Perturbation Analysis of Whitening-Rotation-based Semi-Blind MIMO Channel Estimation

Feng Wan, Wei-Ping Zhu and M.N.S. Swamy

Centre for Signal Processing and Communications
Dept. of Electrical and Computer Engineering, Concordia University
Montreal, Quebec, Canada H3G 1M8

***Abstract*— The whitening-rotation (WR)-based semi-blind methods have been shown to achieve much better channel estimation performance than the conventional training-based methods for MIMO systems. In this paper, the performance analysis is conducted for an ideal WR-based method, in which the knowledge of the ideal whitening matrix is known a priori. This analysis is a valuable study since the ideal WR-based method provides the upper bound of the channel estimation performance for WR-based methods. First, the expression of the error of the rotation matrix is derived by using a perturbation analysis. This result is then utilized to derive a closed-form expression of mean square error (MSE) of the ideal WR-based method. The simulation studies have shown that the theoretical MSE values are consistent with the simulation results, confirming the high accuracy of the derivation of the MSE expression.**

I. Introduction

Channel estimation is a key part for the development of MIMO systems. Based on known pilots, the MIMO channel can be estimated by employing different kinds of training-based algorithms such as the least square (LS), the maximum likelihood (ML), the maximum a posteriori (MAP) and the minimum mean square error (MMSE) algorithms [1]. In contrast to training-based methods, blind channel estimation algorithms like those proposed in [2]–[4], can achieve a better spectral efficiency by use of the second-order statistics, correlative coding or other properties of user data. By combining the training-based and blind algorithms, semi-blind channel estimation techniques can potentially enhance the quality of MIMO channel estimation [5]–[7]. With a small number of training symbols, problems such as ambiguities and misconvergence of the blind methods can be solved by semi-blind techniques. On the other hand, the use of the available information data yields an improved accuracy of the channel estimation.

More recently, a whitening-rotation (WR)-based semi-blind algorithm has been proposed for frequency-flat MIMO channel estimation [8]. The WR-based semi-blind algorithm consists of two steps: (1) estimation of a whitening matrix utilizing information data; and (2) estimation of a unitary rotation matrix using pilots. It is shown that this semi-blind technique can achieve a better channel estimation performance than the conventional LS method, when the number of receive antennas is greater than or equal to the number of transmit antennas. However, the perturbation analysis of the WR-based channel estimation method conducted in our previous work [9] shows that, in the noise-free case, the blind part of the WR-based method is subject to a signal perturbation error, justifying that the WR-based method is efficient only in the case of low SNRs. In [10], we have further developed an enhanced transmit scheme to cancel the signal perturbation error in the receiver, leading to a signal perturbation free WR-based method. This new method has been shown to achieve an channel estimation performance very close to the ideal WR-based method, which obtains the whitening matrix directly from the true channel matrix. Since the ideal WR-based method is a very important reference to the WR-based semi-blind methods, it is necessary to make an theoretical analysis on it. In this paper, we would like to derive a close-form expression for the MSE of the ideal WR-based method.

Throughout the paper, we adopt the following notations:
T Transpose, H Complex conjugate transpose,
$\otimes$ Kronecker product, and $\circ$ Hadamard product.

II. Brief Review of WR-based Channel Estimation Algorithm

Consider a spatial-multiplexed MIMO system with N_T transmit and N_R ($\geq N_T$) receive antennas. Suppose that the frequency-flat fading MIMO channel is characterized by an $N_R \times N_T$ matrix $\mathbf{H}$ whose (i_R, i_T)-th element h_{i_T,i_R} represents the channel response from the i_T-th transmit antenna to the i_R-th receive antenna. Given the transmitted signal vector $\mathbf{x}(n) \triangleq [x_1(n), \cdots, x_{N_T}(n)]^T$ whose elements are independent identically distributed (i.i.d.) random variables with zero mean and unit variance $\delta_x^2 = 1$, the received signal vector $\mathbf{y}(n) \triangleq [y_1(n), \cdots, y_{N_R}(n)]^T$ can be written as

$$\mathbf{y}(n) = \mathbf{H}\mathbf{x}(n) + \mathbf{v}(n) \quad (1)$$

where the noise vector $\mathbf{v}(n) \triangleq [v_1(n), \cdots, v_{N_R}(n)]^T$ is spatio-temporally uncorrelated with variance δ_v^2. Note that, in each block, the transmit pilot matrix $\mathbf{X}_\mathrm{P}$ consisting of the first K of the N slots are used for training purpose and the corresponding received pilot matrix is $\mathbf{Y}_\mathrm{P}$.

