Perturbation Analysis of Subspace-Based Semi-Blind MIMO Channel Estimation Approaches

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Abstract— **In this paper, a perturbation analysis of two subspace-based semi-blind MIMO channel estimation approaches is conducted. Our analysis shows that, in the noise-free case, the whitening-rotation (WR)-based algorithm is subject to a signal perturbation error, while the nulling-based algorithm is a signal perturbation free scheme with an ideal nulling constraint imposed on the channel matrix. This explains why the WR-based method is efficient only in the low SNR case, and concludes that the nulling-based approach is better for moderate to high SNRs. A novel closed-form mean square error (MSE) expression is also derived for the nulling-based blind estimation method, in which an appealing scheme for the determination of the weighting factor is presented. The nulling-based method with the proposed weighting scheme is validated via computer simulations, showing a very high estimation accuracy of our semi-blind solution in terms of the MSE of the channel estimate.**

I. Introduction

The MIMO (Multiple input multiple output) technique has been considered as one of the key technologies for the development of the next-generation wireless communication systems. With multiple transmit and multiple receive antennas, MIMO systems can provide either a diversity gain to combat signal fading or a capacity gain, called spatial multiplexing, to make an efficient use of channel resources. It means that, with MIMO techniques, higher data rate and better performance can be achieved without increasing the total transmission power or bandwidth. On the other hand, the performance of MIMO systems depends largely upon the availability of the knowledge of the channel. Thus, an accurate estimation of the wireless channel is of crucial importance to MIMO systems.

Based on known pilots, the MIMO channel can be estimated by employing different kinds of training-based algorithms such as the least square (LS), the maximum likelihood (ML) and the minimum mean square error (MMSE) algorithms [1]. In contrast to training-based methods, blind channel estimation algorithms like that proposed in [2], can achieve a better spectral efficiency by use of second-order statistics, correlative coding or other properties. With the idea of combining the training-based and the blind algorithms, semi-blind channel estimation techniques can potentially enhance the quality of MIMO channel estimation [3]. With a small number of training symbols, problems such as ambiguities and mis-convergence of the blind methods can be solved. On the other hand, the use of the available data in semi-blind techniques can improve the accuracy of channel estimation.

More recently, a whitening-rotation (WR)-based semi-blind algorithm has been proposed for frequency-flat MIMO channel estimation [4]. This algorithm consists of two phases: (1) estimation of a whitening matrix utilizing information data; and (2) estimation of a unitary rotation matrix using pilots. The Cramer-Rao bound (CRB) of this algorithm shows that it can achieve a much better channel estimation performance than the conventional LS method, when the number of receive antennas is greater than or equal to the number of transmit antennas. However, this method is found to be efficient only in the case of low SNRs.

A nulling based semi-blind MIMO channel estimation approach, which can achieve a better performance in moderate to high SNR cases, was developed in [5]. Instead of estimating the whiting matrix, this method uses the information data to obtain an blind constraint for the channel matrix, which is to be combined with a training-based LS cost function so as to produce a semi-blind solution for the MIMO channel response. This method can be considered as a modified LS solution involving a weighted blind constraint. It has been shown in [5] that the semi-blind method provides a much better channel estimation performance over the pure training-based LS method. However, this superiority has not been theoretically proved, and the weighting factor employed to trade off the least square and the blind criteria has not well been determined.

In this paper, we apply the perturbation theory [6] to the analysis of the above mentioned two semi-blind algorithms, showing that in the noise-free case the blind part of the WR-based method is subject to a signal perturbation error, whereas the nulling-based method gives an ideal nulling constraint on the channel matrix, thus avoiding the signal perturbation error. Our analysis concludes that the nulling-based method is superior to the whitening-rotation-based method in the the moderate to high SNR case. We then derive a novel closed-form expression for the mean square error (MSE) of the blind estimation to facilitate the calculation of the weighting factor in the nulling-based method.

Throughout the paper, we adopt the following notations:
$\dagger$ Pseudo-inverse, $\otimes$ Kronecker product,
T Transpose, H Complex conjugate transpose,
$\| \ \|_F$ Frobenius norm, $\mathrm{vec}\,()$ a stacking of the columns of the involved matrix into a vector with a property:
$\mathrm{vec}\,[\mathbf{AB}] = \left(\mathbf{I}^T \otimes \mathbf{A}\right) \mathrm{vec}\,(\mathbf{B}) = \left(\mathbf{B}^T \otimes \mathbf{I}\right) \mathrm{vec}\,(\mathbf{A})$.

