Channel Estimation of Pulse-Shaped MIMO-OFDM Systems

Feng Wan, W.-P. Zhu and M.N.S. Swamy

Abstract—**Most of the existing MIMO-OFDM channel estimation methods do not take into account the effect of the pulse-shaping filter in the transmitter nor of the matched filter in the receiver, thus leading to an estimation solution for the composite channel including the pulse-shaping and matched filters, instead of for the pure wireless channel. This solution is not sufficiently accurate due to the extra length of the composite channel induced by the two filters especially in the scenario with a small pure channel length. In this paper, we present a novel methodology for the estimation of the pure multipath channels of pulse-shaped MIMO-OFDM systems. By utilizing the knowledge of pulse-shaping and matched filters, we develop two channel estimation approaches, namely, a semi-blind approach for the sampling duration based channels, in which the multipath occurs at the sampling instant, and a training-based least-square technique for the upsampling duration based channels where the multipath may occur in a fraction of sampling duration. A number of computer simulation-based experiments are conducted, and these simulation results confirm the efficacy of the proposed approaches.**

1. Introduction

It is well known that both multiple-input multiple-output (MIMO) and orthogonal frequency division multiplexing (OFDM) are most promising techniques for the next-generation wireless communication systems [1]–[3]. It has been proved that, when perfect knowledge of the wireless channel is available at the receiver, the capacity of MIMO-OFDM systems grows linearly with the number of transmit or receive antennas, whichever is less. In real wireless environments, however, the channel condition is not known. Therefore, channel estimation is required in MIMO-OFDM systems.

Broadly speaking, MIMO-OFDM channel estimation approaches can be categorized into three classes, training-based, blind and semi-blind approaches. First, training-based methods, such as the least-square (LS), the maximum likelihood (ML) and the minimum mean square error (MMSE) methods, employ known training signals to render an accurate channel estimation [4]. Blind MIMO-OFDM channel estimation algorithms, such as those proposed in [5]–[7], which exploit the second-order stationary statistics, correlative coding and other properties, normally have a better spectral efficiency. With a small number of training symbols, semi-blind methods have been proposed in [8]–[10] to estimate the channel ambiguity matrix for space-time coded OFDM systems. It is worth pointing out that most of the existing blind and semi-blind methods for MIMO-OFDM channel estimation are based on the second-order statistics of a long vector of the received frequency-domain signal, whose size is equal to or larger than the number of subcarriers. To estimate the correlation matrix reliably, these techniques need a large number of OFDM symbols, and therefore, they are not suitable for fast time-varying channels. In addition, since the matrices involved in these algorithms are of huge size, their computational complexity is extremely high. In contrast, a linear prediction-based semi-blind algorithm, which is based on the second-order statistics of a short vector with a size only slightly larger than the channel length, has been found to be much more efficient than the conventional LS methods for the estimation of frequency-selective MIMO channels [11]. This method has then been extended for MIMO-OFDM systems in [12]–[14], where the MIMO channel was estimated with a very high accuracy by employing only a few OFDM symbols while the full or partial information of the channel correlation is not needed as in the MMSE methods.

It is well known that the pulse-shaping filter as well as the matched filter are commonly used in digital communication systems. They have also been employed in practical MIMO and OFDM systems to achieve a better spectral efficiency as well as a high signal reception performance [15]–[20]. Perhaps for the sake of simplicity, however, many existing channel estimation methods did not take into consideration either the effect of the pulse-shaping filter in the transmitter or that of the matched filter in the receiver. As such, these methods have actually been developed for the estimation of the composite channel including the pulse-shaping and matched filters. Considering that both filters are known to the receiver and the only unknown part is the discrete-time channel [15], [16], [21], ignoring their existence would lead to less accurate estimation results. By utilizing the information of both filters, some improved channel estimation algorithms have been obtained for OFDM systems [17]–[19] and CDMA systems [22], [23]. Motivated by this observation, in this paper, we will propose a new methodology for the channel estimation of pulse-shaped MIMO-OFDM systems with an emphasis on the development of a linear prediction (LP)-based semi-blind method for multipath channels whose taps are in line with the sampling duration. Another contribution