We now briefly review a whitening-rotation (WR) based semi-blind MIMO channel estimation algorithm [8]. Its idea originates from a decomposition of the channel matrix,

$$\mathbf{H} = \mathbf{W}\mathbf{Q}^H \quad (2)$$

where $\mathbf{W}$ is a whitening matrix and $\mathbf{Q}$ is a unitary rotation matrix. Performing the singular value decomposition (SVD) of $\mathbf{H}$ gives

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{H}. \tag{3}$$

Obviously, one possible choice of $\mathbf{W}$ and $\mathbf{Q}$ can be $\mathbf{U}\boldsymbol{\Sigma}$ and $\mathbf{V}$. Thus, the WR-based channel estimation method can be implemented with two steps:
(i) Estimate the whitening matrix $\mathbf{W}$ in a blind fashion using the autocorrelation matrix of the received signal and a subspace-based method;
(ii) Estimate the unitary rotation matrix $\mathbf{Q}$ by utilizing training pilots and a constrained maximum-likelihood (ML) method.

However, the blind part of the WR-based method is subject to a signal perturbation error [9]. By using a SPF transmit scheme proposed in [10], the signal perturbation error can be removed, leading to a signal perturbation free WR-based method, which can achieve much better performance than the original WR-based method. It is expected that more new signal processing techniques can be proposed to improve the performance of WR-based methods. However, the upper bound of the WR-based methods is given by an ideal WR-based method, in which the ideal whitening matrix $\mathbf{W}$ is known a priori. In fact, the ideal whitening matrix $\mathbf{W}$ can be obtained from the true channel matrix. To investigate the performance of the WR-based methods, it is essential to disclose the MSE expression of the ideal WR-based method. In the following, we would like to derive such a close-form expression.

III. The Perturbation Error of the Rotation Matrix

As $\mathbf{W}$ is ideal, to derive the MSE expression, we only need consider the effect of the perturbation error of $\mathbf{Q}$. In this section, we derive the form of the perturbation error of $\mathbf{Q}$. First, we conduct an analysis for the training-based estimation of $\mathbf{Q}$ given in [8]. In the noise-free case, the rotation matrix $\mathbf{Q}$ can be calculated from

$$\mathbf{Q} = \mathbf{V}_Q\mathbf{U}_Q^H \tag{4}$$

where $\mathbf{U}_Q$ and $\mathbf{V}_Q$ are obtained from an SVD of the matrix

$$\mathbf{Y}_Q \triangleq \frac{1}{K\sigma_x^2}\mathbf{W}^H\mathbf{Y}_P\mathbf{X}_P^H, \tag{5}$$

namely,

$$\mathbf{Y}_Q = \mathbf{U}_Q\boldsymbol{\Sigma}_Q\mathbf{V}_Q^H. \tag{6}$$

As $\mathbf{H}$ has a full column rank, (3) can be rewritten as

$$\mathbf{H} = [\mathbf{U}_S, \mathbf{U}_N]\begin{bmatrix} \boldsymbol{\Sigma}_S & \\ & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{V}_S^H \\ \mathbf{V}_N^H\end{bmatrix} \tag{7}$$

where $\mathbf{U}_S$ and $\mathbf{U}_N$ are the signal subspace and the noise subspace of $\mathbf{U}$, $\mathbf{V}_S$ and $\mathbf{V}_N$ are the signal subspace and the noise subspace of $\mathbf{V}$. It is obvious from (7) that

$$\mathbf{W} = \mathbf{U}_S\boldsymbol{\Sigma}_S. \tag{8}$$

By using (8) and noting that

$$\frac{1}{K\sigma_x^2}\mathbf{Y}_P\mathbf{X}_P^H = \mathbf{H}, \tag{9}$$

(5) can be rewritten as

$$\mathbf{Y}_Q = \boldsymbol{\Sigma}_S^2\mathbf{V}_S^H. \tag{10}$$

As such, one realization of the SVD of $\mathbf{Y}_Q$ is given by

$$\mathbf{U}_Q = \mathbf{I}, \boldsymbol{\Sigma}_Q = \boldsymbol{\Sigma}_S^2, \mathbf{V}_Q = \mathbf{V}_S. \tag{11}$$

