

C. MSE Derivation of the Extrapolated ESPRIT

In this subsection, we conduct the perturbation analysis of the ESPRIT with spatial extrapolation. In this algorithm, the AR coefficients are calculated based on the initial estimate of DOAs, making it difficult to analyze the performance of the algorithm. To simplify the analysis, we assume that the extrapolation coefficients α_ms are known, implying that the true signal samples $u_{N+1},\ldots,u_{N'}$ can be obtained by extrapolation at virtual antennas. Thus, we need to consider only the extrapolation of noises.

Denote the original noise vector and the extrapolated noise vector as

$$\mathbf{v} \stackrel{\Delta}{=} [v_1,\ldots v_N]^{\mathrm{T}}$$

$$\mathbf{v}' \stackrel{\Delta}{=} [v_1,\ldots v_N,\hat{v}_{N+1},\ldots,\hat{v}_{N'}]^{\mathrm{T}}$$

respectively. Noting that

$$(v_{N-M+2},\ldots,v_N,\hat{v}_{N+1})^{\mathrm{T}} = \mathbf{H}(v_{N-M+1},\ldots,v_N)^{\mathrm{T}}$$

where

$$\mathbf{H} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ \alpha_M^* & \alpha_{M-1}^* & \alpha_{M-2}^* & \cdots & \alpha_1^* \end{pmatrix}$$

one can derive

$$(\hat{v}_{N+1},\ldots,\hat{v}_{N+M})^{\mathrm{T}} = \mathbf{H}^M(v_{N-M+1},\ldots,v_N)^{\mathrm{T}}. \tag{35}$$

In the following, we find the relationship between the original noise vector $\mathbf{v}$ and the extrapolated noise vector $\mathbf{v}'$ from (35).

Partitioning $\mathbf{v}$ and $\mathbf{v}'$ as

$$\mathbf{v} = \begin{bmatrix}\mathbf{v}_1 \\ \mathbf{v}_2\end{bmatrix} \tag{36}$$

$$\mathbf{v}' = \begin{bmatrix}\mathbf{v}_1 \\ \mathbf{v}_2'\end{bmatrix} \tag{37}$$

where

$$\mathbf{v}_1 = (v_1, v_2,\ldots,v_{M-N})^{\mathrm{T}}$$

$$\mathbf{v}_2 = (v_{N-M+1}, v_{N-M+2},\ldots,v_N)^{\mathrm{T}}$$

$$\mathbf{v}_2' = (v_{N-M+1},\ldots,v_N,\hat{v}_{N+1},\ldots,\hat{v}_{N'})^{\mathrm{T}}$$

when $N' = (N-M) + gM$, (g is an integer), one can easily find the relationship between $\mathbf{v}_2$ and $\mathbf{v}_2'$ from (35) as

$$\mathbf{v}_2' = \tilde{\mathbf{H}}\mathbf{v}_2 \tag{38}$$

where

$$\tilde{\mathbf{H}} \triangleq [\mathbf{I}, (\mathbf{H}^M)^T, (\mathbf{H}^{2M})^T, \ldots, (\mathbf{H}^{gM})^T]^T.$$

Then from (36), (37), and (38), we have

$$\mathbf{v}' = \mathbf{H}_{ex}\mathbf{v} \tag{39}$$

where

$$\mathbf{H}_{ex} \triangleq \begin{bmatrix} \mathbf{I}_M & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{H}} \end{bmatrix}$$

is the extrapolation matrix. From (39), one can deduce the correlation matrix between the extrapolated noise and the signal as

$$\hat{\mathbf{R}}'_{vs} \triangleq \frac{1}{K}\sum_{k=1}^{K} \mathbf{v}'(k)\mathbf{s}^H(k). \tag{40}$$

In a similar manner, one can obtain the extrapolated array response matrix

$$\mathbf{A}_{ex} = \begin{bmatrix} \mathbf{A}_0 \\ \tilde{\mathbf{H}}\mathbf{A}_1 \end{bmatrix} \tag{41}$$

where

$$\mathbf{A}_0 = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \varphi_1 & \varphi_2 & \cdots & \varphi_M \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_1^{N-M-1} & \varphi_2^{N-M-1} & \cdots & \varphi_M^{N-M-1} \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} \varphi_1^{N-M} & \varphi_2^{N-M} & \cdots & \varphi_M^{N-M} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_1^{N-1} & \varphi_2^{N-1} & \cdots & \varphi_M^{N-1} \end{bmatrix}.$$

Replacing $\hat{\mathbf{R}}_{vs}$ by $\hat{\mathbf{R}}'_{vs}$ in (24) and $\mathbf{A}$ by $\mathbf{A}_{ex}$ in (25), the DOA estimation error of the extrapolated ESPRIT can be derived as

$$\Delta\hat{\theta}'_m = \mathrm{Im}[(\boldsymbol{\alpha}'_m)^H \hat{\mathbf{R}}'_{vs}\boldsymbol{\beta}_m] \tag{42}$$

where

$$(\boldsymbol{\alpha}'_m)^H = c_m \mathbf{e}_m^H (\mathbf{A}_{ex}\downarrow)^{\dagger}[(\mathbf{I}\uparrow)\varphi_m^{-1} - (\mathbf{I}\downarrow)].$$

