Ill Condition Identification for Linear Prediction-based MIMO-OFDM Channel Estimation

Feng Wan, Wei-Ping Zhu and M.N.S. Swamy

Centre for Signal Processing and Communications
Dept. of Electrical and Computer Engineering, Concordia University
Montreal, Quebec, Canada H3G 1M8

***Abstract*— Based on our previously developed signal perturbation free (SPF) transmit scheme, a new linear prediction (LP)-based channel estimation algorithm is proposed for MIMO-OFDM systems. By first using a SPF transmit scheme, the correlation matrix of the received signal without signal perturbation error can be obtained. Then, a linear prediction-based MIMO-OFDM channel estimation algorithm is developed. In particular, the common ill-conditioning problem existing in most LP-based blind methods is properly solved based on two ill-condition test rules. Computer simulations show that the proposed SPF LP-based channel estimation algorithm significantly outperforms the same algorithm without ill condition identification, the nulling-based semi-blind method as well as the LS method.**

I. Introduction

Recent studies have shown that the combination of two powerful technologies, multi-input multi-output (MIMO) and orthogonal frequency division multiplexing (OFDM) is the most promising wireless access scheme for B3G (beyond 3G) systems [1]. The advantages promised by MIMO-OFDM systems rely on the precise knowledge of the channel state information (CSI). The detection of the data and some other signal processing tasks in MIMO-OFDM systems, such as data decoding, space-time processing, limited feedback pre-coding, source and relay power allocation etc, require full or partial knowledge of CSI. It has been proved in [2] that when the channel is Rayleigh fading and perfectly known to the receiver, the capacity of MIMO-OFDM systems grows linearly with the less of transmit and receive antennas. Therefore, channel estimation is of crucial importance in MIMO-OFDM systems.

In general, the MIMO-OFDM channel estimation techniques can be also categorized into three classes, namely, the training-based method, the blind method and the semi-blind one as a combination of the first two methods. First, the training-based methods employ known training signals to render an accurate channel estimation [3]. In contrast to training-based methods, blind MIMO-OFDM channel estimation algorithms, such as those proposed in [4], often use the second-order stationary statistics, correlative coding, or other properties to attain a better spectral efficiency. With a small number of training symbols, semi-blind channel estimation algorithms have been proposed to estimate the channel ambiguity matrix for MIMO-OFDM systems [5].

It is clear that an accurate estimation of the second-order statistics (SOS) of the received signal in the time-domain is essential to blind and semi-blind channel estimation of MIMO-OFDM systems. By using the perturbation theory, our previous study on the SOS-based blind channel estimation in [6] has shown that some conventional SOS-based blind algorithms such as those in [7]–[9] are subject to a signal perturbation error due to the finite data length effect in the calculation of the correlation matrix of the received signal. It means that these algorithms would suffer from a poor performance in the MIMO-OFDM channel estimation if the number of the OFDM symbols is not large enough. In contrast, the semi-blind algorithm proposed in [6] imposes an ideal nulling constraint on the channel matrix in the absence of noise and therefore, gives a better channel estimation performance. In [10], we have proposed for the MIMO-OFDM systems a signal-perturbation-free (SPF) transmit scheme, based on which the signal perturbation error can be cancelled at the receiver. In this paper, by utilizing this SPF transmit scheme, we will develop a linear-prediction-based signal-perturbation-free algorithm for the channel estimation of MIMO-OFDM systems.

Throughout the paper, we adopt the following notations:
T Transpose,
H Complex conjugate transpose,
$\circledast$ circular convolution,
and $\| \|_F$ Frobenius norm.

II. Data Model

Consider a MIMO-OFDM system with the V-BLAST structure, in which there are N_T independent links, each connected to a transmit antenna and containing both pilot and information data. The m-th OFDM symbol in the i_T-th link can be written as a vector of the frequency-domain signals, namely, $\left[X_{i_T}(m,0), X_{i_T}(m,1),, \cdots, \quad X_{i_T}(m,K-1)\right]^T$ where K denotes the number of subcarriers. The IDFT processing gives the time-domain OFDM signal, denoted as $\left[x_{i_T}(m,0), x_{i_T}(m,1), \cdots, x_{i_T}(m,K-1)\right]^T$. After adding a cyclic prefix, each OFDM symbol is then sent out by the corresponding transmit antenna. Suppose there are N_R receive antennas in the MIMO-OFDM receiver. After removing the cyclic prefix in each link, the signal received at the i_R-th antenna can be expressed as $\left[y_{i_R}(m,0), y_{i_R}(m,1), \cdots, y_{i_R}(m,K-1)\right]^T$. Then, the received frequency domain signal after the DFT processing is given by $\left[Y_{i_R}(m,0), Y_{i_R}(m,1), \cdots, Y_{i_R}(m,K-1)\right]^T$.