Using (11) into (4) yields

$$\mathbf{Q} = \mathbf{V}_S. \tag{12}$$

The above discussion indicates that the method in [8] gives an ideal rotation matrix $\mathbf{V}_S$ in the noise-free case. In the following, we derive an expression of the perturbation error $\Delta\mathbf{Q}$ of $\mathbf{Q}$ in the presence of noise.

In the noisy case, (5) should be modified to

$$\hat{\mathbf{Y}}_Q = \frac{1}{K\sigma_x^2}\mathbf{W}^H\mathbf{Y}_P\mathbf{X}_P^H. \tag{13}$$

Noting that $\sigma_x^2 = 1$, one can easily verify that

$$\frac{1}{K}\mathbf{Y}_P\mathbf{X}_P^H = \mathbf{H} + \Delta\mathbf{R}_{xv,P}^H \tag{14}$$

where

$$\Delta\mathbf{R}_{xv,P} \triangleq \frac{1}{K}\sum_{n=1}^{K}\mathbf{x}(n)\,\mathbf{v}^H(n). \tag{15}$$

Using (14), (13) can be expressed as

$$\hat{\mathbf{Y}}_Q = \mathbf{Y}_Q + \Delta\mathbf{Y}_Q = \boldsymbol{\Sigma}_S^2\mathbf{V}_S^H + \mathbf{W}^H\Delta\mathbf{R}_{xv,P}^H. \tag{16}$$

Our goal is to disclose an expression for $\Delta\mathbf{Q}$ in terms of the perturbation error $\Delta\mathbf{Y}_Q$ of $\mathbf{Y}_Q$ that is caused by noise. Let us consider the SVD of $\hat{\mathbf{Y}}_Q$:

$$\begin{aligned}\hat{\mathbf{Y}}_Q &= \hat{\mathbf{U}}_Q\hat{\boldsymbol{\Sigma}}_Q\hat{\mathbf{V}}_Q^H = [(\mathbf{U}_Q + \Delta\mathbf{U}_Q)\,\mathbf{P}] \\ &\quad (\boldsymbol{\Sigma}_Q + \Delta\boldsymbol{\Sigma}_Q)\,[(\mathbf{V}_Q + \Delta\mathbf{V}_Q)\,\mathbf{P}]^H\end{aligned} \tag{17}$$

where $\mathbf{P}$ is a diagonal unitary matrix used to represent a general form of the SVD, since the SVD of $\hat{\mathbf{Y}}_Q$ is not unique. By utilizing $\hat{\mathbf{U}}_Q = (\mathbf{U}_Q + \Delta\mathbf{U}_Q)\,\mathbf{P}$ and $\hat{\mathbf{V}}_Q = (\mathbf{V}_Q + \Delta\mathbf{V}_Q)\,\mathbf{P}$, we have

$$\begin{aligned}\Delta\mathbf{Q} &\triangleq \hat{\mathbf{Q}} - \mathbf{Q} = \hat{\mathbf{V}}_Q\hat{\mathbf{U}}_Q^H - \mathbf{V}_Q\mathbf{U}_Q^H \\ &\approx \Delta\mathbf{V}_Q\mathbf{U}_Q^H + \mathbf{V}_Q\Delta\mathbf{U}_Q^H.\end{aligned} \tag{18}$$

Using (11), (18) reduces to

$$\Delta\mathbf{Q} \approx \Delta\mathbf{V}_Q + \mathbf{V}_S\Delta\mathbf{U}_Q^H. \tag{19}$$

In general, if the noise subspace of $\mathbf{Y}_Q$ exists, the expressions for $\Delta\mathbf{V}_Q$ and $\Delta\mathbf{U}_Q$ can be easily derived in terms of the perturbation error $\Delta\mathbf{Y}_Q$ by utilizing the analysis results of the existing perturbation theory such as that in [11]. However, these results are not applicable to our case, since only the signal subspace exists in $\mathbf{Y}_Q$. To overcome this difficulty, we use the following theorem to derive an expression for $\Delta\mathbf{Q}$ in terms of the perturbation error $\Delta\mathbf{Y}_Q$.

