An Enhanced Scheme for Second-Order-Statistics Estimation in MIMO-OFDM Systems

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***Abstract*— The second-order statistics (SOS) of the received signal are very often used in blind and semi-blind channel estimation. In this paper, an analysis of signal perturbation in SOS of the received signal is first conducted, revealling that, even in the noise-free case, some SOS-based blind and semi-blind algorithms are subject to a signal perturbation error. Based on the analysis, a very efficient transmit scheme that can completely cancel the signal perturbation error at the receiver in the noise-free case is proposed. Computer simulations show that by employing the proposed signal perturbation cancellation approach, the mean square error (MSE) of the SOS estimate can be sufficiently suppressed in the noisy case.**

I. INTRODUCTION

Recent studies have proved that the combination of two powerful technologies, multi-input multi-output (MIMO) and orthogonal frequency division multiplexing (OFDM) is the most promising wireless access scheme for B3G (beyond 3G) systems [1], [2]. This is because the system capacity can be boosted by employing the MIMO technique without increasing the transmit power and signal bandwidth. Meanwhile, the frequency selective fading in MIMO channels can well be overcome by using OFDM. Therefore, the MIMO-OFDM technology has been considered as a strong candidate for B3G systems.

It is well known that channel estimation is of crucial importance to MIMO-OFDM systems. An accurate estimation of the second-order statistics (SOS) of the received signal in the time-domain is essential to blind and semi-blind channel estimation of MIMO-OFDM systems. By using the perturbation theory [3], [4], our previous study on the SOS-based blind channel estimation in [5] has shown that some conventional SOS-based blind algorithms such as those in [6]–[9] are subject to a signal perturbation error due to the finite data length effect in the calculation of the correlation matrix of the received signal. It means that these algorithms would suffer from a poor performance in the MIMO-OFDM channel estimation if the number of the OFDM symbols is not large enough. In contrast, the semi-blind algorithm proposed in [5] imposes an ideal nulling constraint on the channel matrix in the absence of noise and therefore, gives a better channel estimation performance. In [10], we have proposed for the MIMO systems a signal-perturbation-free (SPF) transmit scheme, based on which the signal perturbation error can be cancelled at the receiver, leading to a signal-perturbation-free semi-blind MIMO channel estimation algorithm. In this paper, we will extend the idea of signal perturbation cancellation to MIMO-OFDM systems. By developing a new signal-perturbation-free transmit scheme, we will show that the signal perturbation error can be cancelled in MIMO-OFDM receivers.

Throughout the paper, we adopt the following notations:
T Transpose,
H Complex conjugate transpose,
$\circledast$ circular convolution,
$\| \ \|_F$ Frobenius norm,
and $\phi(k) = \mathrm{e}^{-j2\pi\frac{k}{K}}$.

II. THE SIGNAL PERTURBATION IN SECOND ORDER STATISTICS (SOS) OF RECEIVED SIGNAL

Consider a MIMO-OFDM system with N_T transmit and N_R receive antennas. The channel can be characterized by an array of L-tap FIR filters given by a number of $N_R \times N_T$ matrices $\mathbf{H}(n)$ $(n = 0, 1, \cdots, L-1)$, whose (i_R, i_T)-th element $h_{i_T,i_R}(n)$ represents the channel response from the i_T-th transmit antenna to the i_R-th receive antenna. Define $x_{i_T}(m,n)$ as the transmit time-domain signal of the m-th OFDM symbol at the i_T-th antenna. If the length of the cyclic is not less than the channel length L, after removing the cyclic prefix at the receiver, the i_R-th received signal can be written as

$$y_{i_R}(m,n) = \sum_{i_T=1}^{N_T} h_{i_R,i_T}(n) \circledast x_{i_T}(m,n) + v_{i_R}(m,n), \quad m \in \{0, \cdots, g-1\} \tag{1}$$

where g is the OFDM block size, i.e., the number of OFDM symbols within which the channel remains unchanged, and $v_{i_R}(m,n) \in \mathcal{C}^{N_R \times 1}$ is a spatio-temporally uncorrelated noise with zero-mean and variance σ_v^2.