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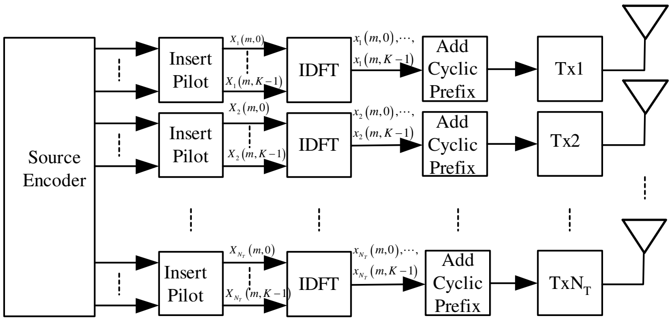

Fig. 1. MIMO-OFDM transmitter

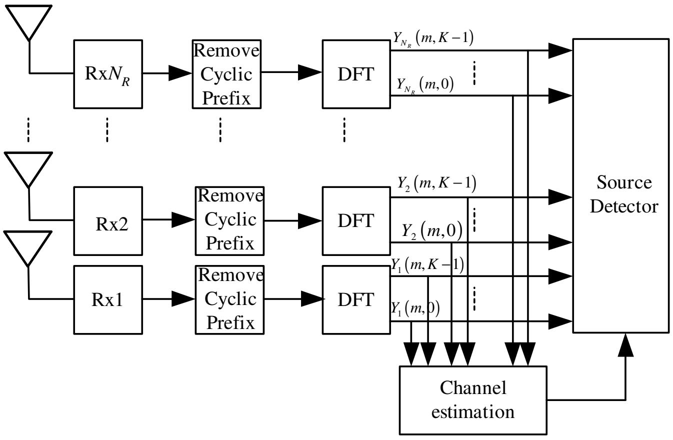

Fig. 2. MIMO-OFDM receiver

of this paper is to present an improved least-square (LS) method for estimating the multipath channels whose taps are in a fraction of the sampling duration. The novelty of the proposed method lies in the investigation, for the first time, of the complicated composite wireless channel model such that the semi-blind and the training-based LS methods can be applied to the estimation of the pure multipath channel. As will be shown by Monte Carlo simulations, our proposed method significantly outperforms the conventional semi-blind and LS channel estimation methods without considering pulse shaping and matched filtering.

The structure of this paper is briefed as follows. Section 2 presents the data model of MIMO-OFDM systems and formulates the semi-blind channel estimation problem. Section 3 deals with the channel estimation of MIMO-OFDM systems with pulse-shaping. By utilizing the knowledge of both the pulse-shaping filter and the matched filter, two improved channel estimation algorithms are developed. Section 4 comprises a number of experimentations validating the effectiveness of the proposed method. Finally, Section 5 summarizes the main contributions of the proposed work.

Throughout the paper, we adopt the following notations:

$\dagger$ Pseudo-inverse, $\otimes$ Kronecker product,

T Transpose, H Complex conjugate transpose,

$*$ linear convolution, $\circledast$ circular convolution,

$\| \; \|_F$ Frobenius norm, and $\mathrm{vec}\,()$ a stacking of the columns of the involved matrix into a vector.

2. Preliminary

In this section, the general signal model for MIMO-OFDM systems with pulse-shaping is first presented. A brief review of linear prediction based semi-blind MIMO-OFDM channel estimation is then provided to facilitate the development of the new channel estimation approach in the following sections.

2.1 Data Model

Fig. 1 shows the block diagram of a typical transmitter in a MIMO-OFDM system with the V-BLAST structure, in which there are N_T independent transmit chains, each connected to a transmit antenna. For the i_T-th transmit chain, the pilots are added to the information data. The m-th OFDM symbol can be written as a vector of the frequency-domain signals, namely,

$$\mathbf{X}_{i_T}(m) \triangleq \left[X_{i_T}(m,0), X_{i_T}(m,1),, \cdots, X_{i_T}(m,K-1)\right]^T .$$

where K denotes the number of subcarriers. The output of the IDFT gives the time-domain OFDM signal,

$$\mathbf{x}_{i_T}(m) \triangleq \left[x_{i_T}(m,0), x_{i_T}(m,1), \cdots, x_{i_T}(m,K-1)\right]^T .$$

After adding a cyclic prefix, each OFDM signal passes through a pulse-shaping filter and is then sent out by the corresponding antenna.