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II. Brief Overview of Subspace-Based Semi-Blind MIMO Channel Estimation

Consider an MIMO system with N_T transmit and $N_R\ (\geq N_T)$ receive antennas. Suppose that the frequency-flat fading MIMO channel is characterized by an $N_R \times N_T$ matrix $\mathbf{H}$ whose (i_R, i_T)-th element h_{i_T,i_R} represents the channel response from the i_T-th transmit antenna to the i_R-th receive antenna. Given the transmitted signal vector $\mathbf{x}(n) \triangleq [x_1(n), \cdots, x_{N_T}(n)]^T$ whose elements are independent identically distributed (i.i.d.) Gaussian random variables with zero mean and unit variance $\delta_x^2 = 1$, the received signal vector $\mathbf{y}(n) \triangleq [y_1(n), \cdots, y_{N_R}(n)]^T$ can be written as

$$\mathbf{y}(n) = \mathbf{H}\mathbf{x}(n) + \mathbf{v}(n) \tag{1}$$

where the noise vector $\mathbf{v}(n) \triangleq [v_1(n), \cdots, v_{N_R}(n)]^T$ is spatio-temporally uncorrelated with variance δ_v^2. Note that, in each block, the first K of the N slots are used for training purpose.

We now briefly review two subspace-based semi-blind MIMO channel estimation algorithms. The first one is the WR-based semi-blind method [4]. Its idea originates from a decomposition of the channel matrix,

$$\mathbf{H} = \mathbf{W}\mathbf{Q}^H, \tag{2}$$

where $\mathbf{W}$ is a whitening matrix and $\mathbf{Q}$ is a unitary rotation matrix. Performing the singular value decomposition (SVD) of $\mathbf{H}$ gives

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H. \tag{3}$$

Obviously, one possible choice of $\mathbf{W}$ and $\mathbf{Q}$ can be $\mathbf{U}\boldsymbol{\Sigma}$ and $\mathbf{V}$. Thus, the WR-based channel estimation method can be implemented with two steps:

(i) Estimate the whitening matrix $\mathbf{W}$ in a blind fashion using the autocorrelation matrix of the received signal and a subspace-based method;

(ii) Estimate the unitary rotation matrix $\mathbf{Q}$ by utilizing training pilots and a constrained maximum-likelihood (ML) method.

Although this method is superior to some of the training-based methods in the low SNR case, its advantage loses with the increase of SNR. In a higher SNR case, alternatively, a nulling-based semi-blind approach can be considered [5]. Instead of estimating the whitening matrix $\mathbf{W}$, this approach uses the subspace method to obtain an estimate of the nulling subspace of the channel matrix, $\hat{\mathbf{U}}_{\text{null}}$. By utilizing $\hat{\mathbf{U}}_{\text{null}}$ in conjunction with a training based least square (LS) criterion, a semi-blind cost function can then be formulated as

$$\min_{\mathbf{H}} \Delta = \|\mathbf{Y}_{\text{P}} - \mathbf{H}\mathbf{X}_{\text{P}}\|_F^2 + \alpha \left\|\hat{\mathbf{U}}_{\text{null}}^H \mathbf{H}\right\|_F^2 \tag{4}$$

where $\mathbf{X}_{\text{P}} \triangleq [\mathbf{x}(1), \cdots, \mathbf{x}(K)]$ is the pilot signal matrix, $\mathbf{Y}_{\text{P}} \triangleq [\mathbf{y}(1), \cdots, \mathbf{y}(K)]$ the corresponding received signal matrix, and $\alpha > 0$ is a weighting factor. It should be pointed out that the superiority of this approach in the high SNR case has not been theoretically justified, and the determination of the weighting factor α employed to trade off the least square and the blind criteria has not been well addressed.

III. Perturbation Analysis of Subspace-based Blind Channel Estimation

It is known that the solution of subspace based methods is always perturbed by various sources, such as the finite data length, the measurement noise etc. The perturbation theory has been employed for the analysis of subspace based methods in [6]. In this section, the first-order perturbation analysis is employed to evaluate the performance of the two subspace-based channel estimation methods.