Further, based on (42), we can obtain a closed-form expression for $\mathrm{Var}(\Delta\hat{\theta}'_m)$ as (see Appendix C for details)

$$\mathrm{Var}(\Delta\hat{\theta}'_m) = \frac{1}{2K}\|\mathbf{H}_{ex}^H\boldsymbol{\alpha}'_m\|_F^2 \delta_v^2 \hat{\mathbf{R}}_s^{-1}(m,m). \tag{43}$$

Like (29), (43) can also be rewritten as a product of two terms involving the array response matrix and the power of the signal source, namely,

$$\mathrm{Var}(\Delta\hat{\theta}'_m) = \omega_{ex1}(\mathbf{A}_{ex}, m)\omega_2(\hat{\mathbf{R}}_s, m) \tag{44}$$

where

$$\omega_{ex1}(\mathbf{A}_{ex}, m) \triangleq \|\mathbf{e}_m^H(\mathbf{A}_{ex}\downarrow)^{\dagger}[(\mathbf{I}\uparrow)\varphi_m^{-1} - (\mathbf{I}\downarrow)]\mathbf{H}_{ex}\|_F^2. \tag{45}$$

It is clear from (29) and (44) that to evaluate the performance of the ESPRIT with spatial extrapolation as opposed to the conventional ESPRIT, one only needs to compare $\omega_{ex1}(\mathbf{A}, m)$ and $\omega_1(\mathbf{A}, m)$. From (30) and (45), we can see that the difference between $\omega_{ex1}(\mathbf{A}, m)$ and $\omega_1(\mathbf{A}, m)$ is that the array response matrix $\mathbf{A}$ has been replaced by the extrapolated array response matrix $\mathbf{A}_{ex}$, and the extrapolation matrix $\mathbf{H}_{ex}$ is involved. As the expressions of $\omega_{ex1}(\mathbf{A}, m)$ and $\omega_1(\mathbf{A}, m)$ are rather complicated, it is difficult to have an analytical comparison. Therefore, we resort to a simulation experimentation to show their numerical values. As seen in Section V, $\omega_{ex1}(\mathbf{A}, m)$ is much smaller than $\omega_1(\mathbf{A}, m)$. Thus, we can conclude that the ESPRIT with ideal spatial extrapolation produces a much better performance than the original ESPRIT algorithm. In other words, the angular resolution of the ESPIRIT algorithm can be enhanced by using the spatial extrapolation when the ideal AR coefficients are known.

V. SIMULATION RESULTS AND DISCUSSIONS

In this section, several computer simulation-based experimentations are conducted to demonstrate the effectiveness of the proposed DOA estimation algorithm. The estimation performance is evaluated in terms of the estimated MSE of DOA estimation given by

$$\mathrm{MSE} = \frac{1}{N_{MC}}\sum_{n=1}^{N_{MC}}\sum_{m=1}^{M}|\hat{\theta}_{m,n} - \theta_{m,n}|^2 \tag{46}$$

where N_{MC} is the number of Monte Carlo iterations, and θ_m, $\hat{\theta}_m$ are true and estimated DOA with respect to the nth Monte Carlo iteration for the mth sources.

Experiment 1. Estimated MSE Versus the Angle of Separation: We first consider a ULA that has 6 isotropic antennas spaced $\lambda/2$ apart and two signal sources with DOAs given by $\theta_1 = -40°$ and $\theta_2 = -40° + \Delta\theta_{sep}$, where $\Delta\theta_{sep} \in [0.6°, 2.5°]$. It is assumed that there are 32 additional extrapolated antennas, and the two signal sources are generated as $s_m(n) = \sigma_{s_m} e^{j\phi_m(n)}$, where $\{\phi_m(n)\}$ is independent and uniformly distributed over $[-\pi \;\; \pi]$, with unit power, i.e., $\sigma_{s_m} = 1$.

Fig. 2 shows the estimated MSE plots of the DOA estimation resulting from the conventional and the proposed methods based on 500 snapshots and 10,000 Monte Carlo iterations for an input SNR of 20 dB. The estimated MSE of the proposed method is smaller than that of the conventional method for $\Delta\theta_{sep} < 2.2°$. This result implies that one can use the proposed method to improve the DOA estimation performance when the angle of separation is very small. When the angle of separation is not very small, the conventional estimation algorithm can be used in order to achieve

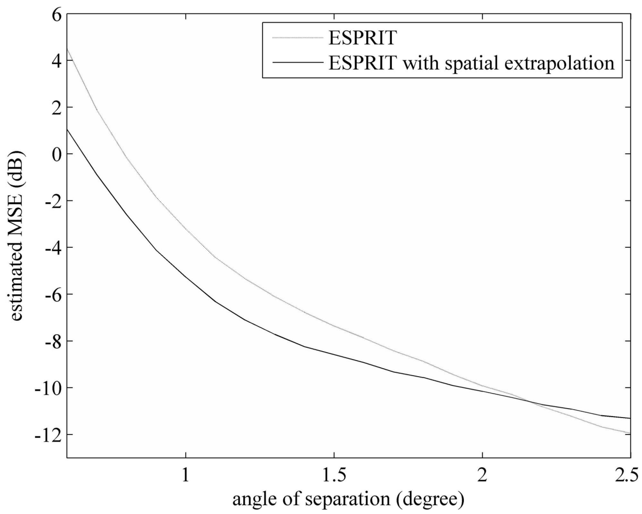

Fig. 2. Estimated MSE versus angle of separation for two close sources under SNR = 20 dB.

