Semi-Blind Channel Estimation of MIMO-OFDM Systems with Pulse Shaping

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Abstract— **Most of the existing MIMO-OFDM channel estimation methods do not take into account the effect of the pulse-shaping filter in the transmitter nor of the matched filter in the receiver, thus leading to an estimation solution for the composite channel including the pulse-shaping and matched filters, instead of for the pure wireless channel. This solution is neither directly applicable to practical communication systems nor sufficiently accurate due to the extra length of the composite channel induced by the two filters. In this paper, a semi-blind channel estimation method is proposed for pulse-shaped MIMO-OFDM systems. By utilizing the knowledge of pulse-shaping and matched filters, a time domain semi-blind estimation method is developed for the pure multi-path channel. In order to reduce the computational burden of the time-domain algorithm, a frequency-domain alternative is derived. A number of computer simulation-based experimentations are conducted, and these simulations confirm the effectiveness of the proposed semi-blind method.**

I. Introduction

Mobile wireless communication is coming to a new era with promises of higher data rate, integrated multimedia services and wide internet accessibility. Due to the distinct advantages of both multiple-input multiple-output (MIMO) and orthogonal frequency division multiplexing (OFDM), MIMO-OFDM as a combination of both technologies has been considered as a strong candidate for future wireless communication systems. When perfect knowledge of the wireless channel is available at the receiver, the capacity of MIMO-OFDM systems grows linearly with the number of transmit or receive antennas, whichever is less. In real wireless environments, however, the channel condition is not known. Therefore, channel estimation is required in MIMO-OFDM systems.

Broadly speaking, MIMO-OFDM channel estimation approaches can be categorized into three classes, training-based method, blind method and semi-blind approach. First, training-based methods, such as the least-square (LS) and the minimum mean square error (MMSE) methods, employ known training signals to render an accurate channel estimation. Blind MIMO-OFDM channel estimation algorithms which exploit the second-order stationary statistics or other properties, normally have a better spectral efficiency. Combining the advantages of both the training-based and the blind algorithms, a semi-blind MIMO channel estimation technique has been proposed in [1] and then extended to MIMO-OFDM systems [2], [3]. With a small number of training pilots, problems such as ambiguities and mis-convergence of the blind methods can be solved. On the other hand, the use of statistical information contained in the available data can improve the accuracy of channel estimation.

It is well known that the pulse-shaping filter as well as the matched filter are very commonly used in digital communication systems. Perhaps for the sake of simplicity, however, many existing channel estimation methods did not take into consideration either the effect of the pulse-shaping filter in the transmitter or the matched filter in the receiver. As such, these methods have actually been developed for the estimation of the composite channel including the pulse-shaping and matched filters. Considering that both filters are known to the receiver and the only unknown part is the pure multipath channel [4], ignoring their existence would lead to less accurate estimation results. By utilizing the information of both filters, some improved channel estimation algorithms have been obtained for OFDM systems [5], [6] and CDMA systems [7], [8].

In this paper, we propose for MIMO-OFDM systems a new semi-blind channel estimation algorithm that can eliminate the effect of pulse-shaping and matched filtering, and thus give a better channel estimation performance.

Throughout the paper, we adopt the following notations:
$\dagger$ Pseudo-inverse, $\otimes$ Kronecker product,
T Transpose, H Complex conjugate transpose,
$*$ linear convolution, $\circledast$ circular convolution,
$\| \, \|_F$ Frobenius norm, and
$\mathrm{vec}\,()$ a stacking of the columns of the involved matrix into a vector.