Fig. 2 shows the block diagram of the MIMO-OFDM receiver including N_R receive antennas as well as the channel estimation unit. After removing cyclic prefix in each receive chain, the received signal at the i_R-th chain can be described as

$$\mathbf{y}_{i_R}(m) \triangleq \left[y_{i_R}(m,0), y_{i_R}(m,1), \cdots, y_{i_R}(m,K-1)\right]^T . \tag{1}$$

Then, the received frequency domain signal after the DFT processing is given by

$$\mathbf{Y}_{i_R}(m) \triangleq \left[Y_{i_R}(m,0), Y_{i_R}(m,1), \cdots, Y_{i_R}(m,K-1)\right]^T .$$

Considering that MIMO-OFDM systems are designed for broadband wireless communications, the signal bandwidth is always larger than the coherence bandwidth, implying that the channel is frequency-selective. Depending on the geometry of the antenna array and scatterers, there are two different multipath MIMO channel models (or a combination of the two), namely, the beamforming model and the diversity model. Nevertheless, both of these channel models can be considered as a combination of L_c multi-paths, namely,

$$\mathbf{H}_c(t) = \sum_{l=0}^{L_c-1} \boldsymbol{\Gamma}_l \delta(t-t_l)$$

where t_l is the delay of the l-th path and $\boldsymbol{\Gamma}_l$ is an $N_R \times N_T$ attenuation matrix. As a matter of fact, in practical communication systems, the pulse-shaping filter $g_t(t)$ and the matched filter $g_r(t)$ are normally used, leading to a composite channel which can be represented by an $N_R \times N_T$ matrix $\mathbf{H}(t)$, with its (i_R, i_T)-th element as

$$h_{i_R,i_T}(t) = h_{i_R,i_T,c}(t) * g_t(t) * g_r(t) \tag{2}$$

where $h_{i_R,i_T,c}(t)$ is the (i_R, i_T)-th element of $\mathbf{H}_c(t)$. Most of the existing channel estimation methods focus on the composite discrete-time channel, i.e. the sampled version of the continuous-time channel response. Thus, each element of the discrete-time MIMO-FIR channel is an L-tap FIR filter. Moreover, the channel can be considered constant during one OFDM symbol, even though it may change over different symbols. Therefore, for the l-th tap, the channel matrix is given by

$$\mathbf{H}(m,l) \triangleq \begin{bmatrix} h_{1,1}(m,l) & h_{1,2}(m,l) & \cdots & h_{1,N_T}(m,l) \\ h_{2,1}(m,l) & h_{2,2}(m,l) & \cdots & h_{2,N_T}(m,l) \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R,1}(m,l) & h_{N_R,2}(m,l) & \cdots & h_{N_R,N_T}(m,l) \end{bmatrix} \in C^{N_R \times N_T}$$

where $h_{i_R,i_T}(m,l), (0 \le l \le L-1)$ represents the composite channel response between the i_R-th receive antenna and i_T-th transmit antenna for the l-th tap for the m-th OFDM symbol.

For notational simplicity, the index m of OFDM symbols can be dropped without loss of clarity. Thus, $\mathbf{H}(m,l)$, $x_{i_T}(m,n)$ and $y_{i_R}(m,n)$ reduce to $\mathbf{H}(l)$, $x_{i_T}(n)$ and $y_{i_R}(n)$, respectively. If the length of the cyclic is not less than the channel length L, the time-domain signal model for the frequency-selective fading channel can be written as

$$y_{i_R}(n) = \sum_{i_T=1}^{N_T} h_{i_R,i_T}(n) \circledast x_{i_T}(n) + v_{i_R}(n) \tag{3}$$

where the noise $v_{i_R}(n) \in C^{N_R \times 1}$ is a spatio-temporally uncorrelated noise with zero mean and variance δ_v^2.