Theorem 4.1:Given an SVD of a full rank $M \times M$ matrix $\mathbf{Z}$, $\mathbf{Z} = \mathbf{U}_Z\boldsymbol{\Sigma}_Z\mathbf{V}_Z^H$, where $\boldsymbol{\Sigma}_Z = \mathrm{diag}\,(\sigma_{z_1}, \sigma_{z_2}, \cdots, \sigma_{z_M})$, for a small perturbed term $\Delta\mathbf{Z}$, the SVD of $\hat{\mathbf{Z}} = \mathbf{Z} + \Delta\mathbf{Z}$ is defined as

$$\begin{aligned}\hat{\mathbf{Z}} &= \hat{\mathbf{U}}_Z\hat{\boldsymbol{\Sigma}}_Z\hat{\mathbf{V}}_Z^H = [(\mathbf{U}_Z + \Delta\mathbf{U}_Z)\,\mathbf{P}_Z]\\ &\quad(\boldsymbol{\Sigma}_Z + \Delta\boldsymbol{\Sigma}_Z)\,[(\mathbf{V}_Z + \Delta\mathbf{V}_Z)\,\mathbf{P}_Z]^H \end{aligned} \tag{20}$$

where $\mathbf{P}_Z$ is a diagonal unitary matrix whose role is the same as $\mathbf{P}$ in (17). Then, it can be shown that

$$\begin{aligned}\boldsymbol{\Omega} &\triangleq \Delta\mathbf{U}_Z^H\mathbf{U}_Z + \mathbf{V}_Z^H\Delta\mathbf{V}_Z\\ &\approx \boldsymbol{\Gamma}_Z \circ \left(\mathbf{V}_Z^H\Delta\mathbf{Z}^H\mathbf{U}_Z - \mathbf{U}_Z^H\Delta\mathbf{Z}\mathbf{V}_Z\right)\end{aligned} \tag{21}$$

where

$$\boldsymbol{\Gamma}_Z = \begin{bmatrix} \frac{1}{2\sigma_{z_1}} & \frac{1}{\sigma_{z_1}+\sigma_{z_2}} & \cdots & \frac{1}{\sigma_{z_1}+\sigma_{z_M}}\\ \frac{1}{\sigma_{z_2}+\sigma_{z_1}} & \frac{1}{2\sigma_{z_2}} & \cdots & \frac{1}{\sigma_{z_2}+\sigma_{z_M}}\\ \vdots & \vdots & \ddots & \vdots\\ \frac{1}{\sigma_{z_M}+\sigma_{z_1}} & \frac{1}{\sigma_{z_M}+\sigma_{z_2}} & \cdots & \frac{1}{2\sigma_{z_M}} \end{bmatrix}. \tag{22}$$

The proof of the theorem is omitted due to lack of space.

In order to use *Theorem* 4.1, we first rewrite (19) as

$$\Delta\mathbf{Q} \approx \mathbf{V}_S\left(\Delta\mathbf{U}_Q^H\mathbf{I} + \mathbf{V}_S^H\Delta\mathbf{V}_Q\right). \tag{23}$$