II. Data Model

Consider a MIMO-OFDM system with N_T transmit and N_R receive antennas. The frequency-selective channel can be considered as a combination of L_c multi-paths, namely,

$$\mathbf{H}_c\,(t) = \sum_{l=0}^{L_c-1} \mathbf{\Gamma}_l \delta\,(t - t_l)$$

where t_l is the delay of the l-th path and $\mathbf{\Gamma}_l$ is an $N_R \times N_T$ attenuation matrix. Assuming the transmit pulse-shaping filter $g_t\,(t)$ and the receive matched filter $g_r\,(t)$, the entire or composite channel can be represented by an $N_R \times N_T$ matrix $\mathbf{H}\,(t)$, with its (i_R, i_T)-th element as

$$h_{i_R,i_T}\,(t) = h_{i_R,i_T,c}\,(t) * g_t\,(t) * g_r\,(t) \qquad (1)$$

where $h_{i_R,i_T,c}(t)$ is the (i_R, i_T)-th element of $\mathbf{H}_c(t)$. Most of the existing channel estimation literatures focus on the entire discrete-time channel, i.e. the sampled version of the continuous-time channel response. Thus, the channel can be regarded as an array of L-tap FIR filters characterized by a number of $N_R \times N_T$ matrices $\mathbf{H}(n)\,(n = 0, 1, \cdots, L-1)$ with $h_{i_T,i_R}(n)$ being its (i_R, i_T)-th element. The time-domain signal received at the i_R-th antenna can be written as:

$$y_{i_R}(n) = \sum_{i_T=1}^{N_T} h_{i_R,i_T}(n) \circledast x_{i_T}(n) + v_{i_R}(n) \tag{2}$$

where $x_{i_T}(n)$ is the transmit time-domain signal at the i_T-th antenna and the noise $v_{i_R}(n) \in \mathcal{C}^{N_R \times 1}$ is a spatio-temporally uncorrelated noise with zero mean and variance δ_v^2.

III. Proposed Semi-Blind Channel Estimation Algorithm

In this section, we utilize the knowledge of both the pulse-shaping filter and the matched filter to obtain an improved version of our semi-blind MIMO-OFDM channel estimation approach previously developed in [2], [3].

A. Brief Overview of Linear Prediction based Semi-Blind MIMO-OFDM Channel Estimation

Based on the second-order statistics, the MIMO linear prediction method can be employed to estimate a blind constraint $\hat{\mathbf{B}}$ for the channel vector $\mathbf{h} \triangleq \mathrm{vec}\left[\mathbf{h}_1, \cdots, \mathbf{h}_{N_R}\right]$ [2], [3], where

$$\mathbf{h}_{i_R} \triangleq \left[h_{i_R,1}(0), \cdots, h_{i_R,1}(L-1), \cdots, h_{i_R,N_T}(L-1)\right]^T.$$

By combining the blind constraint with a training-based LS criterion, a semi-blind cost function for the channel estimation can be formulated as

$$\min_{\hat{\mathbf{h}}} \Delta = \left\|\mathbf{Y}_{\text{pilot}} - \tilde{\mathbf{A}}\hat{\mathbf{h}}\right\|_F^2 + \alpha\left\|\hat{\mathbf{B}}\hat{\mathbf{h}}\right\|_F^2 \tag{3}$$

where $\tilde{\mathbf{A}}$ is a pilot signal matrix, $\mathbf{Y}_{\text{pilot}}$ is the corresponding signal vector and $\alpha > 0$ is a weighting factor. The solution to this optimization problem is given by

$$\hat{\mathbf{h}} = \left(\tilde{\mathbf{A}}^H\tilde{\mathbf{A}} + \alpha\hat{\mathbf{B}}^H\hat{\mathbf{B}}\right)^{\dagger}\tilde{\mathbf{A}}^H\mathbf{Y}_{\text{pilot}}. \tag{4}$$

B. The Semi-Blind Algorithm with Pulse-Shaping

As the pulse-shaping filter and the matched filter are normally pre-determined in a communication system, their knowledge can be exploited to improve the channel estimation accuracy. The most commonly used pulse-shaping filter in communication systems has the following raised-cosine impulse response [9]

$$g(t) = \mathrm{sinc}\left(\frac{\pi t}{T}\right)\frac{\cos\left(\frac{\beta\pi t}{T}\right)}{1-\left(\frac{2\beta t}{T}\right)^2}$$

where β is the roll-off factor and T the symbol period. In practical systems, the pulse-shaping filter is often realized by an up-sampled raised-cosine FIR filter. Thus, the composite