Most of the existing channel estimation methods work on a discrete-time MIMO-FIR channel, whose element can be characterized as an L-tap FIR filter. Assuming that the channel is constant during a number of consecutive OFDM symbols, the channel matrix for the l-th tap can be written as

$$\mathbf{H}(l) \triangleq \begin{bmatrix} h_{1,1}(l) & h_{1,2}(l) & \ldots & h_{1,N_T}(l) \\ h_{2,1}(l) & h_{2,2}(l) & \ldots & h_{2,N_T}(l) \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R,1}(l) & h_{N_R,2}(l) & \ldots & h_{N_R,N_T}(l) \end{bmatrix} \quad (1)$$

where $h_{i_R,i_T}(l), (0 \le l \le L-1)$ represents the composite channel response between the i_R-th receive antenna and the i_T-th transmit antenna for the l-th tap. If the length of the cyclic prefix is not less than the channel length L, the time-domain signal model can be written as

$$y_{i_R}(m,n) = \sum_{i_T=1}^{N_T} h_{i_R,i_T}(n) \circledast x_{i_T}(m,n) + v_{i_R}(m,n), \quad m \in \{0, \cdots, g-1\} \quad (2)$$

where g is the OFDM block size, i.e., the number of OFDM symbols within which the channel remains unchanged, and $v_{i_R}(m,n) \in \mathcal{C}^{N_R \times 1}$ is a spatio-temporally uncorrelated noise with zero-mean and variance σ_v^2.

III. Linear Prediction-based MIMO-OFDM Channel Estimation

In the following, the index m is omitted for the sake of notational convenience. Define

$$\mathbf{y}(n) \triangleq \left[y_1(n), \cdots, y_{N_R}(n)\right]^T, \quad (3)$$

$$\mathbf{y}_P(n-1) \triangleq \left[\mathbf{y}^T(n-1), \cdots, \mathbf{y}^T(n-P)\right]^T, \quad (4)$$

$$\tilde{\mathbf{R}}_{n-1} \triangleq \mathrm{E}\left\{\mathbf{y}_P(n-1)\,\mathbf{y}_P^H(n-1)\right\}, \quad (5)$$

$$\ddot{\mathbf{R}}_n \triangleq \mathrm{E}\left\{\mathbf{y}(n)\,\mathbf{y}_P^H(n-1)\right\}. \quad (6)$$

The MIMO linear predictor can be written as

$$\mathbf{P}_P \triangleq \left[\mathbf{P}_P(1), \mathbf{P}_P(2), \cdots, \mathbf{P}_P(P)\right] = \ddot{\mathbf{R}}_n \tilde{\mathbf{R}}_{n-1}^{-1} \quad (7)$$

where $\mathbf{P}_P(n), (n = 1, \cdots, P)$, is an $N_R \times N_R$ matrix representing the n-th tap of the prediction filter. The covariance matrix of the prediction error is then given by

$$\boldsymbol{\delta}_{\tilde{\mathbf{y}},P}^2 = \mathbf{R}_y(0) - \mathbf{P}_P \ddot{\mathbf{R}}_n^H \quad (8)$$

where

$$\mathbf{R}_y(0) \triangleq \mathrm{E}\left[\mathbf{y}(n)\,\mathbf{y}^H(n)\right]. \quad (9)$$

Further, define

$$\mathbf{P}_P(z) = \mathbf{I}_{N_R} - \sum_{i=1}^{P} \mathbf{P}_P(i)\, z^{-i},$$

$$\mathbf{H}(z) = \sum_{i=0}^{L-1} \mathbf{H}(i)\, z^{-i}.$$

It can be shown [7], [11] that if the transmitted signals are uncorrelated and moreover, $PN_R \ge (L+P-1)N_T$, we have

$$\mathbf{P}_P(z)\,\mathbf{H}(z) = \mathbf{H}(0), \quad (10)$$

$$\boldsymbol{\delta}_{\tilde{\mathbf{y}},P}^2 = \mathbf{H}(0)\,\mathbf{H}^H(0). \quad (11)$$

Based on (10) and (11), some blind algorithms have been proposed for MIMO channel estimation [7]–[9]. The basic idea is to first acquire an estimate of $\mathbf{H}(0)$ from that of $\boldsymbol{\delta}_{\tilde{\mathbf{y}},P}^2$ according to (11), and then use (10) to obtain an estimate of the channel matrix $\mathbf{H}(z)$.