An accurate estimation of the second order statistics (SOS) of the received signal in the time-domain, $y_{i_R}(n)$, is essential to blind and semi-blind MIMO-OFDM channel estimation algorithms [5]. For the sake of simplicity, only one OFDM symbol with K subcarriers is considered. By letting

$$\mathbf{y}(n) \triangleq [y_1(n), \cdots, y_{N_R}(n)]^T,$$

$$\mathbf{y}_{P+1}(n) \triangleq \left[\mathbf{y}^T(n), \mathbf{y}^T(n-1), \cdots, \mathbf{y}^T(n-P)\right]^T,$$

the 2nd-order statistics in terms of the correlation matrix $\mathbf{R}_T$ of $\mathbf{y}_{P+1}(n)$ can be estimated by

$$\hat{\mathbf{R}}_{\mathrm{T}} = \frac{1}{K}\sum_{n=0}^{K-1}\mathbf{y}_{P+1}(n)\,\mathbf{y}_{P+1}^H(n)\,. \tag{2}$$

Note that $\mathbf{y}_{P+1}(n)$ for $n = 0, 1, \cdots, P-1$ can be obtained using $\mathbf{y}(n-j) \triangleq \mathbf{y}(K+n-j)$ for $n < j$ due to the circular convolution. It is obvious from (2) that $\hat{\mathbf{R}}_{\mathrm{T}}$ depends on the estimate of the correlation matrices

$$\hat{\mathbf{R}}(l) = \frac{1}{K}\sum_{n=0}^{K-1}\mathbf{y}(n)\mathbf{y}^H(n-l),\ (l = 0, \cdots, P)\,. \tag{3}$$

We now give a brief analysis of the signal perturbation in $\hat{\mathbf{R}}(l)$, $(l = 0, \cdots, P)$, for the noise-free case. Letting

$$\mathbf{H}_A \triangleq [\mathbf{H}(0), \mathbf{H}(1), \cdots, \mathbf{H}(L-1)]$$

$$\mathbf{x}(n) \triangleq [x_1(n), \cdots, x_{N_T}(n)]^T$$

$$\mathbf{x}_L(n) \triangleq \left[\mathbf{x}^T(n) \cdots \mathbf{x}^T(n-L+1)\right]^T, (n = 0, 1, \cdots, K-1)$$

where $\mathbf{x}(n) = \mathbf{x}(K+n)$ for $n < 0$, the circular convolution (1) can be rewritten in the matrix form as

$$\mathbf{y}(n) = \mathbf{H}_A\mathbf{x}_L(n)\,. \tag{4}$$

Using (4), the estimate of $\mathbf{R}_y(l)$ can be expressed as

$$\hat{\mathbf{R}}_y(l) = \mathbf{H}_A\hat{\mathbf{R}}_{x,L}(l)\,\mathbf{H}_A^H \tag{5}$$

where

$$\hat{\mathbf{R}}_{x,L}(l) \triangleq \frac{1}{K}\sum_{n=0}^{K-1}\mathbf{x}_L(n)\,\mathbf{x}_L^H(n-l)\,.$$

It is now clear that $\hat{\mathbf{R}}(l)$, $(l = 0, \cdots, P)$ is determined by the channel matrix $\mathbf{H}_A$ and the estimate of the correlation matrix of the transmitted signal,

$$\hat{\mathbf{R}}_x(l) \triangleq \frac{1}{K}\sum_{n=0}^{K-1}\mathbf{x}(n)\,\mathbf{x}^H(n-l) \tag{6}$$

where $l \in [1-L, P+L-1]$. It has been proved in [5] that, when the transmitted frequency-domain signal can be considered as an i.i.d. Gaussian process with zero mean and unit variance, the transmitted time-domain MIMO-OFDM signal is uncorrelated, i.e., $\mathbf{R}_x(l) = \delta(l)\,\mathbf{I}_{N_T}$. Thus, (6) can be rewritten as

$$\hat{\mathbf{R}}_x(l) = \mathbf{R}_x(l) + \Delta\mathbf{R}_x(l) \tag{7}$$

where $\Delta\mathbf{R}_x(l)$ is the perturbation term of $\mathbf{R}_x(l)$ as given by

$$\Delta\mathbf{R}_x(l) \triangleq \frac{1}{K}\sum_{n=0}^{K-1}\mathbf{x}(n)\,\mathbf{x}^H(n-l) - \delta(l)\,\mathbf{I}_{N_T}. \tag{8}$$

Note that, when multiple OFDM symbols are used, $\hat{\mathbf{R}}_x(l)$ as well as $\hat{\mathbf{R}}_{x,L}(l)$ can be easily calculated by averaging the results obtained from each OFDM symbol.