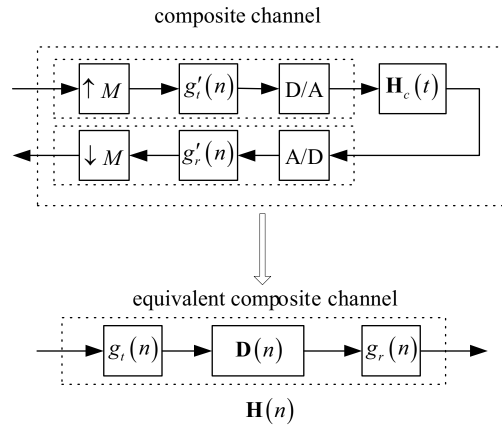

Fig. 1. Discrete-time channel model with pulse shaping

channel model should include the pulse-shaping filter, the analog multi-path channel $\mathbf{H}_c(t)$ and the matched filter as shown in Fig. 1. In this model, an upsampling is usually implemented by inserting $M-1$ zeros between the consecutive input samples prior to pulse-shaping. The transmit filter $g_t(t)$ and the receive filter $g_r(t)$ in (1) are then replaced by two root raised-cosine FIR filters $g_t'(n)$ and $g_r'(n)$, whose sampling period is $\frac{T}{M}$. A discrete-time model equivalent to the composite MIMO-OFDM channel can be formulated as an $N_R \times N_T$ matrix $\mathbf{H}(n)$, whose (i_R, i_T)-th element is given by

$$h_{i_R,i_T}(n) = g_t(n) * d_{i_R,i_T}(n) * g_r(n) \tag{5}$$

where $g_t(n) = g_t'(Mn)$, $g_r(n) = g_r'(Mn)$ and $d_{i_R,i_T}(n)$ is the (i_R, i_T)-th element of the equivalent multi-path channel matrix $\mathbf{D}(n)$. It should be mentioned that a common assumption used in many existing algorithms is $g_t(n)*g_r(n) = \delta(n)$, which implies $\mathbf{H}(n) = \mathbf{D}(n)$. In this sense, therefore, the pulse-shaping effect has been neglected by some researchers. However, this assumption is not true in practical systems. Letting $g(n) \triangleq g_t(n) * g_r(n)$, (5) can be written as

$$h_{i_R,i_T}(n) = g(n) * d_{i_R,i_T}(n) \tag{6}$$

with $g(n) = 0,\ n \notin [0, L_g - 1]$, $d_{i_R,i_T}(n) = 0, n \notin [0, L_d - 1]$, and $L = L_g + L_d - 1$. In what follows, we will improve the semi-blind algorithm proposed in [2], [3] by using (6), namely, we will estimate $\mathbf{D}(n)$, instead of the large-dimensional matrix $\mathbf{H}(n)$, with the information of $g(n)$. Since the number of channel parameters has been considerably decreased, the estimation performance of the new approach is expected to be much better than that of those focusing only on the estimation of the composite channel $\mathbf{H}(n)$. The idea of the new semi-blind algorithm is presented below.

(i) *A time-domain solution*

Using (6), the channel link between the i_R-th receive antenna and i_T-th transmit antenna can be described as the following vector,

$$\begin{aligned}\mathbf{h}_{i_R,i_T} &\triangleq \left[h_{i_R,i_T}(0), \cdots, h_{i_R,i_T}(L-1)\right]^T & (7)\\ &= \mathbf{\Lambda}\mathbf{d}_{i_R,i_T} & (8)\end{aligned}$$

where $\mathbf{\Lambda}$ is an $L \times L_d$ circulant matrix with its first column given by $\left[g(0), \cdots, g(L_g), \mathbf{0}_{1\times(L-L_g)}\right]^T$ and $\mathbf{d}_{i_R,i_T} \triangleq [d_{i_R,i_T}(0), \cdots, d_{i_R,i_T}(L_d-1)]^T$. From (8), the i_R-th partition of the composite channel vector $\mathbf{h}$ can be written as