With replacements $\Delta\mathbf{U}_Z = \Delta\mathbf{U}_Q$, $\mathbf{U}_Z = \mathbf{I}$, $\mathbf{V}_Z = \mathbf{V}_S$, $\Delta\mathbf{V}_Z = \Delta\mathbf{V}_Q$, $\boldsymbol{\Sigma}_Z = \boldsymbol{\Sigma}_S^2$ and $\Delta\mathbf{Z} = \Delta\mathbf{Y}_Q$ in (21), we immediately obtain

$$\Delta\mathbf{Q} \approx \mathbf{V}_S\left(\boldsymbol{\Gamma}_Q \circ \boldsymbol{\Pi}\right) \tag{24}$$

where

$$\boldsymbol{\Gamma}_Q = \begin{bmatrix} \frac{1}{2\sigma_{S_1}^2} & \frac{1}{\sigma_{S_1}^2+\sigma_{S_2}^2} & \cdots & \frac{1}{\sigma_{S_1}^2+\sigma_{S_{N_T}}^2}\\ \frac{1}{\sigma_{S_2}^2+\sigma_{S_1}^2} & \frac{1}{2\sigma_{S_2}^2} & \cdots & \frac{1}{\sigma_{S_2}^2+\sigma_{S_{N_T}}^2}\\ \vdots & \vdots & \ddots & \vdots\\ \frac{1}{\sigma_{S_{N_T}}^2+\sigma_{S_1}^2} & \frac{1}{\sigma_{S_{N_T}}^2+\sigma_{S_2}^2} & \cdots & \frac{1}{2\sigma_{S_{N_T}}^2} \end{bmatrix}, \tag{25}$$

$$\boldsymbol{\Pi} = \mathbf{V}_S^H\Delta\mathbf{Y}_Q^H - \Delta\mathbf{Y}_Q\mathbf{V}_S. \tag{26}$$

Note that σ_{S_i}, $(i = 1, 2, \cdots, N_T)$ is the diagonal element of $\boldsymbol{\Sigma}_S$. Substituting (16) into (26), we obtain

$$\boldsymbol{\Pi} = \mathbf{V}_S^H\Delta\mathbf{R}_{xv,P}\mathbf{U}_S\boldsymbol{\Sigma}_S - \boldsymbol{\Sigma}_S\mathbf{U}_S^H\Delta\mathbf{R}_{xv,P}^H\mathbf{V}_S. \tag{27}$$

As a result, the perturbation error of $\mathbf{Q}$ has been expressed in terms of the perturbation error $\Delta\mathbf{R}_{xv,P}$. In the next section, we will derive the MSE of the ideal WR method by utilizing the expression of $\Delta\mathbf{Q}$ as given by (24).

IV. MSE of the Ideal WR-based Method

The estimation error of the channel matrix $\mathbf{H}$ due to the perturbation error can be written as

$$\Delta\mathbf{H} \triangleq \hat{\mathbf{H}} - \mathbf{H} = \mathbf{W}\Delta\mathbf{Q}^H. \tag{28}$$

Substituting (24) into (28) gives

$$\Delta\mathbf{H} \approx \mathbf{U}_S\boldsymbol{\Sigma}_S\left(\boldsymbol{\Gamma}_Q \circ \boldsymbol{\Pi}^H\right)\mathbf{V}_S^H. \tag{29}$$

Thereby, we have

$$\text{vec}\,(\Delta\mathbf{H}) = \text{vec}\,[\mathbf{V}_S^* \otimes (\mathbf{U}_S\boldsymbol{\Sigma}_S)]\left[\text{vec}\,(\boldsymbol{\Gamma}_Q) \circ \text{vec}\left(\boldsymbol{\Pi}^H\right)\right]. \tag{30}$$

Then, the MSE of the channel estimate can be expressed as

$$\begin{aligned}\text{MSE}_{\text{WR,ideal}} &= \text{Trace}\,\{\text{E}\,\{[\mathbf{V}_S^* \otimes (\mathbf{U}_S\boldsymbol{\Sigma}_S)]\,\{[\text{vec}\,(\boldsymbol{\Gamma}_Q)\\ &\text{vec}^H(\boldsymbol{\Gamma}_Q)] \circ [\text{vec}\left(\boldsymbol{\Pi}^H\right)\text{vec}^H\left(\boldsymbol{\Pi}^H\right)]\}\\ &[\mathbf{V}_S^* \otimes (\mathbf{U}_S\boldsymbol{\Sigma}_S)]^H\}\}. \end{aligned} \tag{31}$$

In the following, we simplify the calculation of $\text{MSE}_{\text{WR,ideal}}$.