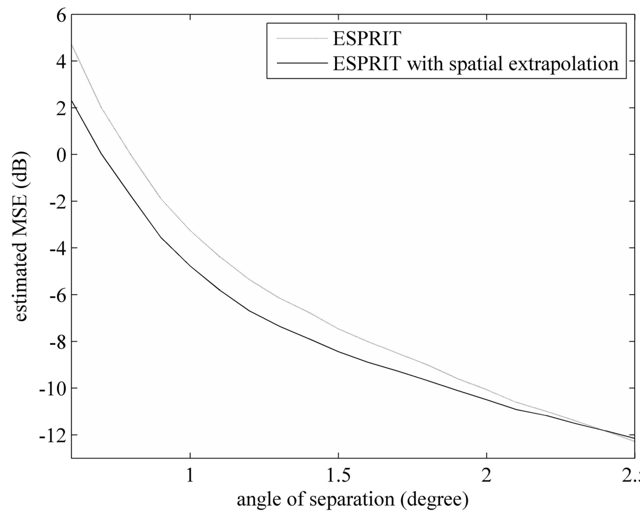

Fig. 4. Estimated MSE versus angle of separation of two close sources in third source scenario under SNR = 20 dB.

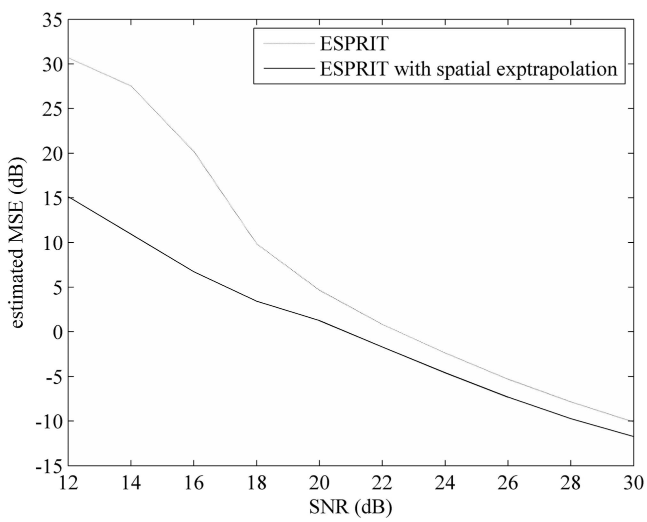

Fig. 3. Estimated MSE versus SNR for two sources with angle of separation 0.6°.

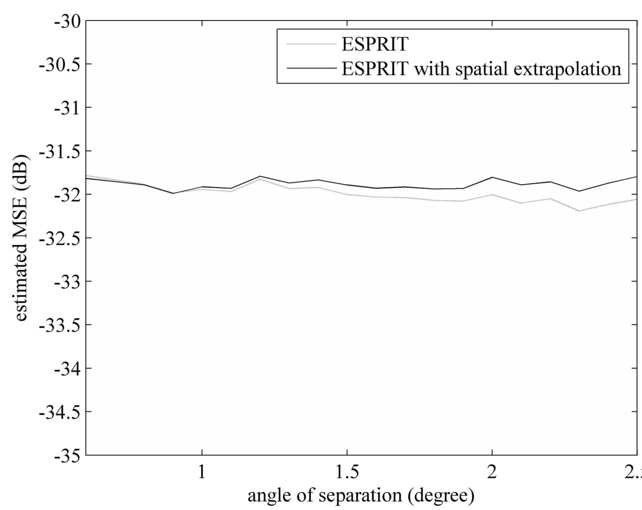

Fig. 5. Estimated MSE of third source versus angle of separation of the other two close sources under SNR = 20 dB.

a good compromise between the complexity and the estimation performance.

Experiment 2. Estimated MSE versus SNR: In this experiment, we investigate the DOA estimation performance versus the input SNR varying between 12 dB and 30 dB when the angle of separation is set to 0.6°. Fig. 3 shows the DOA estimation results for 10,000 Monte Carlo iterations. It is seen that the performance of the proposed method is consistently better than that of the conventional method. Moreover, the proposed method is more advantageous for a smaller SNR value.

Experiment 3. A scenario with Three Sources, Two of Which Have Close DOAs: In the third experiment, we consider the performance of the spatial extrapolation algorithm where there exists a third source in addition to the two sources with close DOAs. The conditions in Experiment 1 are used except that the third source with DOA $\theta_3 = 50^\circ$ is added. Fig. 4 shows the estimated MSE of the proposed DOA estimation as well as that of the conventional method for the two close sources from 10,000 Monte Carlo iterations, which exhibits a good similarity to that of Experiment 1. The DOA estimation result of the third source is given in Fig. 5, showing that the performance of the proposed method is very close to that of the conventional one. This experiment indicates that the proposed method can improve the DOA estimation performance for the sources with close DOAs, while not deteriorating the DOA estimation of other sources.