$$\begin{aligned} \mathbf{h}_{i_R} &\triangleq \left[\mathbf{h}_{i_R,1}^T, \cdots, \mathbf{h}_{i_R,N_T}^T\right]^T \\ &= (\mathbf{I}_{N_T} \otimes \mathbf{\Lambda})\, \mathbf{d}_{i_R} \end{aligned} \tag{9}$$

where $\mathbf{d}_{i_R} \triangleq \left[\mathbf{d}_{i_R,1}^T, \cdots, \mathbf{d}_{i_R,i_T}^T\right]^T$. Using (9), one can obtain

$$\mathbf{h} = \mathbf{\Psi}\mathbf{d} \tag{10}$$

where $\mathbf{\Psi} \triangleq [\mathbf{I}_{N_R} \otimes (\mathbf{I}_{N_T} \otimes \mathbf{\Lambda})]$ and $\mathbf{d} \triangleq \left[\mathbf{d}_1^T, \cdots, \mathbf{d}_{N_R}^T\right]^T$. Thus, (10) gives the relationship between the composite channel vector and the pure multipath channel vector.

Substituting (10) into (3), a new semi-blind cost function for MIMO-OFDM channel estimation with pulse-shaping can be formulated as

$$\min_{\hat{\mathbf{h}}} \Delta = \left\|\mathbf{Y}_{\text{pilot}} - \tilde{\mathbf{A}}'\mathbf{d}\right\|_F^2 + \alpha\left\|\hat{\mathbf{B}}'\mathbf{d}\right\|_F^2 \tag{11}$$

where $\tilde{\mathbf{A}}' \triangleq \tilde{\mathbf{A}}\mathbf{\Psi}$ and $\hat{\mathbf{B}}' \triangleq \hat{\mathbf{B}}\mathbf{\Psi}$. Similar to (4), the estimate of the channel vector can be derived as

$$\hat{\mathbf{d}} = \left(\left(\tilde{\mathbf{A}}'\right)^H \tilde{\mathbf{A}}' + \alpha\left(\hat{\mathbf{B}}'\right)^H \hat{\mathbf{B}}'\right)^{\dagger} \left(\tilde{\mathbf{A}}'\right)^H \mathbf{Y}_{\text{pilot}}. \tag{12}$$

The above equation gives a time-domain semi-blind solution for the channel estimation of MIMO-OFDM systems with pulse-shaping. Note that the computational complexity of $\hat{\mathbf{d}}$ depends on the size of the matrices $\tilde{\mathbf{A}}'$ and $\hat{\mathbf{B}}'$, which are determined by the length of the pulse-shaping and matched filters as well as the length of the pure multipath channel. When the length L_g of the filter $g(n)$ is relatively small as compared to the channel length L_d, (12) gives an efficient channel estimate. For a large value of L_g, however, the total length $L = L_g + L_d - 1$ of the composite channel can be very large which may incur a high complexity in the computation of (12). In what follows, we propose a very efficient frequency-domain estimation approach regardless of the relative size of L_g.

(ii) *A frequency-domain method*

The frequency-domain signal model between the i_R-th receive antenna and the i_T-th transmit antenna can be represented by

$$Y_{i_R}(k) = H_{i_R,i_T}(k)\, X_{i_T}(k)\,, \text{for}\, k \in [0, K-1] \tag{13}$$

where $H_{i_R,i_T}(k) \triangleq \mathbf{f}_{0k}\left[\mathbf{h}_{i_R,i_T}^T, \mathbf{0}_{1\times(K-L)}\right]^T$ and $\mathbf{f}_{0k}$ is the k-th row of the $K \times K$ DFT matrix $\mathbf{F}_0$. From (6), we can derive

$$H_{i_R,i_T}(k) = G(k)\, D_{i_R,i_T}(k) \tag{14}$$

where $G(k) \triangleq \mathbf{f}_{0k}\left[g(0), \cdots, g(L_g-1), \mathbf{0}_{1\times(L-L_g)}\right]^T$ and $D_{i_R,i_T}(k) \triangleq \mathbf{f}_{0k}\left[\mathbf{d}_{i_R,i_T}^T, \mathbf{0}_{1\times(K-L_b)}\right]^T$. Using (14) into (13), we can obtain the received signal after removing the effect of the pulse-shaping and matched filters,

$$\begin{aligned} Y'_{i_R}(k) &\triangleq G^{-1}(k)\, Y_{i_R}(k) \\ &= D_{i_R,i_T}(k)\, X_{i_T}(k)\,. \end{aligned} \tag{15}$$

Now the previous semi-blind algorithm as stated in Subsection III-A can be applied to (15) to obtain a frequency-domain channel estimate.