A. Analysis of the WR-based Method

In this paper, we consider only the perturbation due to the finite data length in the computation of correlation matrices. Our objective is to show that the whitening matrix would be perturbed even in the absence of noise. Using (1), the autocorrelation matrix of the received signal, $\hat{\mathbf{R}}_{\mathbf{Y}}$, with such a perturbation can be written as

$$\hat{\mathbf{R}}_{\mathbf{Y}} = \hat{\mathrm{E}}\left[\mathbf{y}(n)\mathbf{y}^H(n)\right] - \delta_v^2\mathbf{I} = \mathbf{H}\left[\mathbf{I} + \Delta\mathbf{R}_x\right]\mathbf{H}^H + \Delta\mathbf{R}_v \tag{5}$$

where $\Delta\mathbf{R}_x$ denotes the the signal perturbation matrix,

$$\Delta\mathbf{R}_x \triangleq \frac{1}{N}\sum_{n=1}^{N}\mathbf{x}(n)\mathbf{x}^H(n) - \delta_x^2\mathbf{I}, \tag{6}$$

and $\Delta\mathbf{R}_v$ the perturbation matrix introduced by the noise,

$$\Delta\mathbf{R}_v \triangleq \mathbf{H}\Delta\mathbf{R}_{xv} + \Delta\mathbf{R}_{xv}^H\mathbf{H}^H + \Delta\mathbf{R}_{vv} \tag{7}$$

with

$$\Delta\mathbf{R}_{xv} \triangleq \frac{1}{N}\sum_{n=1}^{N}\mathbf{x}(n)\mathbf{v}^H(n), \tag{8}$$

$$\Delta\mathbf{R}_{vv} \triangleq \frac{1}{N}\sum_{n=1}^{N}\mathbf{v}(n)\mathbf{v}^H(n) - \delta_v^2\mathbf{I}.$$

In the noise-free case, all the perturbation terms introduced by noise would disappear. Then, (5) reduces to

$$\hat{\mathbf{R}}_{\mathbf{Y}} = \mathbf{H}\left[\mathbf{I} + \Delta\mathbf{R}_x\right]\mathbf{H}^H. \tag{9}$$

Based on the above perturbed expressions, we can derive the estimate of $\mathbf{W}$. Using (3) with (9), one can get

$$\hat{\mathbf{R}}_{\mathbf{Y}} = \mathbf{U}\mathbf{I}_C\mathbf{T}\mathbf{I}_C^H\mathbf{U}^H \tag{10}$$

$$\mathbf{I}_C \triangleq \begin{bmatrix} \mathbf{I}_{N_T \times N_T} \\ \mathbf{0}_{(N_R - N_T)\times N_T} \end{bmatrix}, \tag{11}$$

$$\mathbf{T} \triangleq \boldsymbol{\Sigma}_S^2 + \boldsymbol{\Sigma}_S\mathbf{V}^H\Delta\mathbf{R}_x\mathbf{V}\boldsymbol{\Sigma}_S, \tag{12}$$

where $\boldsymbol{\Sigma}_S$ is a diagonal matrix satisfying $\boldsymbol{\Sigma} = \mathbf{I}_C\boldsymbol{\Sigma}_S$. As $\mathbf{T}$ is Hermitian, its SVD can be written as

$$\mathbf{T} \triangleq \boldsymbol{\Pi}_S\left(\boldsymbol{\Sigma}_S + \Delta\boldsymbol{\Sigma}_S\right)^2\boldsymbol{\Pi}_S^H \tag{13}$$

where $\boldsymbol{\Pi}_S$ is a unitary matrix and $\Delta\boldsymbol{\Sigma}_S$ is the perturbed error of $\boldsymbol{\Sigma}_S$. Substituting (13) into (10), comparing it with (9) and noting that $\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}$, one can obtain an estimate of the whitening matrix $\mathbf{W}$ as given below

$$\hat{\mathbf{W}} = \hat{\mathbf{U}}\hat{\boldsymbol{\Sigma}} = \mathbf{U}\mathbf{I}_C\boldsymbol{\Pi}_S\left(\boldsymbol{\Sigma}_S + \Delta\boldsymbol{\Sigma}_S\right). \tag{14}$$

From (14), one can find that even in the noise-free case $\hat{\mathbf{W}}$ consists of two perturbation terms $\mathbf{\Pi}_S$ and $\Delta\mathbf{\Sigma}_S$, which are dictated by the signal perturbation matrix $\Delta\mathbf{R}_x$. This could explain why the performance of the WR-based method is very poor in the moderate to high SNR cases.