It is clear that $\text{MSE}_{\text{WR,ideal}}$ can be rewritten as

$$\begin{aligned}&\text{MSE}_{\text{WR,ideal}} =\\ &\quad\text{Trace}\left\{[\mathbf{V}_S^* \otimes (\mathbf{U}_S\boldsymbol{\Sigma}_S)]\,\boldsymbol{\Upsilon}\,[\mathbf{V}_S^* \otimes (\mathbf{U}_S\boldsymbol{\Sigma}_S)]^H\right\}\end{aligned} \tag{32}$$

where

$$\boldsymbol{\Upsilon} \triangleq \text{E}\left\{\left[\text{vec}\,(\boldsymbol{\Gamma}_Q)\,\text{vec}^H(\boldsymbol{\Gamma}_Q)\right] \circ \left[\text{vec}\left(\boldsymbol{\Pi}^H\right)\text{vec}^H\left(\boldsymbol{\Pi}^H\right)\right]\right\}. \tag{33}$$

By using (27) along with (15), it can be proved that

$$\begin{aligned}&\text{E}\left[\text{vec}\left(\boldsymbol{\Pi}^H\right)\text{vec}^H\left(\boldsymbol{\Pi}^H\right)\right]\\ &= \frac{\sigma_v^2}{K}\left(\boldsymbol{\Sigma}_S^2 \otimes \mathbf{I}_{N_T} + \mathbf{I}_{N_T} \otimes \boldsymbol{\Sigma}_S^2\right). \end{aligned} \tag{34}$$

Utilizing (25) and (34), one can easily verify that $\boldsymbol{\Upsilon}$ is a diagonal matrix with diagonal elements being given by

$$\boldsymbol{\rho}\,(l = i + N_T\,(j-1)) = \frac{\sigma_v^2}{K}\frac{1}{\sigma_{S_i}^2 + \sigma_{S_j}^2} \quad (i, j = 1, 2, \cdots, N_T). \tag{35}$$

Letting $\mathbf{u}_{S_i}$ and $\mathbf{v}_{S_j}$ be the i-th and j-th column vectors of $\mathbf{U}_S$ and $\mathbf{V}_S$, respectively, one can find the l-th column vector of $\mathbf{V}_S^* \otimes (\mathbf{U}_S\boldsymbol{\Sigma}_S)$ as $\sigma_{S_i}\mathbf{v}_{S_j}^* \otimes \mathbf{u}_{S_i}$. Therefore, (32) can be expressed as

$$\begin{aligned}\text{MSE}_{\text{WR,ideal}} = &\frac{\sigma_v^2}{K}\sum_{j=1}^{N_T}\sum_{i=1}^{N_T}\frac{\sigma_{S_i}^2}{\sigma_{S_i}^2 + \sigma_{S_j}^2}\text{Trace}\\ &\left[\left(\mathbf{v}_{S_j}^* \otimes \mathbf{u}_{S_i}\right)\left(\mathbf{v}_{S_j}^* \otimes \mathbf{u}_{S_i}\right)^H\right]. \end{aligned} \tag{36}$$

Since $\text{Trace}\left[\left(\mathbf{v}_{S_j}^* \otimes \mathbf{u}_{S_i}\right)\left(\mathbf{v}_{S_j}^* \otimes \mathbf{u}_{S_i}\right)^H\right] = \left\|\mathbf{v}_{S_j}^* \otimes \mathbf{u}_{S_i}\right\|_F^2 = 1$, from (36), we obtain the closed-form expression of MSE of the ideal WR-based method

$$\text{MSE}_{\text{WR,ideal}} = \frac{\sigma_v^2}{K}\sum_{j=1}^{N_T}\sum_{i=1}^{N_T}\frac{\sigma_{S_i}^2}{\sigma_{S_i}^2 + \sigma_{S_j}^2} = \frac{N_T^2\sigma_v^2}{2K}. \tag{37}$$