It is obvious that the existence of the signal perturbation terms $\Delta\mathbf{R}_x(l)$ may in general introduce the perturbation error to the SOS-based blind or semi-blind algorithms. By conducting a perturbation analysis, it has been shown in [5] that, even in the noise-free case, the linear prediction-based blind channel estimation algorithms are subject to a signal perturbation error. In order to improve the performance of these algorithms, in the next section, we will propose an efficient signal-perturbation-free transmit scheme to cancel the signal perturbation error in the estimation of $\hat{\mathbf{R}}(l)$ for MIMO-OFDM systems. Our idea is to send information of the signal perturbation matrix $\Delta\mathbf{R}_x(l)$ to the receiver. The received version of this information will be then exploited to cancel the signal perturbation error.

III. THE SIGNAL PERTURBATION CANCELLATION SCHEME

Let us consider the computation of the signal perturbation matrix $\Delta\mathbf{R}_x(l)$ at the transmitter by using the frequency-domain data. Define the transmitted frequency-domain signal vector at the k-th subcarrier as

$$\mathbf{X}(k) \triangleq [X_1(k), \cdots, X_{N_T}(k)]^T\,. \tag{9}$$

It can proved that the estimated correlation matrix of the time-domain signal $\mathbf{x}(n)$, $\hat{\mathbf{R}}_x(l)$, can be calculated as

$$\hat{\mathbf{R}}_x(l) = \frac{1}{K}\sum_{k=0}^{K-1}\mathbf{X}(k)\,\mathbf{X}^H(k)\,\phi^{-l}(k) \tag{10}$$

which, using (8), leads to

$$\Delta\mathbf{R}_x(l) = \frac{1}{K}\sum_{k=0}^{K-1}\mathbf{X}(k)\,\mathbf{X}^H(k)\,\phi^{-l}(k) - \delta(l)\,\mathbf{I}_{N_T}. \tag{11}$$

Interestingly, $\hat{\mathbf{R}}_x(l)$ given by (10) can be regarded as the IDFT of $\mathbf{X}(k)\,\mathbf{X}^H(k)$. Accordingly, the estimate of the correlation matrix of the received signal $\mathbf{y}(n)$, $\hat{\mathbf{R}}_y(l)$, can be represented as an IDFT of $\mathbf{Y}(k)\,\mathbf{Y}^H(k)$, where $\mathbf{Y}(k)$ is the frequency-domain version of $\mathbf{y}(n)$. In order to develop a new transmit scheme to cancel the perturbation error $\Delta\mathbf{R}_y(l) = \hat{\mathbf{R}}_y(l) - \mathbf{R}_y(l)$ due to $\Delta\mathbf{R}_x(l)$, we would like to express $\Delta\mathbf{R}_x(l)$ as an IDFT of a set of data $\mathbf{T}(k)$ for $-L_1 \leq l \leq L_2$, where $L_1 = L-1, L_2 = P+L-1$. As such, the size of the IDFT should be at least $K_T = L_1 + L_2 + 1$. Considering that the total number of subcarriers usually satisfies $K \gg L_1 + L_2 + 1$, it would be sufficient and convenient to choose $K_T = \frac{K}{M} \geq L_1 + L_2 + 1$ where M is the largest possible integer. By using the periodicity property of IDFT, we have

$$\Delta\mathbf{R}_x(l) = \begin{cases} \sum\limits_{k=0}^{K_T-1}\mathbf{T}(k)\,\mathrm{e}^{j2\pi kl/K_T}, \\ \qquad (l = 0, 1, \cdots L_2) \\ \sum\limits_{k=0}^{K_T-1}\mathbf{T}(k)\,\mathrm{e}^{j2\pi k(K_T+l)/K_T}, \\ \qquad (l = -L_1, 1-L_1, \cdots -1) \end{cases}. \tag{12}$$