Experiment 4. MSE of the ESPRIT with Ideal Extrapolation: In this experiment, we compare the performance of the conventional ESPRIT algorithm and that of the ESPRIT with ideal spatial extrapolation using the theoretical expression of $\omega_{\text{ex}1}$ and $\omega_1(\mathbf{A},m)$ given in Section IV. The same simulation conditions as in Experiment 1 are used. For comparison, we also

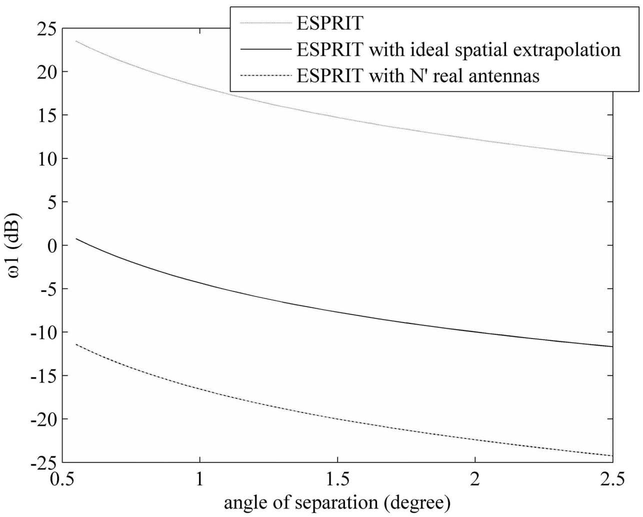

Fig. 6. Theoretical values of ω_1 resulting from three ESPRIT algorithms.

give the simulation result of ω_1 for the conventional ESPRIT with N' real antennas. From Fig. 6, one can see that the value of $\omega_{\text{ex}1}$ of the ESPRIT with ideal spatial extrapolation using 6 real and 32 extrapolated antennas is nearly 22 dB lower than that of ω_1 in the conventional ESPRIT with 6 real antennas only, although its performance is not as good as that of the conventional ESPRIT with $N' = 38$ real antennas.

The above simulation results show that the proposed spatial extrapolation-based blind DOA estimation consistently outperforms that of the conventional ESPRIT algorithm for closely-spaced sources. In other words, when the angle of separation of the sources is small, the DOA estimation performance of the ESPRIT can be enhanced by employing the proposed spatial extrapolation. It should be pointed out that, just like the interleaving-based resolution enhancement technique [12], our technique cannot improve the DOA estimation accuracy for all cases. In particular, when the angle of separation of the sources is very large, the performance of the proposed algorithm is even worse than that of the conventional ESPRIT algorithm. However, this problem can easily be solved by combining the conventional ESPRIT algorithm with the proposed extrapolation scheme. For example in Experiment 1, an initial DOA estimate can be obtained using the ESPRIT. If the angle of separation is less than 2.2°, the spatial extrapolation method can be employed to improve the DOA estimation; otherwise, the initial estimate of the DOA is kept as the ultimate result.

VI. CONCLUSION

A blind DOA estimation method for closely-spaced sources that is based on the spatial extrapolation of the correlation matrix has been presented. The spatial extrapolation has been implemented using the AR coefficients that are estimated by an initial DOA estimation method. It has been shown through computer simulations that using the extrapolated correlation matrix in a subspace-based DOA estimation algorithm such as ESPRIT would improve the DOA estimation performance for closely-spaced sources. Moreover, by conducting a perturbation analysis, a closed-form expression for the MSE of the DOA estimation of the ESPRIT with the ideal spatial extrapolation has been derived, justifying to a certain degree the advantage of the proposed method. In addition, extensive experimentations have confirmed the superiority of our method over the conventional DOA estimation techniques without employing the extrapolated correlation matrix.

APPENDIX A. DERIVATION OF $\mathrm{Var}(\Delta\hat{\theta}_m)$ IN (28)

From (24), it can be shown that [28]

$$
\begin{aligned}
\mathrm{Var}(\Delta\hat{\theta}_m) &= \mathrm{Var}(\mathrm{Im}[\boldsymbol{\alpha}_m^{\mathrm{H}}\hat{\mathbf{R}}_{vs}\boldsymbol{\beta}_m]) \\
&= -\mathrm{E}\left\{\left[\frac{\boldsymbol{\alpha}_m^{\mathrm{H}}\hat{\mathbf{R}}_{vs}\boldsymbol{\beta}_m - \boldsymbol{\beta}_m^{\mathrm{H}}\hat{\mathbf{R}}_{vs}^{\mathrm{H}}\boldsymbol{\alpha}_m}{2}\right]^2\right\} \\
&= \frac{1}{4}(E_1 + E_2 - E_3 - E_4) \qquad (47)
\end{aligned}
$$