It should be mentioned that for the computation of the blind constraint $\hat{\mathbf{B}}$ in (4), the correlation matrices $\tilde{\mathbf{R}}_{n-1}$, $\ddot{\mathbf{R}}_n$ and $\mathbf{R}(0)$ need to be computed. In general, this can be done via the time-domain correlation estimation method. Utilizing the property

$$\tilde{\mathbf{R}}_n = \begin{bmatrix} \mathbf{R}(0) & \ddot{\mathbf{R}}_n \\ \ddot{\mathbf{R}}_n^H & \tilde{\mathbf{R}}_{n-1} \end{bmatrix}, \tag{16}$$

the estimates $\hat{\tilde{\mathbf{R}}}_{n-1}$, $\hat{\ddot{\mathbf{R}}}_n$ and $\hat{\mathbf{R}}(0)$ can be computed from

$$\hat{\tilde{\mathbf{R}}}_n = \frac{1}{K}\sum_{n=0}^{K-1} \mathbf{y}_{P+1}(n)\, \mathbf{y}_{P+1}^H(n) \tag{17}$$

where $\mathbf{y}_{P+1}(n) \triangleq [y_1(n), \cdots, y_{N_R}(n), \cdots, y_{N_R}(n-P)]$ $(n = 0, \cdots, P-1)$, can be obtained using $\mathbf{y}(n-j) \triangleq \mathbf{y}(K+n-j)$ for $n < j$, due to the circular convolution. Since the effect of pulse-shaping and matched filtering is eliminated, the practical scheme of determining the weighting factor α suggested in [2] can be used directly in this frequency-domain method.

It should also be noted that the received signal, after removing the effect of pulse-shaping and matched filters, is only available in the frequency-domain. In order to derive the correlation matrices via the above method, an FFT process is needed to convert the frequency-domain signal to the time-domain version.

IV. Simulation Results

Here, we consider a MIMO-OFDM system with 2 transmit and 4 receive antennas. The number of subcarriers is set to 512, the length of cyclic prefix is 10, and the length of the linear predictor is $P = 4$. In our simulation, the QPSK modulation is used and a Rayleigh channel modelled by a 3-tap MIMO-FIR filter is assumed, in which each tap corresponds to a 2×4 random matrix whose elements are i.i.d. complex Gaussian variables with zero mean and unit variance. A square root raised cosine filter with order 16, oversampling rate 4 and rolloff factor 0.15 is used for pulse-shaping. As shown in [2], [3], the channel estimation performance is associated with $\mathbf{H}(0)$. Accordingly, we define the metric

$$\eta \triangleq \frac{\|\mathbf{H}(0)\|_F^2}{\sum_{n=0}^{2} \|\mathbf{H}(n)\|_F^2}$$

and conduct a simulation study with respect to different ranges of η.

In the experiments, for the purpose of comparison, the composite channel vector $\mathbf{h}$ is first estimated by the LS method. As for the estimation of the pure multi-path channel vector $\mathbf{d}$, we consider the time-domain LS, the frequency-domain LS and the proposed frequency-domain semi-blind method, all with pulse shaping. For simplicity, we call these four methods as the basic LS, enhanced time-domain