2.2 Brief review of Linear-prediction-based Semi-Blind Channel Estimation

Here we briefly review our previous work on the linear prediction-based semi-blind MIMO-OFDM channel estimation [12], [13]. The main idea is to employ the MIMO linear predictor to obtain a blind constraint on the channel vector $\mathbf{h} \triangleq \text{vec}[\mathbf{h}_1, \cdots, \mathbf{h}_{N_R}]$ [12], [13], whose sub-vector is given by

$$\mathbf{h}_{i_R} \triangleq [h_{i_R,1}(0), \cdots, h_{i_R,1}(L-1), \cdots, h_{i_R,N_T}(L-1)]^T.$$

By defining

$$\mathbf{y}(n) \triangleq [y_1(n), \cdots, y_{N_R}(n)]^T, \tag{4}$$

$$\mathbf{y}_P(n-1) \triangleq \left[\mathbf{y}^T(n-1), \cdots, \mathbf{y}^T(n-P)\right]^T, \tag{5}$$

$$\tilde{\mathbf{R}}_{n-1} \triangleq \mathrm{E}\left\{\mathbf{y}_P(n-1)\mathbf{y}_P^H(n-1)\right\}, \tag{6}$$

$$\ddot{\mathbf{R}}_n \triangleq \mathrm{E}\left\{\mathbf{y}(n)\mathbf{y}_P^H(n-1)\right\}, \tag{7}$$

the MIMO linear predictor can be written as [12], [13]

$$\mathbf{P}_P \triangleq [\mathbf{P}_P(1), \mathbf{P}_P(2), \cdots, \mathbf{P}_P(P)] = \ddot{\mathbf{R}}_n \tilde{\mathbf{R}}_{n-1}^{-1} \tag{8}$$

where $\mathbf{P}_P(n), (n = 1, \cdots, P)$, is an $N_R \times N_R$ matrix representing the n-th tap of the prediction filter. By using (8), a blind constraint on the channel vector $\mathbf{h}$ can be derived as [12], [13]

$$\mathbf{B} = (\mathbf{I} \otimes \mathbf{P}_\Sigma)\mathbf{E}_P \tag{9}$$

where $\mathbf{I}$ is an identity matrix, $\mathbf{E}_P$ is a known permutation matrix, and $\mathbf{P}_\Sigma$ is a matrix determined by the null subspace of $\mathbf{H}(0)$ and a block Toeplitz matrix consisting of $\mathbf{P}_P(n), (n = 1, \cdots, P)$.

By combining the blind constraint with a training-based LS criterion, a time-domain semi-blind channel estimation problem can be formulated as

$$\min_{\hat{\mathbf{h}}} \Delta = \left\|\mathbf{Y}_{\text{pilot}} - \tilde{\mathbf{A}}\hat{\mathbf{h}}\right\|_F^2 + \alpha\left\|\hat{\mathbf{B}}\hat{\mathbf{h}}\right\|_F^2 \tag{10}$$

where $\tilde{\mathbf{A}}$ is a pilot signal matrix, $\mathbf{Y}_{\text{pilot}}$ is the corresponding received signal vector, $\hat{\mathbf{B}}$ is an estimate of the blind constraint and $\alpha > 0$ is a weighting factor. The solution to this optimization problem is given by

$$\hat{\mathbf{h}} = \left(\tilde{\mathbf{A}}^H\tilde{\mathbf{A}} + \alpha\hat{\mathbf{B}}^H\hat{\mathbf{B}}\right)^\dagger \tilde{\mathbf{A}}^H\mathbf{Y}_{\text{pilot}} \tag{11}$$

where the value of α can be determined using the scheme presented in [13].

3. Proposed Channel Estimation with Pulse-Shaping

As the pulse-shaping filter and the matched filter are normally pre-determined in a communication system, their knowledge could be exploited to improve the channel estimation accuracy. However, many of the existing channel estimation methods such as [4], [5], [13] have not yet taken into account the pulse-shaping in the transmitter and the matched filtering in the receiver. In this section, by utilizing the knowledge of pulse shaping and matched filtering, two improved channel estimation approaches are proposed for pulse-shaped MIMO-OFDM systems.