Clearly, (12) gives a K_T-size IDFT of $\mathbf{T}(k)$ with a gain of K_T. Thus, $\mathbf{T}(k)$ can be calculated as a K_T-size DFT of

$\Delta\mathbf{R}_x(l)$ with a gain of $1/K_T$,

$$\mathbf{T}(k) = \frac{1}{K_T}\left[\sum_{l=0}^{L_2} \Delta\mathbf{R}_x(l)\,\mathrm{e}^{-j2\pi kl/K_T} + \sum_{l=K_T-L_1}^{K_T-1} \Delta\mathbf{R}_x(l-K_T)\,\mathrm{e}^{-j2\pi kl/K_T}\right] (k=0,1,\cdots,K_T-1). \quad (13)$$

In what follows, we show that as long as the $N_T \times N_T$ matrix $\mathbf{T}(k)$ can be factorized into

$$\mathbf{T}(k) = \mathbf{T}_{\mathrm{L}}(k)\,\mathbf{T}_{\mathrm{R}}^H(k) \quad (14)$$

and $\mathbf{T}_{\mathrm{L}}(k)$ and $\mathbf{T}_{\mathrm{R}}^H(k)$ are transmitted to the receiver, the signal perturbation error in $\hat{\mathbf{R}}_y(l)$ can be eliminated in the absence of noise. Noting that $\mathrm{e}^{j2\pi k(l+K_T)/K_T} = \mathrm{e}^{j2\pi kl/K_T} = \mathrm{e}^{j2\pi kMl/K}$, (12) can be rewritten as

$$\Delta\mathbf{R}_x(l) = \sum_{k=0}^{K_T-1} \mathbf{T}(k)\,\phi^{-Ml}(k),\ (l = -L_1, -L_1+1, \cdots L_2). \quad (15)$$

Interestingly, (15) corresponds to an M-rate decimated version of the K-size DFT of $\mathbf{T}'(k)$, where $\mathbf{T}'(k)$ is an up-sampled version of $\mathbf{T}(k)$ by a factor of M, i.e., it can easily be obtained by inserting $M-1$ zero matrices following each $\mathbf{T}(k)$. This observation gives us an idea that $\mathbf{T}_{\mathrm{L}}(k)$ and $\mathbf{T}_{\mathrm{R}}(k)$ should be transmitted over the kM-th subcarriers only $(k = 0, 1, \cdots, K_T - 1)$.

On the one hand, one can prove that the estimated correlation matrix of the received signal can be expressed in terms of the channel matrix and the estimated correlation matrix of the transmitted signal as

$$\hat{\mathbf{R}}_y(l) = \sum_{l_1=0}^{L-1}\sum_{l_2=0}^{L-1} \mathbf{H}(l_1)\,\hat{\mathbf{R}}_x(l-l_1+l_2)\,\mathbf{H}^H(l_2). \quad (16)$$

On the other hand, by using the received noise-free version of the user specific data $\mathbf{T}_{\mathrm{L}}(k)$ and $\mathbf{T}_{\mathrm{R}}(k)$, denoted as

$$\mathbf{Y}_{\mathrm{TL}}(k) = \mathbf{H}_{\mathrm{F}}(kM)\,\mathbf{T}_{\mathrm{L}}(k), \quad (17)$$

$$\mathbf{Y}_{\mathrm{TR}}(k) = \mathbf{H}_{\mathrm{F}}(kM)\,\mathbf{T}_{\mathrm{R}}(k), \quad (18)$$

where $\mathbf{H}_{\mathrm{F}}(k),\ (k = 0, 1, \cdots, K-1)$ is the frequency-domain channel matrix, we can construct matrices $\mathbf{R}_{\mathrm{YT}}(l),\ (l = 0, 1, \cdots, L-1)$ as

$$\mathbf{R}_{\mathrm{YT}}(l) = \sum_{k=0}^{K_T-1} \mathbf{Y}_{\mathrm{TL}}(k)\,\mathbf{Y}_{\mathrm{TR}}^H(k)\,\phi^{-Ml}(k). \quad (19)$$