It was proved in [10] that the LP-based blind channel estimation is subject to a signal perturbation error due to the finite data length in the computation of second-order statistics. Furthermore, a signal-perturbation-free (SPF) transmit scheme that can completely cancel the signal perturbation error at the receiver in the noise-free case was proposed in [10]. Here, we first utilize the SPF transmit scheme to cancel the signal perturbation error. Then, we develop a SPF linear prediction-based method for MIMO-OFDM channel estimation. Following the linear prediction process [9], we can obtain a time-domain expression of (10) as [6]

$$\left[\mathbf{I}_{N_R}, -\mathbf{P}_P\right]\mathbf{H}_D = \left[\mathbf{H}(0), \mathbf{0}, \cdots, \mathbf{0}\right] \quad (12)$$

where $\mathbf{P}_P$ is given by (7) and $\mathbf{H}_D$ is a $(P+1)N_R \times (L+P)N_T$ block Toeplitz matrix with the first block row as

$$\left[\mathbf{H}(0), \cdots, \mathbf{H}(L-1), \mathbf{0}, \cdots, \mathbf{0}\right].$$

Letting

$$\mathbf{H}_F \triangleq \begin{bmatrix} \mathbf{H}(0) \\ \vdots \\ \mathbf{H}(L-1) \end{bmatrix}, \quad (13)$$

$$\mathbf{P}_Q \triangleq \begin{bmatrix} \mathbf{I}_{N_R} & & & \mathbf{0} \\ -\mathbf{P}_P(1) & \ddots & & \\ \vdots & \ddots & \ddots & \\ -\mathbf{P}_P(P) & \vdots & \ddots & \mathbf{I}_{N_R} \\ & \ddots & \vdots & -\mathbf{P}_P(1) \\ & & \ddots & \vdots \\ \mathbf{0} & & & -\mathbf{P}_P(P) \end{bmatrix}, \quad (14)$$

(12) can be rewritten as

$$\mathbf{P}_Q \mathbf{H}_F = \begin{bmatrix} \mathbf{H}(0) \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}. \quad (15)$$

On the other hand, it is clear from (11) that the estimate $\hat{\mathbf{H}}(0)$ can be easily obtained by using a semi-blind technique [12]. Therefore, the MIMO channel matrix can be estimated by using (15).

IV. Identification of the Ill Condition

It should be mentioned that, in the noisy case the blind estimation of the proposed channel estimation algorithm suffers from the common ill-conditioning problem as many other LP-based blind methods like the one proposed in [8]. As mentioned in [8], the ill condition arises from the pseudo-inverse of the matrix $\hat{\tilde{\mathbf{R}}}_{n-1}$ in (5). Performing singular value decomposition (SVD) of $\hat{\tilde{\mathbf{R}}}_{n-1}$ gives

$$\tilde{\mathbf{R}}_{n-1} = \mathbf{U}_{Rn}\mathbf{\Sigma}_{Rn}\mathbf{V}_{Rn}^{H}. \tag{16}$$

Based on (16), the pseudo-inverse of the matrix $\hat{\tilde{\mathbf{R}}}_{n-1}$ can be obtained from

$$\tilde{\mathbf{R}}_{n-1}^{\dagger} = \mathbf{V}_{Rn}\mathbf{\Sigma}_{Rn}^{-1}\mathbf{U}_{Rn}^{H}. \tag{17}$$

Occasionally, $\mathbf{\Sigma}_{Rn}$ is quite ill-conditioned. To solve this problem, a regularized pseudo-inverse

$$\tilde{\mathbf{R}}_{n-1}^{\dagger} = \mathbf{V}_{Rn}\left(\mathbf{\Sigma}_{Rn} + \sigma_{Rn}\mathbf{I}\right)^{-1}\mathbf{U}_{Rn}^{H} \tag{18}$$

with $\sigma_{Rn} > 0$, has been adopted in [8]. However, it has been found through a large amount of computer simulations that in ill-conditioned cases, the performance of the LP-based methods using the regularized scheme is still much worse than the traditional LS method, leading to a poor overall channel estimation performance. Therefore, we suggest replacing the LP-based channel estimation solution with our previously developed nulling-based semi-blind algorithm [6], once the ill-condition case is identified. Thus, what remains is to identify the ill conditions of the proposed signal-perturbation-free LP-based method.