3.1 Channel Modelling of Pulse-Shaped MIMO-OFDM Systems

A typical pulse-shaping filter in communication systems has the following raised-cosine impulse response [23], [24]

$$g(t) = \text{sinc}\left(\frac{\pi t}{T}\right)\frac{\cos\left(\frac{\beta\pi t}{T}\right)}{1-\left(\frac{2\beta t}{T}\right)^2}$$

where β is the roll-off factor and T the symbol period. In digital communications, the pulse-shaping filter is often realized by an up-sampled raised-cosine FIR filter [18], [20], [22], [25]. Thus, the composite channel model should include the pulse-shaping filter, the analog multi-path channel $\mathbf{H}_c(t)$ and the matched filter as shown in Fig. 3 [17], [18], [21]–[23]. In this model, an upsampling is implemented by inserting $M-1$ zeros between any two consecutive input samples prior to pulse-shaping. The transmit filter $g_t(t)$ and the receive filter $g_r(t)$ in (2) are replaced by two root raised-cosine FIR filters $g_t(n)$ and $g_r(n)$, whose sampling period is $\frac{T}{M}$. In the upsampling domain, namely, the discrete-time domain with a

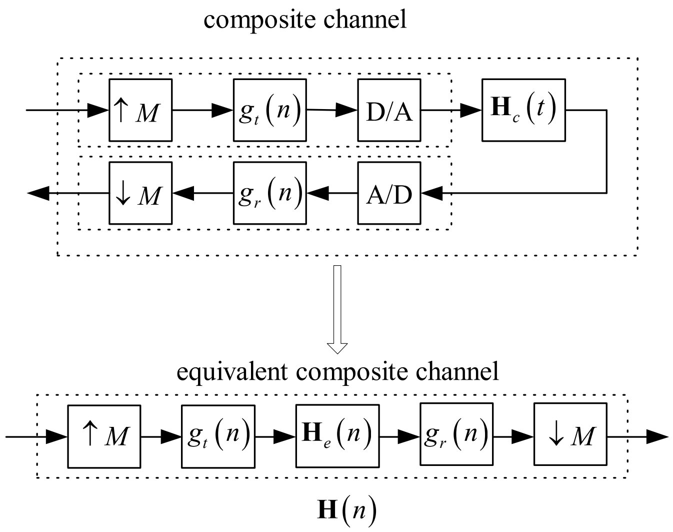

Fig. 3. Discrete-time channel model with pulse-shaping

sampling duration of $\frac{T}{M}$, the combination of the D/A converter, the multipath channel $\mathbf{H}_c(t)$ and the A/D converter can be represented by an equivalent discrete-time "multipath" channel $\mathbf{H}_e(n)$. In general, the channel path may not arrive at the exact sampling time, but it can be considered as an equivalent path occurring at the sampling instant synchronized to $\mathbf{H}_e\left(\frac{T}{M}\right)$, since the waveform of the D/A converter can normally be assumed as $p(t) = 1,\, 0 \le t < \frac{T}{M}$. Thus, in the case of L_d paths, the discrete-time channel $\mathbf{H}_e(n)$ can be represented by

$$\mathbf{H}_e(n) = \sum_{i=0}^{L_d-1} \mathbf{D}(i)\,\delta(n - l_i) \tag{12}$$

where $\mathbf{D}(i)$ and l_i are the channel matrix and the delay with respect to the i-th path. Now the composite discrete-time channel $\mathbf{H}(n)$ can be regarded as a downsamped version of the convolution of the transmit pulse-shaping filter $g_t(n)$, the discrete-time multipath channel $\mathbf{H}_e(n)$ and the received matched filter $g_r(n)$. It should be mentioned that the delay l_i can be determined prior to channel estimation. In particular, in advanced wireless networks, the time of arrivals (TOA) are often estimated at the start of communication and updated periodically [26]–[28]. For example, the TOA estimation is conducted by using the ranging techniques for the uplink synchronization phase of the OFDMA (OFDM Access) systems [26] or for some geolocation applications [27]. On the other hand, TOA are known to be a slow fading parameter, compared with the fast fading parameter (i.e. complex fading amplitude), which means that once an estimate of TOA is obtained, it can be used to estimate the fading amplitude for a relatively larger period of time. In the following, based on the knowledge of TOAs, i.e., l_i, $(i = 0, 1, \cdots, L_d - 1)$, we develop two improved channel estimation approaches for the MIMO-OFDM systems.