Using (17) and (18) into (19) yields

$$\mathbf{R}_{\mathrm{YT}}(l) = \sum_{l_1=0}^{L-1}\sum_{l_2=0}^{L-1} \mathbf{H}(l_1) \sum_{k=0}^{K_T-1} \left[\mathbf{T}_{\mathrm{L}}(k)\,\mathbf{T}_{\mathrm{R}}^H(k)\,\phi^{-M(l-l_1+l_2)}(k)\right] \mathbf{H}^H(l_2). \quad (20)$$

By using (14) and (15), (20) can be written as

$$\mathbf{R}_{\mathrm{YT}}(l) = \sum_{l_1=0}^{L-1}\sum_{l_2=0}^{L-1} \mathbf{H}(l_1)\,\Delta\mathbf{R}_x(l-l_1+l_2)\,\mathbf{H}^H(l_2). \quad (21)$$

From (16) and (21), and noting that $\Delta\mathbf{R}_x(l) = \hat{\mathbf{R}}_x(l) - \mathbf{R}_x(l)$ and $\Delta\mathbf{R}_y(l) = \hat{\mathbf{R}}_y(l) - \mathbf{R}_y(l)$, it is now clear that $\mathbf{R}_{\mathrm{YT}}(l)$ gives exactly the signal perturbation error $\Delta\mathbf{R}_y(l)$. Therefore, the correlation matrix of the received signal without the signal perturbation error can be calculated by

$$\hat{\mathbf{R}}_y'(l) = \hat{\mathbf{R}}_y(l) - \mathbf{R}_{\mathrm{YT}}(l) \quad (22)$$

$$= \sum_{l_1=0}^{L-1}\sum_{l_2=0}^{L-1} \mathbf{H}(l_1)\,\mathbf{R}_x(l-l_1+l_2)\,\mathbf{H}^H(l_2) = \mathbf{R}_y(l). \quad (23)$$

The above discussion shows that via the transmission of $\mathbf{T}_{\mathrm{L}}(k)$ and $\mathbf{T}_{\mathrm{R}}(k)$, the signal perturbation error in the receiver has been completely eliminated in the noise free case.

In the end, we give a brief discussion on the construction of $\mathbf{T}_{\mathrm{L}}(k)$ and $\mathbf{T}_{\mathrm{R}}(k)$ for the implementation of the new transmit scheme. Our idea is to obtain $\mathbf{T}_{\mathrm{L}}(k)$ and $\mathbf{T}_{\mathrm{R}}(k)$ by using the singular value decomposition (SVD) technique. Performing the SVD on $\mathbf{T}(k)$ gives

$$\mathbf{T}(k) = \mathbf{U}_T(k)\,\mathbf{\Sigma}_T(k)\,\mathbf{V}_T^H(k) \quad (24)$$

where $\mathbf{U}_T(k) = [\mathbf{u}_{T,1}(k), \mathbf{u}_{T,2}(k), \cdots, \mathbf{u}_{T,N_T}(k)]$, $\mathbf{V}_T(k) = [\mathbf{v}_{T,1}(k), \mathbf{v}_{T,2}(k), \cdots, \mathbf{v}_{T,N_T}(k)]$, and $\mathbf{\Sigma}_T(k)$ is a diagonal matrix conposed of the singular values $\sigma_{T,i}(k),\ (i = 1, 2, \cdots, N_T)$ of $\mathbf{T}(k)$. Following a similar manner in the derivation of the signal-perturbation-free transmit structure for the MIMO systems in our previous work [10], the matrices $\mathbf{T}_{\mathrm{L}}(k)$ and $\mathbf{T}_{\mathrm{R}}(k)$ can be constructed using the singular values $\sigma_{T,i}(k)$ and the singular vectors $\mathbf{u}_{T,i}(k)$ and $\mathbf{v}_{T,i}(k)$.