B. Analysis of the Nulling-Based Method

In the nulling-based method proposed in [5], a nulling constraint $\hat{\mathbf{U}}_{\text{null}}$ on the channel matrix $\mathbf{H}$ is obtained from the SVD of $\hat{\mathbf{R}}_{\mathbf{Y}}$. It can be verified from (9) that in the absence of noise, the nulling constraint $\hat{\mathbf{U}}_{\text{null}}$ is ideal, namely,

$$\hat{\mathbf{U}}_{\text{null}}^H\mathbf{H} = \mathbf{0}.$$

This implies that in the noise-free case the blind part of the nulling-based method is not affected by the signal perturbation terms, and therefore, the nulling-based method is superior to the WR-based method. In the following, we derive a closed-form expression for the MSE of the blind estimate in the nulling-based method for the noisy case.

The MSE of the blind estimate is defined as

$$\text{MSE}_{\text{B}} \triangleq \text{E}\left\{\left\|\hat{\mathbf{U}}_{\text{null}}^H\mathbf{H}\right\|_F^2\right\}. \tag{15}$$

Using (5) and (7) along with the first-order approximation [6], one can get an estimate of $\mathbf{U}_{null}$,

$$\hat{\mathbf{U}}_{\text{null}} \approx \mathbf{U}_{\text{null}} - \left(\mathbf{H}\mathbf{H}^H\right)^\dagger \mathbf{H}\Delta\mathbf{R}_{xv}\mathbf{U}_{\text{null}}. \tag{16}$$

For the medium to high SNR, say SNR $\geq$ 5dB, the perturbation matrix $\Delta\mathbf{R}_{vv}$ of the noise autocorrelation can be neglected. Thus, substituting (16) into (15), and noting that $\mathbf{H}^H\left(\mathbf{H}\mathbf{H}\right)^\dagger\mathbf{H} = \mathbf{I}$ when $\mathbf{H}$ has a full column rank and $N_R > N_T$, one can obtain

$$\text{MSE}_{\text{B}} = \text{E}\left\{\left\|\hat{\mathbf{U}}_{\text{null}}^H\Delta\mathbf{R}_{xv}^H\right\|_F^2\right\}. \tag{17}$$

From the fact that $\|\mathbf{A}\|_F^2 = \text{vec}\left(\mathbf{A}\right)\text{vec}^H\left(\mathbf{A}\right)$, MSE_{B} can be calculated by

$$\text{MSE}_{\text{B}} = \text{Trace}[\left(\mathbf{I}\otimes\mathbf{U}_{\text{null}}^H\right)\mathbf{R}_{\Delta xv}\left(\mathbf{I}\otimes\mathbf{U}_{\text{null}}\right)] \tag{18}$$

where $\mathbf{R}_{\Delta xv} \triangleq \text{E}\left\{\text{vec}\left(\Delta\mathbf{R}_{xv}^H\right)\text{vec}^H\left(\Delta\mathbf{R}_{xv}^H\right)\right\}$.

Prior to the determination of MSE_{B}, we first compute $\mathbf{R}_{\Delta xv}$ in terms of the correlation of the signal and the correlation of the noise. Since (8) can be rewritten as

$$\text{vec}\left(\Delta\mathbf{R}_{xv}^H\right) = \frac{1}{N}\sum_{n=1}^{N}\left\{\left[\mathbf{x}^*\left(n\right)\otimes\mathbf{I}_{N_R}\right]\text{vec}\left[\mathbf{v}\left(n\right)\right]\right\}, \tag{19}$$

one can derive

$$\begin{aligned}\mathbf{R}_{\Delta xv} \;=\; & \frac{1}{N^2}\text{E}(\sum_{n_1=1}^{N}\sum_{n_2=1}^{N}\left[\mathbf{x}^*\left(n_1\right)\otimes\mathbf{I}_{N_R}\right] \\ & \text{E}\left[\mathbf{v}\left(n_1\right)\mathbf{v}^H\left(n_2\right)\right]\left[\mathbf{x}^T\left(n_2\right)\otimes\mathbf{I}_{N_R}\right])\end{aligned} \tag{20}$$

which can then be further simplified as

$$\mathbf{R}_{\Delta xv} = \frac{\delta_v^2}{N}\left\{\text{E}\left[\frac{1}{N}\sum_{n=1}^{N}\mathbf{x}^*\left(n\right)\mathbf{x}^T\left(n\right)\right]\otimes\mathbf{I}_{N_R}\right\}.$$