3.2 Semi-Blind Estimation for Sampling Duration-based Channels

Let us consider first the case of sampling duration-based channels in which the path delay is measured by a multiple of the sampling duration T, i.e., $l_i = iM$. In this case, the (i_R, i_T)-th element of the composite discrete-time channel $\mathbf{H}(n)$ is given by

$$h_{i_R,i_T}(n) = g_0(n) * d_{i_R,i_T}(n) \tag{13}$$

where $d_{i_R,i_T}(n)$ is the (i_R, i_T)-th element of the discrete-time multipath channel $\mathbf{H}_e(n) = \mathbf{D}(n)$, and $g_0(n) = g(Mn)$ with $g(n) = g_t(n) * g_r(n)$. Note that, in (13), $g_0(n) = 0,\ n \notin [0, L_g - 1]$, $d_{i_R,i_T}(n) = 0, n \notin [0, L_d - 1]$, and $L = L_g + L_d - 1$. It should be mentioned that a common assumption used in many existing algorithms is $g_0(n) = \delta(n)$, which implies $\mathbf{H}(n) = \mathbf{D}(n)$. In this sense, therefore, the pulse-shaping effect has been neglected. However, this assumption is not true in practical systems. In what follows, we will improve the semi-blind algorithm proposed in [12], [13] by using (13), namely, we will estimate $\mathbf{D}(n)$, instead of the large-dimensional matrix $\mathbf{H}(n)$, with the information of $g_0(n)$. Since the number of channel parameters has been considerably decreased, the estimation performance of the new approach is expected to be much better than that of those focusing only on the estimation of the composite channel $\mathbf{H}(n)$.

Using (13), the channel link between the i_R-th receive antenna and i_T-th transmit antenna can be described by the following vector,

$$\mathbf{h}_{i_R,i_T} \triangleq \left[h_{i_R,i_T}(0), \cdots, h_{i_R,i_T}(L-1)\right]^T = \mathbf{\Lambda}\mathbf{d}_{i_R,i_T} \tag{14}$$

where $\mathbf{\Lambda}$ is an $L \times L_d$ circulant matrix with its first column given by $\left[g_0(0), \cdots, g_0(L_g), \mathbf{0}_{1\times(L-L_g)}\right]^T$ and $\mathbf{d}_{i_R,i_T} \triangleq \left[d_{i_R,i_T}(0), \cdots, d_{i_R,i_T}(L_d - 1)\right]^T$. From (14), the i_R-th partition of the composite channel vector $\mathbf{h}$ can be written as

$$\mathbf{h}_{i_R} \triangleq \left[\mathbf{h}_{i_R,1}^T, \cdots, \mathbf{h}_{i_R,N_T}^T\right]^T = \left(\mathbf{I}_{N_T} \otimes \mathbf{\Lambda}\right)\mathbf{d}_{i_R} \tag{15}$$

where $\mathbf{d}_{i_R} \triangleq \left[\mathbf{d}_{i_R,1}^T, \cdots, \mathbf{d}_{i_R,N_T}^T\right]^T$. Using (15), one can obtain

$$\mathbf{h} = \mathbf{\Psi}\mathbf{d} \tag{16}$$

where $\mathbf{\Psi} \triangleq \left[\mathbf{I}_{N_R} \otimes \left(\mathbf{I}_{N_T} \otimes \mathbf{\Lambda}\right)\right]$ and $\mathbf{d} \triangleq \left[\mathbf{d}_1^T, \cdots, \mathbf{d}_{N_R}^T\right]^T$. Thus, (16) gives the relationship between the composite channel vector and the pure multipath channel vector.

Substituting (16) into (10), a new semi-blind cost function for MIMO-OFDM channel estimation with pulse-shaping can be formulated as

$$\min_{\hat{\mathbf{h}}} \Delta = \left\|\mathbf{Y}_{\text{pilot}} - \tilde{\mathbf{A}}'\mathbf{d}\right\|_F^2 + \alpha\left\|\hat{\mathbf{B}}'\mathbf{d}\right\|_F^2 \tag{17}$$

where $\tilde{\mathbf{A}}' \triangleq \tilde{\mathbf{A}}\mathbf{\Psi}$ and $\hat{\mathbf{B}}' \triangleq \hat{\mathbf{B}}\mathbf{\Psi}$. Similar to (11), the estimate of the channel vector can be derived as

$$\hat{\mathbf{d}} = \left(\left(\tilde{\mathbf{A}}'\right)^H \tilde{\mathbf{A}}' + \alpha\left(\hat{\mathbf{B}}'\right)^H \hat{\mathbf{B}}'\right)^\dagger \left(\tilde{\mathbf{A}}'\right)^H \mathbf{Y}_{\text{pilot}}. \tag{18}$$