where

$$
\begin{aligned}
E_1 &= \boldsymbol{\alpha}_m^{\mathrm{H}}\mathrm{E}\{\hat{\mathbf{R}}_{vs}\boldsymbol{\beta}_m\boldsymbol{\beta}_m^{\mathrm{H}}\hat{\mathbf{R}}_{vs}^{\mathrm{H}}\}\boldsymbol{\alpha}_m \\
E_2 &= \boldsymbol{\beta}_m^{\mathrm{H}}\mathrm{E}\{\hat{\mathbf{R}}_{vs}^{\mathrm{H}}\boldsymbol{\alpha}_m\boldsymbol{\alpha}_m^{\mathrm{H}}\hat{\mathbf{R}}_{vs}\}\boldsymbol{\beta}_m \\
E_3 &= \boldsymbol{\alpha}_m^{\mathrm{H}}\mathrm{E}\{\hat{\mathbf{R}}_{vs}\boldsymbol{\beta}_m\boldsymbol{\alpha}_m^{\mathrm{H}}\hat{\mathbf{R}}_{vs}\}\boldsymbol{\beta}_m \\
E_4 &= \boldsymbol{\beta}_m^{\mathrm{H}}\mathrm{E}\{\hat{\mathbf{R}}_{vs}^{\mathrm{H}}\boldsymbol{\alpha}_m\boldsymbol{\beta}_m^{\mathrm{H}}\hat{\mathbf{R}}_{vs}^{\mathrm{H}}\}\boldsymbol{\alpha}_m.
\end{aligned}
$$

To compute E_1, we first perform $\mathrm{E}\{\hat{\mathbf{R}}_{vs}\boldsymbol{\beta}_m\boldsymbol{\beta}_m^{\mathrm{H}}\hat{\mathbf{R}}_{vs}^{\mathrm{H}}\}$. Using (19) and (27), and noting that $\mathrm{E}\{v_n(k)v_n^*(k)\} = \frac{1}{2}\delta_v^2 + \frac{1}{2}\delta_v^2 = \delta_v^2$, we obtain

$$
\begin{aligned}
&E\{\hat{\mathbf{R}}_{vs}\boldsymbol{\beta}_m\boldsymbol{\beta}_m^{\mathrm{H}}\hat{\mathbf{R}}_{vs}^{\mathrm{H}}\} \\
&\quad = E\left\{\frac{1}{K^2}\sum_{k=1}^{K}\mathbf{v}(k)[\mathbf{s}^{\mathrm{H}}(k)\boldsymbol{\beta}_m\boldsymbol{\beta}_m^{\mathrm{H}}\mathbf{s}(k)]\mathbf{v}^{\mathrm{H}}(k)\right\} \\
&\quad = E\left\{\frac{1}{K^2}\sum_{k=1}^{K}\mathbf{v}(k)[\hat{\mathbf{R}}_s^{-2}(m,m)\mathbf{s}^{\mathrm{H}}(k)\mathbf{e}_m\mathbf{e}_m^{\mathrm{H}}\mathbf{s}(k)]\mathbf{v}^{\mathrm{H}}(k)\right\} \\
&\quad = \frac{1}{K^2}\hat{\mathbf{R}}_s^{-2}(m,m)\delta_v^2\left\{\sum_{k=1}^{K}[\mathbf{s}^{\mathrm{H}}(k)\mathbf{e}_m\mathbf{e}_m^{\mathrm{H}}\mathbf{s}(k)]\right\} \\
&\quad = \frac{1}{K}\hat{\mathbf{R}}_s^{-1}(m,m)\delta_v^2\mathbf{I}.
\end{aligned}
$$

Accordingly,

$$E_1 = \boldsymbol{\alpha}_m^{\mathrm{H}} \left[\frac{1}{K}\hat{\mathbf{R}}_s^{-1}(m,m)\delta_v^2 \mathbf{I}\right] \boldsymbol{\alpha}_m = \frac{1}{K}\|\boldsymbol{\alpha}_m\|_{\mathrm{F}}^2 \delta_v^2 \hat{\mathbf{R}}_s^{-1}(m,m). \tag{48}$$

Similarly, for E_2, we can compute first

$$\begin{aligned}
&\mathrm{E}\{\hat{\mathbf{R}}_{vs}^{\mathrm{H}}\boldsymbol{\alpha}_m\boldsymbol{\alpha}_m^{\mathrm{H}}\hat{\mathbf{R}}_{vs}\} \\
&\quad = \frac{1}{K^2}\sum_{k=1}^{K}\mathbf{s}(k)\mathrm{E}\{[\mathbf{v}^{\mathrm{H}}(k)\boldsymbol{\alpha}_m\boldsymbol{\alpha}_m^{\mathrm{H}}\mathbf{v}(k)]\}\mathbf{s}^{\mathrm{H}}(k) \\
&\quad = \frac{1}{K^2}\|\boldsymbol{\alpha}_m\|_{\mathrm{F}}^2\delta_v^2\sum_{k=1}^{K}\mathbf{s}(k)\mathbf{s}^{\mathrm{H}}(k) = \frac{1}{K}\|\boldsymbol{\alpha}_m\|_{\mathrm{F}}^2\delta_v^2\hat{\mathbf{R}}_s.
\end{aligned}$$

Then,

$$E_2 = \boldsymbol{\beta}_m^{\mathrm{H}}\frac{1}{K}\|\boldsymbol{\alpha}_m\|_{\mathrm{F}}^2\delta_v^2\hat{\mathbf{R}}_s\boldsymbol{\beta}_m = \frac{1}{K}\|\boldsymbol{\alpha}_m\|_{\mathrm{F}}^2\delta_v^2\hat{\mathbf{R}}_s^{-1}(m,m). \tag{49}$$