Using the above expression, one can finally obtain

$$\mathbf{R}_{\Delta xv} = \frac{1}{N}\delta_x^2\delta_v^2\mathbf{I}_{N_RN_T}. \tag{21}$$

Substituting (21) into (18) yields

$$\begin{aligned}\text{MSE}_{\text{B}} \;&= \frac{1}{N}\delta_x^2\delta_v^2\text{Trace}\left[\mathbf{I}_{N_T}\otimes\left(\mathbf{U}_{\text{null}}^H\mathbf{U}_{\text{null}}\right)\right] \\ &= \frac{1}{N}\delta_x^2\delta_v^2 N_T\left(N_R - N_T\right).\end{aligned} \tag{22}$$

In the next section, the above closed-form expression of MSE_{B} will be utilized for the calculation of the weighting factor in the nulling-based method.

IV. A New Nulling-Based Semi-blind Algorithm

We now derive a closed-form solution for the minimization problem in (4). This can be easily done by calculating the derivative of the cost function in (4) with respect to the channel vector, $\mathbf{h} \triangleq \text{vec}\left(\mathbf{H}\right)$, namely,

$$\begin{aligned}\frac{\partial\Delta}{\partial\mathbf{h}^H} \;=\; & -\left(\mathbf{X}_{\text{P}}^*\otimes\mathbf{I}_{N_R}\right)\left[\text{vec}\left(\mathbf{Y}_{\text{P}}\right) - \left(\mathbf{X}_{\text{P}}^T\otimes\mathbf{I}_{N_R}\right)\mathbf{h}\right] \\ & + \alpha\left(\mathbf{I}_{N_T}\otimes\hat{\mathbf{U}}_{\text{null}}\right)\left[\left(\mathbf{I}_{N_T}\otimes\hat{\mathbf{U}}_{\text{null}}^H\right)\mathbf{h}\right].\end{aligned} \tag{23}$$

Letting $\frac{\partial\Delta}{\partial\mathbf{h}^H}$ be zero gives a closed-form solution for the channel vector,

$$\begin{aligned}\hat{\mathbf{h}} \;=\; & \left\{\left[\left(\mathbf{X}_{\text{P}}^*\mathbf{X}_{\text{P}}^T\right)\otimes\mathbf{I}_{N_R}\right] + \alpha\left[\mathbf{I}_{N_T}\otimes\left(\hat{\mathbf{U}}_{\text{null}}\hat{\mathbf{U}}_{\text{null}}^H\right)\right]\right\}^\dagger \\ & \left(\mathbf{X}_{\text{P}}^*\otimes\mathbf{I}_{N_R}\right)\text{vec}\left(\mathbf{Y}_{\text{P}}\right).\end{aligned} \tag{24}$$

The weight factor α can be chosen as [7], [8]

$$\alpha = \frac{\text{MSE}_{\text{T}}}{\text{MSE}_{\text{B}}}\frac{\|\mathbf{X}_P\|_F^2}{\left\|\hat{\mathbf{U}}_{\text{null}}\right\|_F^2}, \tag{25}$$

where MSE_{T} represents the MSE of the training-based LS estimation of the channel vector. For orthogonal training pilots, it can be verified that

$$\|\mathbf{X}_{\text{P}}\|_F^2 = KN_T\delta_x^2, \tag{26}$$

$$\text{MSE}_{\text{T}} \triangleq \text{E}\left\{\left\|\hat{\mathbf{H}}_{\text{T}} - \mathbf{H}\right\|_F^2\right\} = \frac{N_RN_T\delta_v^2}{K\delta_x^2}, \tag{27}$$

$$\left\|\hat{\mathbf{U}}_{\text{null}}\right\|_F^2 = N_R - N_T. \tag{28}$$

Thus, by substituting (22), (26), (27) and (28) into (25), the weighting factor α of the nulling-based semi-blind method can be calculated as

$$\alpha = \frac{NN_TN_R}{\left(N_R - N_T\right)^2\delta_x^2}. \tag{29}$$

Thus, using (29) into (24), a new nulling-based semi-blind algorithm with the proposed weighting scheme can be obtained.