The above equation gives a time-domain semi-blind solution for the channel estimation of MIMO-OFDM systems with pulse-shaping. Note that the computational complexity of $\hat{\mathbf{d}}$ depends on the sizes of the matrices $\tilde{\mathbf{A}}'$ and $\hat{\mathbf{B}}'$, which are determined by the length of the pulse-shaping and matched filters as well as the length of the pure multipath channel. When the length L_g of the filter $g(n)$ is relatively small as compared to the channel length L_d, (18) gives an efficient channel estimate. For a large value of L_g, however, the total length $L = L_g + L_d - 1$ of the composite channel can be very large which may incur a high complexity in the computation of (18). In what follows, we propose a very efficient frequency-domain estimation approach regardless of the relative size of L_g.

The frequency-domain signal model between the i_R-th receive antenna and the i_T-th transmit antenna can be represented by

$$Y_{i_R}(k) = H_{i_R,i_T}(k) X_{i_T}(k), \text{for } k \in [0, K-1] \quad (19)$$

where $H_{i_R,i_T}(k) \triangleq \mathbf{f}_{0k}\left[\mathbf{h}_{i_R,i_T}^T, \mathbf{0}_{1\times(K-L)}\right]^T$ and $\mathbf{f}_{0k}$ is the k-th row of the $K \times K$ DFT matrix $\mathbf{F}_0$. From (13), we can derive

$$H_{i_R,i_T}(k) = G(k) D_{i_R,i_T}(k) \quad (20)$$

where $G(k) \triangleq \mathbf{f}_{0k}\left[g_0(0), \cdots, g_0(L_g-1), \mathbf{0}_{1\times(L-L_g)}\right]^T$ and $D_{i_R,i_T}(k) \triangleq \mathbf{f}_{0k}\left[\mathbf{d}_{i_R,i_T}^T, \mathbf{0}_{1\times(K-L_b)}\right]^T$. Using (20) into (19), we can obtain the received signal after removing the effect of the pulse-shaping and matched filters,

$$\begin{aligned} Y'_{i_R}(k) &\triangleq G^{-1}(k) Y_{i_R}(k) \\ &= D_{i_R,i_T}(k) X_{i_T}(k) + V'(k) \end{aligned} \quad (21)$$

where $V'(k)$ is the frequency-domain noise. Clearly, by using the IFFT to (21), a time-domain version of the received signal without the effect of pulse shaping and matched filters can be obtained as

$$y'_{i_R}(n) = \sum_{i_T=1}^{N_T} d_{i_R,i_T}(n) \circledast x_{i_T}(n) + v'_{i_R}(n). \quad (22)$$

Interestingly, (22) is similar to the time-domain signal model in (3). Therefore, a semi-blind solution for the discrete-time multipath channel $\hat{\mathbf{d}}$ can be obtained by using the semi-blind method discussed in Subsection 2.2.

Before closing this subsection, we would like to evaluate the complexity of the proposed frequency-domain semi-blind method as opposed to the regular semi-blind estimation method. Here, we mainly consider three major steps involved in semi-blind methods. Note that a complexity of W^3 multiplications is considered for a regular inverse of a $W \times W$ matrix. A detailed comparison of the two methods is given in Table I. It is obvious that the frequency-domain semi-blind method has a smaller complexity than the regular semi-blind method, while the former becomes more advantageous for a small L_d.

3.3 LS Estimation for Upsampling Duration-based Channels

Now, we consider the general upsampling-duration-based channels. By using Fig. 3 and (12) and noting that the equivalent composite channel $\mathbf{H}(n)$ is a down-sampled version of the discrete-time multipath channel $\mathbf{H}_e(n)$ given by (12), one can express $\mathbf{H}(n)$ as

$$\mathbf{H}(n) = \sum_{i=0}^{L_d-1} \mathbf{D}(i) b_i(n - m_i) \quad (23)$$

where

$$m_i = \lfloor \frac{l_i + M - 1}{M} \rfloor, \quad (24)$$

$$b_i(n) = g_q(n) = g(Mn + q), \quad q = m_i M - l_i. \quad (25)$$

As $l_i\,(i = 0, 1, \cdots, L_d - 1)$ are known, m_i and $b_i(n), (i = 0, 1, \cdots, L_d - 1)$ can be obtained by using (24) and (25), respectively. Now, we propose an enhanced LS algorithm for the estimation of $\mathbf{D}(i)\,(i = 0, 1, \cdots, L_d - 1)$, with the information of m_i and $b_i(n)\,(i = 0, 1, \cdots, L_d - 1)$.