By noting that $\mathrm{E}\{v_n(k)v_n(k)\} = \frac{1}{2}\delta_v^2 - \frac{1}{2}\delta_v^2 = 0$, one can easily verify that $E_3 = E_4 = 0$. As a consequence, using (48) and (49) into (47) yields

$$\mathrm{Var}(\Delta\hat{\theta}_m) = \frac{1}{4}(E_1 + E_2) = \frac{1}{2K}\|\boldsymbol{\alpha}_m\|_{\mathrm{F}}^2\delta_v^2\hat{\mathbf{R}}_s^{-1}(m,m).$$

APPENDIX B. DERIVATION OF THE SINGULAR VALUES OF MATRIX A

Given a very small angle of separation $\Delta\theta_{\mathrm{sep}} = \theta_2 - \theta_1$, $(\Delta\theta_{\mathrm{sep}} \to 0)$,

$$\sin\theta_2 = \sin(\theta_1 + \Delta\theta_{\mathrm{sep}}) \approx \sin\theta_1 + \cos\theta_1\Delta\theta_{\mathrm{sep}}. \tag{50}$$

Substituting (50) into (3) and (4) gives

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ e^{-j\pi\sin\theta_1} & e^{-j\pi\sin\theta_1}e^{-j\pi\cos\theta_1\Delta\theta_{\mathrm{sep}}} \\ e^{-j2\pi\sin\theta_1} & e^{-j2\pi\sin\theta_1}e^{-j2\pi\cos\theta_1\Delta\theta_{\mathrm{sep}}} \\ \vdots & \vdots \\ e^{-j(N-1)\pi\sin\theta_1} & e^{-j(N-1)\pi\sin\theta_1}e^{-j(N-1)\pi\cos\theta_1\Delta\theta_{\mathrm{sep}}} \end{bmatrix}. \tag{51}$$

To obtain the singular values of $\mathbf{A}$, we can calculate the eigenvalues of $\mathbf{A}\mathbf{A}^{\mathrm{H}}$. It is clear from (51) that the rank of $\mathbf{A}\mathbf{A}^{\mathrm{H}}$ is 2, suggesting that the characteristic equation

$$|\mathbf{A}\mathbf{A}^{\mathrm{H}} - \lambda\mathbf{I}| = 0 \tag{52}$$

be reduced to

$$\lambda^{N-2}(\lambda - \lambda_1)(\lambda - \lambda_2) = 0 \tag{53}$$

where λ_1 and λ_2 are two non-zero eigenvalues. In what follows, we derive the coefficients of three highest order terms of the polynomial on the left-hand side (LHS) of (52) to obtain λ_1 and λ_2.

Defining a matrix $\boldsymbol{\Pi}_1 \triangleq \mathbf{A}\mathbf{A}^{\mathrm{H}} - \lambda\mathbf{I}$ with its first row being $\boldsymbol{\pi}_1$ and the partitioned form being

$$\boldsymbol{\Pi}_1 = \begin{bmatrix} \boldsymbol{\pi}_1(1) & \boldsymbol{\pi}_1(2,\ldots,N) \\ \boldsymbol{\pi}_1^{\mathrm{H}}(2,\ldots,N) & \boldsymbol{\Pi}_2 \end{bmatrix}. \tag{54}$$

From (54), the determinant of $\boldsymbol{\Pi}_1$ can be calculated by expanding along the first row $\boldsymbol{\pi}_1$ and keeping three highest order terms of the polynomial, yielding

$$|\boldsymbol{\Pi}_1| = (2-\lambda)|\boldsymbol{\Pi}_2| + T_1 \tag{55}$$

where

$$T_1 = (-1)\sum_{n=2}^{N}(2 + e^{j(n-1)\pi\cos\theta_1\Delta\theta_{\mathrm{sep}}} + e^{-j(n-1)\pi\cos\theta_1\Delta\theta_{\mathrm{sep}}})(2-\lambda)^{N-2}. \tag{56}$$

Similarly, letting the first row of $\boldsymbol{\Pi}_2$ be $\boldsymbol{\pi}_2$, $\boldsymbol{\Pi}_2$ can be partitioned as

$$\boldsymbol{\Pi}_2 = \begin{bmatrix} \boldsymbol{\pi}_2(1) & \boldsymbol{\pi}_2(2,\ldots,N-1) \\ \boldsymbol{\pi}_2^{\mathrm{H}}(2,\ldots,N-1) & \boldsymbol{\Pi}_3 \end{bmatrix}. \tag{57}$$

Then, by keeping three highest order terms, one can derive

$$(2-\lambda)|\boldsymbol{\Pi}_2| = (2-\lambda)^2|\boldsymbol{\Pi}_3| + T_2 \tag{58}$$

where

$$T_2 = (-1)\sum_{n=2}^{N-1}(2 + e^{j(n-1)\pi\cos\theta_1\Delta\theta_{\mathrm{sep}}} + e^{-j(n-1)\pi\cos\theta_1\Delta\theta_{\mathrm{sep}}})(2-\lambda)^{N-2}. \tag{59}$$