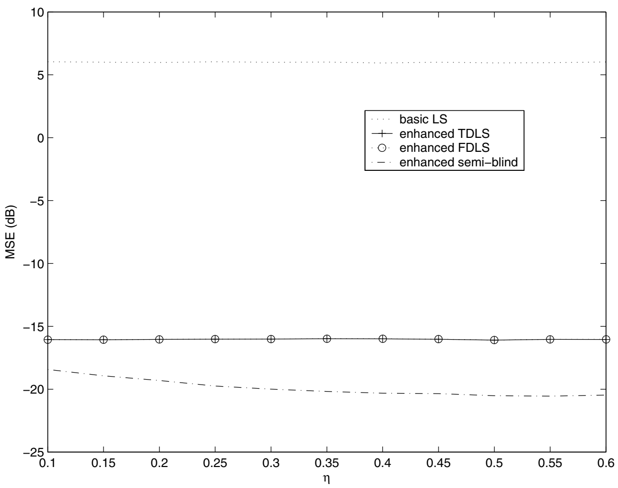

Fig. 2. MSE versus η.

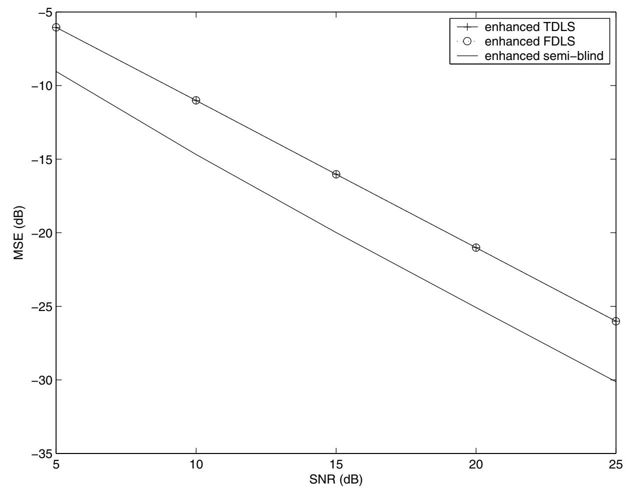

Fig. 3. MSE versus SNR

LS (TDLS), enhanced frequency-domain LS (FDLS) and enhanced semi-blind methods. Note that the LS methods can be easily obtained by setting α to zero in the proposed two semi-blind methods with pulse-shaping.

Experiment 1:*MSE versus* η

In the first experiment, the channel estimation performance in terms of the MSE versus η is investigated. The simulation is undertaken by 1000 Monte Carlo runs of the transmission of one OFDM symbol at 512 subcarriers under an SNR of 15dB. Fig. 2 shows the MSE plots resulting from the proposed enhanced TDLS, FDLS and semi-blind frequency-domain methods as well as the basic LS estimation. Obviously, the MSE performance of the proposed three enhanced methods are at least 20dB-30dB better than that of the basic LS, indicating that the new approach focusing on the pure multipath channel vector significantly outperforms that for composite channel vector irrespective of pulse-shaping. One can find a high consistency between the enhanced TDLS and the enhanced FDLS methods. Also, the semi-blind frequency-enhanced method significantly outperforms the two enhanced LS methods. In addition, the performance of the semi-blind method improves with the increase of η when $\eta < 0.3$, and it remains almost the same when η is in the range of 0.3 to 0.6, which represents typical mobile communication scenarios where the first arrived path is comparable to or stronger than other paths.

Experiment 2: *MSE versus SNR*

Now we investigate the channel estimation performance versus the SNR. The simulation involves 5000 Monte Carlo runs of the transmission of one OFDM symbol. Fig. 3 shows the channel estimation results of the three enhanced methods when $\eta > 0.2$. It is seen that the performances of the two LS methods are almost the same and the semi-blind frequency-enhanced method can achieve nearly 3∼4 dB gains over the two LS methods, when the SNR varies from 5 to 25 dB, respectively.

V. Conclusions

The semi-blind channel estimation issue of pulse-shaped MIMO-OFDM systems has been investigated. By exploiting the pulse-shaping filter available in the transmitter and the matched filter in the receiver, a time-domain semi-blind solution was first developed. To reduce the computational complexity of the time-domain method, an equivalent frequency-domain algorithm has then been proposed. The effectiveness of the new channel estimation algorithm has been confirmed by computer simulations.

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