IV. SIMULATION RESULTS

We consider a MIMO-OFDM system with 2 transmit and 4 receive antennas. The number of subcarriers is set to 512. In this paper, the QPSK modulation is used and an SUI-3 type MIMO channel is considered. In particular, the channel is modelled as a 3-tap MIMO-FIR filter, in which each tap corresponds to a 2×4 random matrix whose elements are i.i.d. complex Gaussian variables with zero mean and an equal variance. Moreover, the channel has an exponentially decaying profile, giving 0 dB, -5 dB and -10 dB powers for the first, second and third taps, respectively. In our simulation, M, P and K_{T} are set to be 64, 3 and 8, respectively, suggesting that the SPF data are transmitted only in the subcarriers indexed by $64 \times k$, $(k = 0, 1, \cdots, 7)$, which, for the convenience, are referred to as the SPF subcarriers.

The estimation performance is evaluated in terms of the MSE of the estimate of the correlation matrix given by

$$\mathrm{MSE} = \sum_{n=1}^{N_{\mathrm{MC}}}\sum_{l=0}^{P} \left\|\hat{\mathbf{R}}_n(l) - \mathbf{R}_n(l)\right\|_F^2$$

where N_{MC} is the number of Monte Carlo iterations, and $\hat{\mathbf{R}}_n(l)$ and $\mathbf{R}_n(l)$ are the estimated correlation matrix and the ideal correlation matrix with respect to the n-th Monte Carlo iteration, respectively. For easy citation, we call the correlation matrix estimation method with the proposed signal

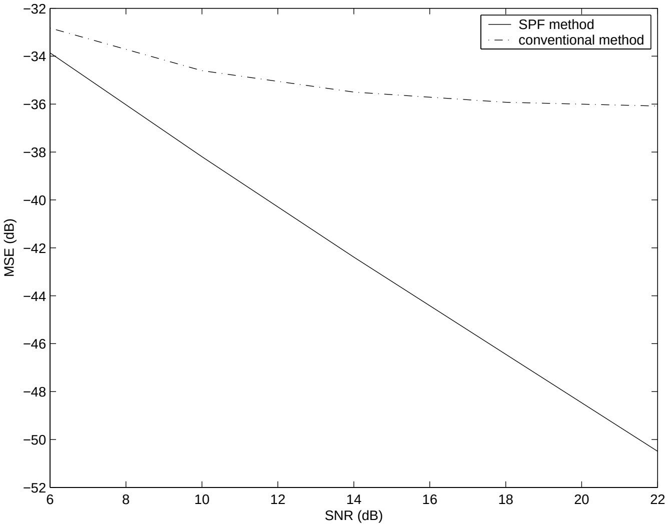

Fig. 1. MSE of the estimate of the correlation matrix of the received signal versus SNR

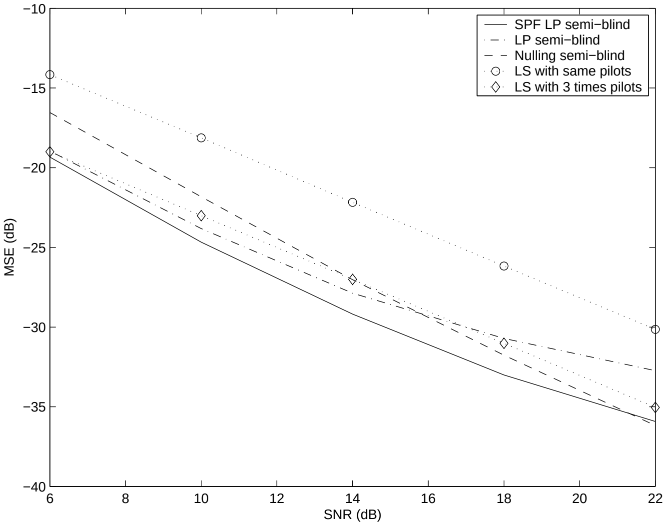

Fig. 2. MSE of the channel estimate versus SNR

perturbation cancellation algorithm as the SPF method and the method without using the proposed algorithm as the conventional method.