In the same manner, using (56) and (59), one can finally derive

$$\begin{aligned}
|\boldsymbol{\Pi}_1| &= T_1 + T_2 + \cdots T_{N-1} + (2-\lambda)^N \\
&= (-1)\sum_{n=2}^{N}\sum_{m=2}^{n}(2 + e^{j(m-1)\pi\cos\theta_1\Delta\theta_{\mathrm{sep}}} + e^{-j(m-1)\pi\cos\theta_1\Delta\theta_{\mathrm{sep}}}) \\
&\quad \times (2-\lambda)^{N-2} + (2-\lambda)^N.
\end{aligned} \tag{60}$$

By leaving only the coefficients with respect to three highest order terms in (60), one can obtain

$$\lambda^{N-2}\Bigg[\lambda^2 - 2N\lambda + N^2 - N - \left(\sum_{n=2}^{N}\sum_{m=2}^{n}e^{j(m-1)\pi\cos\theta_1\Delta\theta_{\mathrm{sep}}} + e^{-j(m-1)\pi\cos\theta_1\Delta\theta_{\mathrm{sep}}}\right)\Bigg] = 0. \tag{61}$$

Moreover, one can easily verify that

$$B^2 \overset{\Delta}{=} N + \left(\sum_{n=2}^{N} \sum_{m=2}^{n} e^{j(m-1)\pi\cos\theta_1 \Delta\theta_{\text{sep}}} + e^{-j(m-1)\pi\cos\theta_1\Delta\theta_{\text{sep}}} \right)$$

$$= \begin{cases} \left[1 + \sum_{n=1}^{(N-1)/2} (e^{jn\pi\cos\theta_1\Delta\theta_{\text{sep}}} + e^{-jn\pi\cos\theta_1\Delta\theta_{\text{sep}}}) \right]^2; & \text{if } N \text{ is odd} \\ \left[\sum_{n=0}^{(N/2)-1} (e^{j(n+1/2)\pi\cos\theta_1\Delta\theta_{\text{sep}}} + e^{-j(n+1/2)\pi\cos\theta_1\Delta\theta_{\text{sep}}}) \right]^2; & \text{if } N \text{ is even} \end{cases}. \tag{62}$$

From (61) and (62), one can calculate the two eigenvalues of $\mathbf{AA}^{\text{H}}$ as

$$\lambda_1 = N + B \tag{63}$$

$$\lambda_2 = N - B. \tag{64}$$

By noting that when $\Delta\theta_{\text{sep}} \to 0$, $B \to N$, (63) and (64) can be approximated to

$$\lambda_1 \approx 2N$$

$$\lambda_2 = N - B$$

$$\approx \begin{cases} \sum_{n=1}^{(N-1)/2} 4\pi^2 n^2 \cos^2\theta_1 \Delta\theta_{\text{sep}}^2; & N \text{ odd} \\ \sum_{n=0}^{(N/2)-1} 4\pi^2 (n+\frac{1}{2})^2 \cos^2\theta_1 \Delta\theta_{\text{sep}}^2; & N \text{ even} \end{cases}.$$

Obviously, the singular values of $\mathbf{A}$ are readily given by the square root of the eigenvalues λ_1 and λ_2 of $\mathbf{AA}^{\text{H}}$.

APPENDIX C. DERIVATION OF $\text{Var}(\Delta\hat{\theta}'_m)$ IN (43)

In a manner similar to obtaining (47), by using (42), $\text{Var}(\Delta\hat{\theta}'_m)$ can be written as

$$\text{Var}(\Delta\hat{\theta}'_m) = \tfrac{1}{4}(E'_1 + E'_2 - E'_3 - E'_4) \tag{65}$$

where

$$E'_1 = (\boldsymbol{\alpha}'_m)^{\text{H}} E\{\hat{\mathbf{R}}'_{vs}\boldsymbol{\beta}_m\boldsymbol{\beta}_m^{\text{H}}(\hat{\mathbf{R}}'_{vs})^{\text{H}}\}\boldsymbol{\alpha}'_m$$

$$E'_2 = \boldsymbol{\beta}_m^{\text{H}} E\{(\hat{\mathbf{R}}'_{vs})^{\text{H}}\boldsymbol{\alpha}'_m(\boldsymbol{\alpha}'_m)^{\text{H}}\hat{\mathbf{R}}'_{vs}\}\boldsymbol{\beta}_m$$

$$E'_3 = (\boldsymbol{\alpha}'_m)^{\text{H}} E\{\hat{\mathbf{R}}'_{vs}\boldsymbol{\beta}_m(\boldsymbol{\alpha}'_m)^{\text{H}}\hat{\mathbf{R}}'_{vs}\}\boldsymbol{\beta}_m$$

$$E'_4 = \boldsymbol{\beta}_m^{\text{H}} E\{(\hat{\mathbf{R}}'_{vs})^{\text{H}}\boldsymbol{\alpha}'_m\boldsymbol{\beta}_m^{\text{H}}(\hat{\mathbf{R}}'_{vs})^{\text{H}}\}\boldsymbol{\alpha}'_m.$$