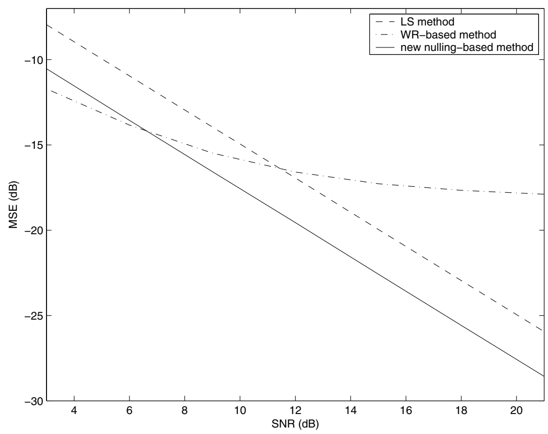

Fig. 1. MSE versus SNR

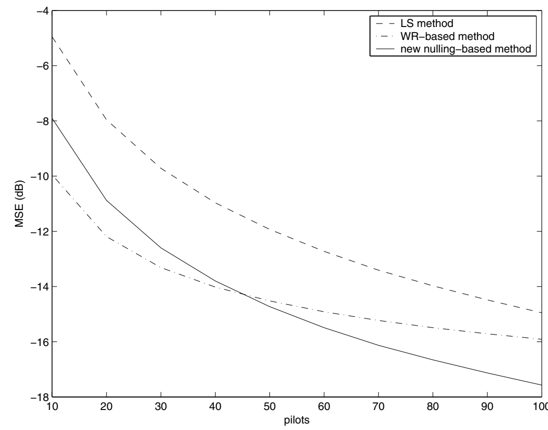

Fig. 2. MSE versus pilot length

V. SIMULATION RESULTS

We consider a MIMO system with 4 transmit and 8 receive antennas, in which the QPSK modulation is used and a Rayleigh channel, whose elements are i.i.d. complex Gaussian variables with zero mean and unit variance, is assumed. Here, the orthogonal pilots are generated. It is noted that, a joint optimization approach was proposed in [4] to improve the estimation of the whitening matrix. Although this approach can improve the estimation accuracy of the whitening matrix slightly, the complexity of this method is extremely high since it involves many iterations in the computation of the "fminunc" function in MATLAB, making its implementation very difficult for real-world applications. Thus, in our experiments, this approach is not considered and only the general WR-based method is simulated.

Experiment 1: *MSE versus SNR*

In the first experiment, the channel estimation performance in terms of the plot of the MSE versus the SNR is investigated. The simulation is undertaken based on 20000 Monte Carlo runs of the transmission of one data frame with 1000 slots out of which 100 are used as pilots. Fig. 1 shows the MSE plots of the LS method, the WR-based method and the nulling-based method with the proposed weighting scheme. It is seen that the performance of the WR-based method is the best among the three methods when the SNR is less than 7dB, while that of the nulling-based method becomes the best when the value of SNR is larger than 7dB. Moreover, the nulling-based method can achieve a gain of nearly 2.6 dB over the LS method regardless of the level of the SNR, which significantly outperforms the WR-based method when the value of SNR is moderately large.

Experiment 2: *MSE versus pilot length*

Here we investigate the channel estimation performance versus the pilot length. Fig. 2 shows the MSE plots from 20000 Monte Carlo runs of the transmission of one data frame of 1000 slots for an SNR of 10 dB. One can find that if $K < 45$, the WR-based method is the best, while if $K > 45$, the nulling-based method becomes the best. For example, when the pilot length is 100, the gain of the nulling-based method over the WR-based method is about 1.7 dB, implying that the nulling-based method is efficient even when SNR is 10dB only.

VI. CONCLUSIONS

A perturbation analysis of two subspace-based semi-blind MIMO channel estimation algorithms has been performed, justifying that the WR-based method is efficient only in the low SNR case, and the nulling-based method is a better choice when the SNR is moderate or high. The analysis has also led to a closed-form expression for the MSE of the blind criterion of the nulling-based method, simplifying the calculation of the weighting factor in the semi-blind solution. The nulling-based method with the proposed weighting scheme has been simulated and compared with the WR-based method as well as the LS method, showing a significant improvement in terms of the MSE of the channel estimate for the moderate to high SNR cases.

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