V. Simulation Results

We consider a MIMO system with 4 transmit and 8 receive antennas, in which the QPSK modulation is used and a Rayleigh channel, whose elements are i.i.d. complex Gaussian variables with zero mean and unit variance, is assumed. Here, the orthogonal pilots are generated by using the scheme proposed in [12]. Here, the ideal WR-based

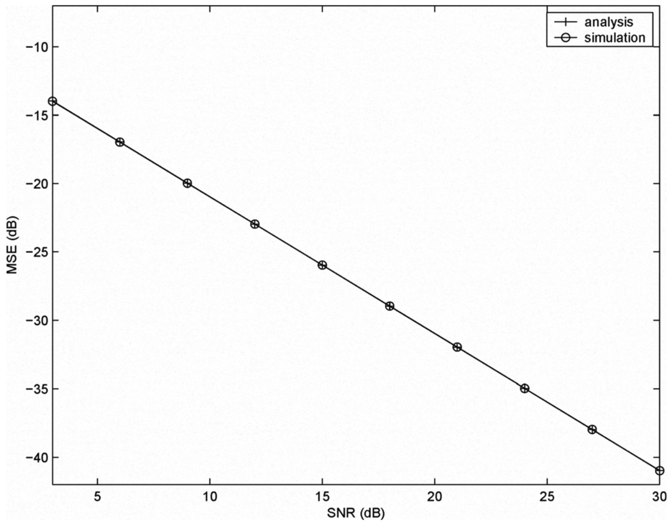

Fig. 1. MSE versus SNR

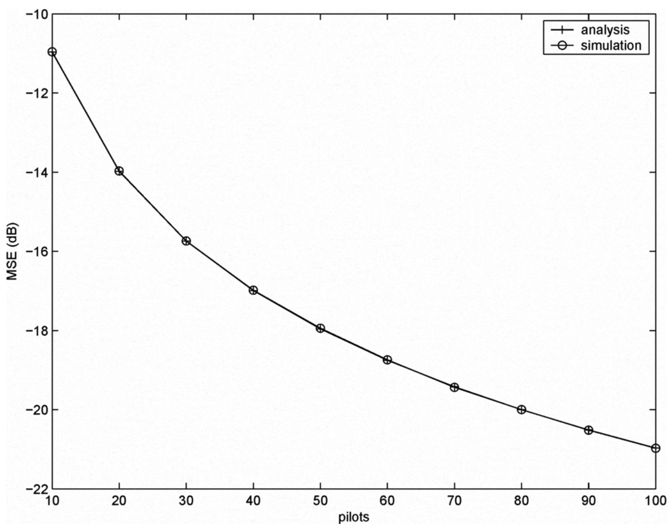

Fig. 2. MSE versus pilot length

method, which obtains the whitening matrix directly from the true channel matrix, is simulated in our experiments. For comparisons, the MSE value calculated from its theoretical MSE expression as given in (37) is also provided.

Experiment 1: *MSE versus SNR*

In the first experiment, the channel estimation performance in terms of the MSE versus the SNR is investigated. The simulation is undertaken based on 20000 Monte Carlo runs of the transmission of one data frame with 1000 slots out of which 100 are used as pilots. Fig. 1 shows the MSE plots of the ideal WR-based method from simulation and analysis, respectively. It is clear the MSE results from simulation are highly consistent with those from analysis at all SNR levels.

Experiment 2: *MSE versus pilot length*

Here, we investigate the channel estimation performance versus the pilot length. Fig. 2 shows the MSE plots from 20000 Monte Carlo runs of the transmission of one data frame of 1000 slots for an SNR of 10 dB. Again, the simulated results are almost the same as the analysis results, confirming the high accuracy of the derivation of the MSE expressions in Section IV.

VI. Conclusions

In this paper, the perturbation analysis is conducted for the WR-based semi-blind MIMO channel estimation method. By using the perturbation analysis, the expression of the perturbation error of the rotation matrix is obtained for the ideal WR-based method, which assumed to have knowledge of the ideal whitening matrix. Then, this expression is used to derive a closed-form expression of MSE of the ideal WR-based method. Simulation results have shown that the theoretical MSE values are consistent with the simulation results, confirming the high accuracy of the derivation of the MSE expression of the ideal WR-based method.

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