To compute E'_1, we first perform $E\{\hat{\mathbf{R}}'_{vs}\boldsymbol{\beta}_m\boldsymbol{\beta}_m^{\text{H}}(\hat{\mathbf{R}}'_{vs})^{\text{H}}\}$. Using (40) and (27), one can obtain

$$\begin{aligned} E\{\hat{\mathbf{R}}'_{vs}\boldsymbol{\beta}_m\boldsymbol{\beta}_m^{\text{H}}(\hat{\mathbf{R}}'_{vs})^{\text{H}}\} &= E\left\{ \frac{1}{K^2}\sum_{k=1}^{K} \mathbf{H}_{\text{ex}}\mathbf{v}(k)[\mathbf{s}^{\text{H}}(k)\boldsymbol{\beta}_m\boldsymbol{\beta}_m^{\text{H}}\mathbf{s}(k)]\mathbf{v}^{\text{H}}(k)\mathbf{H}_{\text{ex}}^{\text{H}} \right\} \\ &= \frac{1}{K^2}\hat{\mathbf{R}}_s^{-2}(m,m)\delta_v^2\mathbf{H}_{\text{ex}}\left\{ \sum_{k=1}^{K}[\mathbf{s}^{\text{H}}(k)\mathbf{e}_m\mathbf{e}_m^{\text{H}}\mathbf{s}(k)] \right\}\mathbf{H}_{\text{ex}}^{\text{H}} \\ &= \frac{1}{K}\hat{\mathbf{R}}_s^{-1}(m,m)\delta_v^2\mathbf{H}_{\text{ex}}\mathbf{H}_{\text{ex}}^{\text{H}}. \end{aligned}$$

As a result,

$$\begin{aligned} E'_1 &= \boldsymbol{\alpha}_m^{\text{H}}\left[\frac{1}{K}\hat{\mathbf{R}}_s^{-1}(m,m)\delta_v^2\mathbf{H}_{\text{ex}}\mathbf{H}_{\text{ex}}^{\text{H}}\right]\boldsymbol{\alpha}_k \\ &= \frac{1}{K}\|\mathbf{H}_{\text{ex}}^{\text{H}}\boldsymbol{\alpha}_m\|_{\text{F}}^2\delta_v^2\hat{\mathbf{R}}_s^{-1}(m,m). \end{aligned} \tag{66}$$

In a similar manner, for E_2, we can compute first

$$\begin{aligned} E\{(\hat{\mathbf{R}}'_{vs})^{\text{H}}\boldsymbol{\alpha}'_m(\boldsymbol{\alpha}'_m)^{\text{H}}\hat{\mathbf{R}}'_{vs}\} &= \frac{1}{K^2}\sum_{k=1}^{K}\mathbf{s}(k)E\{[\mathbf{v}^{\text{H}}(k)\mathbf{H}_{\text{ex}}^{\text{H}}\boldsymbol{\alpha}_m\boldsymbol{\alpha}_m^{\text{H}}\mathbf{H}_{\text{ex}}\mathbf{v}(k)]\}\mathbf{s}^{\text{H}}(k) \\ &= \frac{1}{K^2}\|\mathbf{H}_{\text{ex}}^{\text{H}}\boldsymbol{\alpha}_m\|_{\text{F}}^2\delta_v^2\sum_{k=1}^{K}\mathbf{s}(k)\mathbf{s}^{\text{H}}(k) = \frac{1}{K}\|\mathbf{H}_{\text{ex}}^{\text{H}}\boldsymbol{\alpha}_m\|_{\text{F}}^2\delta_v^2\hat{\mathbf{R}}_s. \end{aligned}$$

Then,

$$\begin{aligned} E'_2 &= \boldsymbol{\beta}_m^{\text{H}}\frac{1}{K}\|\mathbf{H}_{\text{ex}}^{\text{H}}\boldsymbol{\alpha}_m\|_{\text{F}}^2\delta_v^2\hat{\mathbf{R}}_s\boldsymbol{\beta}_m \\ &= \frac{1}{K}\|\mathbf{H}_{\text{ex}}^{\text{H}}\boldsymbol{\alpha}_m\|_{\text{F}}^2\delta_v^2\hat{\mathbf{R}}_s^{-1}(m,m). \end{aligned} \tag{67}$$

Substituting (66) and (67) into (65) and noting that $E_3 = E_4 = 0$, we eventually have

$$\text{Var}(\Delta\hat{\theta}'_m) = \frac{1}{4}(E'_1 + E'_2) = \frac{1}{2K}\|\mathbf{H}_{\text{ex}}^{\text{H}}\boldsymbol{\alpha}_m\|_{\text{F}}^2\delta_v^2\hat{\mathbf{R}}_s^{-1}(m,m).$$

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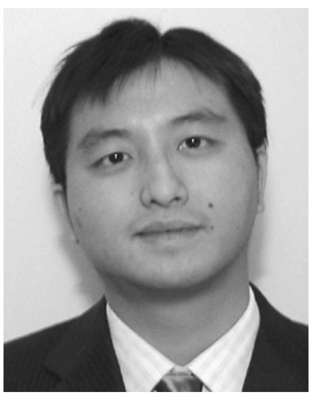

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