In the experiment, the channel estimation performance in terms of the MSE versus the SNR is investigated. The simulation involves 2000 Monte Carlo runs of the transmission of 60 OFDM symbols with pilot length $g_{\mathrm{p}} = 20$. Here, on the average, 5.8 OFDM symbols per SPF subcarrier are used for the transmission of $\mathbf{T}_{\mathrm{L}}(k)$ and $\mathbf{T}_{\mathrm{R}}(k)$. First of all, Fig. 1 shows the MSE of the estimate of the correlation matrices of the received signal from N_{MC} Monte Carlo iterations, by using the norm of the error correlation matrix. Clearly, the conventional correlation matrix estimation without using the proposed SPF cancellation scheme achieves very little gain in the MSE with increasing the SNR level. In contrast, by using the SPF cancellation scheme, the MSE of the estimated correlation matrix has been significantly improved, which is linearly propertional to the increase of the SNR. Fig. 2 shows the channel estimation results of the SPF LP semi-blind, LP semi-blind, nulling semi-blind methods as well as the LS method with 20 pilot symbols. Moreover, the result from the LS method using 60 pilot symbols which is three times the pilot length of other methods is also provided for comparison. It is seen that the SPF LP semi-blind algorithm consistently outperforms the nulling semi-blind method and the LS method. Also, one can find that the performance gain of the SPF LP semi-blind algorithm over the LP semi-blind algorithm becomes larger with increasing SNR value. In particular, the MSE is improved by 3.2 dB when the SNR is 22 dB.

V. CONCLUSIONS

A signal perturbation cancellation algorithm has been proposed to improve the estimation performance of the SOS in MIMO-OFDM systems. An analysis of the signal perturbation in the correlation matrix of the received signal was first conducted. Based on it, a new scheme that transmits user specific data bearing the information of the correlation matrix of the information signal is proposed to cancel the signal perturbation error at the receiver. Simulation results have confirmed that, by using a small number of additional slots for the transmission of the user specific signal-perturbation-free data, the new approach is capable of efficiently suppressing the signal perturbation error.

REFERENCES

[1] G. Stuber, J. R. Barry, S. W. Mclaughlin, Y. Li, M. A. Ingram, and T. G. Pratt, "Broadband MIMO-OFDM wireless communications," *Proceedings of the IEEE*, vol. 92, no. 2, pp. 271–294, 2004.

[2] A. J. Paulraj, D. A. Gore, R. U. Nabar, and H. Bolcskei, "An overview of MIMO communications-a key to gigabit wireless," *Proceedings of the IEEE*, vol. 92, no. 2, pp. 198–218, 2004.

[3] F. Li, H. Liu, and R. J. Vaccaro, "Performance analysis for DOA estimation algorithms: unification, simplification, and observations," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 29, no. 4, pp. 1170–1184, 1993.

[4] Z. Xu, "Perturbation analysis for subspace decomposition with applications in subspace-based algorithms," *IEEE Trans. on Signal Processing*, vol. 50, no. 11, pp. 2820–2830, 2002.

[5] F. Wan, W.-P. Zhu, and M. N. S. Swamy, "A semi-blind channel estimation approach for MIMO-OFDM systems," *IEEE Trans. on Signal Processing*, vol. 56, no. 7, pp. 2821–2834, 2008.

[6] A. Gorokhov and P. Loubaton, "Blind identification of MIMO-FIR systems: A generalized linear prediction approach," *Signal Processing*, vol. 73 (1-2), pp. 105–124, 1999.

[7] S. Houcke and P. Loubaton, "A weighted linear prediction approach for the blind CDMA forward link channel estimation," in *IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 5, 2000, pp. 2937–2940.

[8] J. K. Tugnait and B. Huang, "Multistep linear predictors-based blind identification and equalization of multiple-input multiple-output channels," *IEEE Trans. on Signal Processing*, vol. 48, no. 1, pp. 26–38, 2000.

[9] T. Chow, B. Wang, and K. Ng, "Linear prediction based multipath channel identification algorithm," *IEEE Trans. on Circuits and Systems I: Regular Papers*, vol. 50, no. 6, pp. 769–774, 2003.

[10] F. Wan, W.-P. Zhu, and M. N. S. Swamy, "A signal perturbation free transmit scheme for MIMO channel estimation," in *IEEE Vehicular Technology Conference Fall*, 2008.