Here, we first give an LS-based criterion to identify the ill condition.

1) Calculate the cost of the LS estimate:

$$\Delta_{\text{LS}} = \left\|\mathbf{Y}_{\text{pilot}} - \tilde{\mathbf{A}}\hat{\mathbf{h}}_{\text{LS}}\right\|_F^2 \tag{19}$$

where $\hat{\mathbf{h}}_{\text{LS}}$ is the LS channel estimate resulting from the method in [3].

2) Calculate the cost of the SPF semi-blind estimate:

$$\Delta_{\text{SB}} = \left\|\mathbf{Y}_{\text{pilot}} - \tilde{\mathbf{A}}\hat{\mathbf{h}}_{\text{SB}}\right\|_F^2 \tag{20}$$

where $\hat{\mathbf{h}}_{\text{SB}}$ is the semi-blind channel estimate, which can be obtained in the previous subsection.

3) Define a coefficient ρ_1 to evaluate the deviation of Δ_{SB} from Δ_{LS} as

$$\rho_1 = \frac{\left|\Delta_{\text{SB}} - \Delta_{\text{LS}}\right|}{\Delta_{\text{LS}}}. \tag{21}$$

4) If ρ_1 is larger than a predetermined threshold τ_1, the SPF semi-blind channel estimate is considered as ill conditioned.

As the above criterion may not cover all the ill-conditioned cases, we suggest another testing to further improve the reliability of the ill condition identification.

1) Obtain a difference matrix of $\boldsymbol{\delta}_{\tilde{\mathbf{y}},P}^{2}$ between the LP-method and the LS method as

$$\mathbf{\Upsilon} = \hat{\delta}_{\tilde{\mathbf{y}},P}^{2} - \hat{\mathbf{H}}_{\text{LS}}\left(0\right)\hat{\mathbf{H}}_{\text{LS}}^{H}\left(0\right) \tag{22}$$

where $\hat{\delta}_{\tilde{\mathbf{y}},P}^{2}$ is an estimate of $\boldsymbol{\delta}_{\tilde{\mathbf{y}},P}^{2}$ given in (8), $\hat{\mathbf{H}}_{\text{LS}}\left(0\right)$ is the first tap of the LS channel estimate.

2) Perform the eigenvalue decomposition on $\mathbf{\Upsilon}$, giving the largest and the second largest eigenvalues as $\sigma_{\Upsilon 1}$ and $\sigma_{\Upsilon 2}$.

3) Define a coefficient ρ_2 as

$$\rho_2 = \sigma_{\Upsilon 1}/\sigma_{\Upsilon 2}. \tag{23}$$

4) If ρ_2 is larger than a predetermined threshold τ_2, the SPF semi-blind channel estimate is considered as ill-conditioned.

We will show in the next section that by using both the above ill-condition test rules, the proposed linear prediction-based algorithm performs very well.

V. Simulation Results

We consider a MIMO-OFDM system with 2 transmit and 4 receive antennas. The number of subcarriers is set to 512, the length of cyclic prefix is 10. In this paper, the QPSK modulation is used and an SUI-3 type MIMO channel is considered. In particular, the channel is modelled as a 3-tap MIMO-FIR filter, in which each tap corresponds to a 2×4 random matrix whose elements are i.i.d. complex Gaussian variables with zero mean and an equal variance. Moreover, the channel has an exponentially decaying profile, giving 0 dB, -5 dB and -10 dB powers for the first, second and third taps, respectively. In our simulation, the pilots and SPF data are transmitted only in the subcarriers indexed by $64 \times k$, $(k = 0, 1, \cdots, 7)$, which, for the convenience, are referred to as the SPF subcarriers.

For the purpose of comparison, the channel vector $\mathbf{h}$ is estimated by the proposed SPF LP-based semi-blind algorithm, the SPF LP-based semi-blind algorithm without ill condition identification, the LS and the nulling-based semi-blind methods. For easy citation, we call these three methods as the SPF LP semi-blind, SPF LP semi-blind without ill condition identification, LS, and nulling semi-blind methods. In the SPF LP semi-blind algorithm, the ill condition is identified if $\rho_1 > 0.5$ or $\rho_2 > 5$ or $\rho_1 \times \rho_2 > 0.2$. The estimation performance is evaluated in terms of the MSE of the estimate of the channel matrix given by

$$\text{MSE} = \frac{1}{N_{\text{MC}}}\sum_{n=1}^{N_{\text{MC}}}\left\|\hat{\mathbf{h}}_n - \mathbf{h}_n\right\|^2$$

where N_{MC} is the number of Monte Carlo iterations, and $\mathbf{h}_n$ and $\hat{\mathbf{h}}_n$ are the true and the estimated channel vectors with respect to the n-th Monte Carlo iteration, respectively.

Experiment 1: *MSE versus SNR*

In the first experiment, the channel estimation performance in terms of the MSE versus the SNR is investigated. The simulation involves 2000 Monte Carlo runs of the transmission of 60 OFDM symbols with pilot length $g_{\text{p}} = 20$. On the

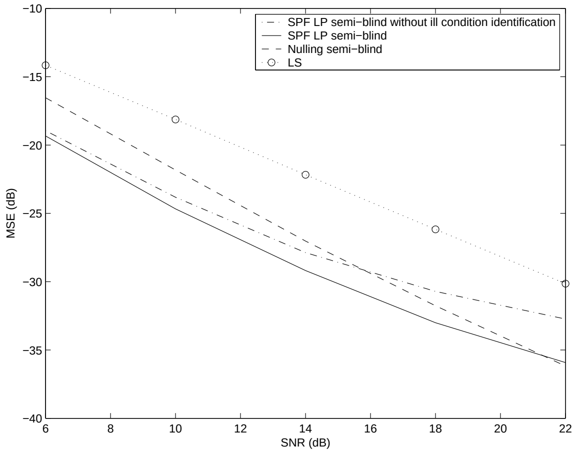

Fig. 1. MSE of the channel estimate versus SNR

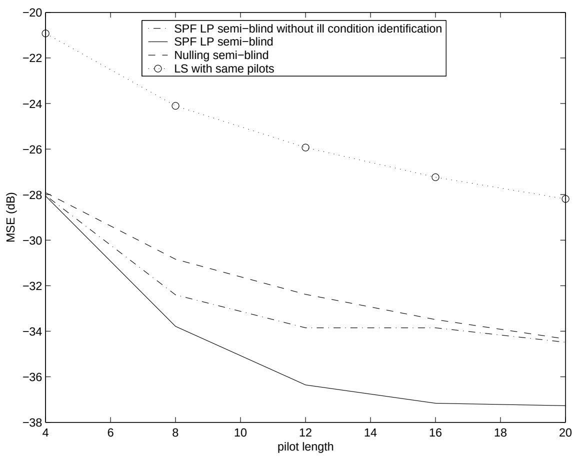

Fig. 2. MSE versus pilot length

average, 5.8 OFDM symbols per SPF subcarrier are used for the transmission of SPF data. Fig. 1 shows the channel estimation results of three semi-blind methods as well as the LS method with 20 pilot symbols. Moreover, the result from the LS method using 60 pilot symbols which is three times the pilot length of other methods is also provided for comparison. It is seen that the SPF LP semi-blind algorithm consistently outperforms the performance of the SPF LP semi-blind method without ill condition identification, the nulling semi-blind method and the LS methods. Also, one can find that the gain of the SPF LP semi-blind method over the version without ill condition identification improves with increasing SNR level, implying the ill condition identification scheme is more advantageous in high SNR levels.

Experiment 2: *MSE versus pilot length*

Here, we investigate the channel estimation performance of the proposed algorithm versus the pilot length. Fig. 2 shows the MSE plots from 500 Monte Carlo iterations for $g = 120$ at an SNR of 15 dB. It is seen that the performance of all the algorithms is improved with increasing pilot length. Again, the SPF LP semi-blind algorithm outperforms the SPF LP semi-blind method without ill condition identification, the nulling semi-blind method as well as the LS method for all pilot lengths. Also, one can find that the SPF LP semi-blind method achieve better performance than the version without ill condition identification by using more pilot, suggesting the ill condition identification scheme is more advantageous for cases with high pilot length.

VI. Conclusions

By using a signal-perturbation-free (SPF) transmit scheme, the correlation matrix of the received signal without signal perturbation error was first obtained. Then, a SPF linear prediction-based MIMO-OFDM channel estimation solution can be derived. In this new channel estimation algorithm, the common ill-conditioning problem in LP-based blind methods is solved. By proposing two ill-condition test rules, the ill-condition cases can be identified, for which our previously developed nulling-based semi-blind algorithm is used. Computer simulations have confirmed that the proposed SPF linear prediction-based channel estimation algorithm significantly outperforms the same algorithm without ill condition identification, the nulling-based semi-blind method and the LS method.

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