Assume that K_p sub-carriers, say from i_{pilot1} to $i_{\text{pilot}K_p}$, of each OFDM symbol carry the pilot signal. The transmitted and the received pilot vectors for each transmit and receive antenna can be defined as

$$\mathbf{X}_{i_T,\text{pilot}}(m) \triangleq \left[X_{i_T}(m, i_{\text{pilot1}}), \cdots, X_{i_T}\left(m, i_{\text{pilot}K_p}\right)\right]^T,$$

$$\mathbf{Y}_{i_R,\text{pilot}}(m) \triangleq \left[Y_{i_R}(m, i_{\text{pilot1}}), \cdots, Y_{i_R}\left(m, i_{\text{pilot}K_p}\right)\right]^T.$$

It should be noted that the pilot signal might not be located at the same position in each OFDM symbol. Letting $\mathbf{F}_1$ consist of the first L columns of a $K \times K$ DFT matrix $\mathbf{F}_0$, for the m-th OFDM symbol, one can form a $K_P \times L$ matrix, say $\mathbf{F}(m)$, by taking only the rows of $\mathbf{F}_1$ associated with the K_p pilot sub-carriers. It was shown in [4] that

$$\mathbf{Y}_{i_R,\text{pilot}}(m) = \sum_{i_T=1}^{N_T} \mathbf{X}_{i_T,\text{diag}}^m \mathbf{F}(m) \mathbf{h}_{i_R,i_T} + \boldsymbol{\xi}_{i_R,\text{pilot}}(m) \quad (26)$$

where $\mathbf{X}_{i_T,\text{diag}}^m \triangleq \text{diag}(\mathbf{X}_{i_T,\text{pilot}}(m))$ and $\boldsymbol{\xi}_{i_R}(m)$ represents the frequency-domain noise corresponding to $v_{i_R}(m,n)$ in (3). From (23), one can obtain

$$\mathbf{h}_{i_R,i_T} = [\mathbf{b}_0, \mathbf{b}_1, \cdots, \mathbf{b}_{L_d-1}] \mathbf{d}_{i_R,i_T} \quad (27)$$

where $\mathbf{d}_{i_R,i_T} = [d_{i_R,i_T}(0), \cdots, d_{i_R,i_T}(L_d-1)]^T$ and $\mathbf{b}_i$ is a vector whose k-th element is given by

$$\mathbf{b}_i(k) = \begin{cases} b_i(k - m_i), & m_i \le k \le L_g + m_i - 1 \\ 0, & \text{otherwise} \end{cases}.$$

From (26) and (27), the frequency-domain pilot signal received at the i_R-th receive antenna with respect to g OFDM symbols can be obtained as

$$\mathbf{Y}_{i_R,\text{pilot}} = \mathbf{A}_{\text{d}} \mathbf{d}_{i_R} + \boldsymbol{\xi}_{i_R,\text{pilot}} \quad (28)$$

where

$$\mathbf{Y}_{i_R,\text{pilot}} \triangleq \left[\mathbf{Y}_{i_R,\text{pilot}}^T(0), \cdots, \mathbf{Y}_{i_R,\text{pilot}}^T(g-1)\right]^T,$$

$$\mathbf{A}_{\text{d}} \triangleq \begin{bmatrix} \mathbf{X}_{1,\text{diag}}^0 \mathbf{F}(0) & \cdots & \mathbf{X}_{N_T,\text{diag}}^0 \mathbf{F}(0) \\ \vdots & \ddots & \vdots \\ \mathbf{X}_{1,\text{diag}}^{g-1} \mathbf{F}(g-1) & \cdots & \mathbf{X}_{N_T,\text{diag}}^{g-1} \mathbf{F}(g-1) \end{bmatrix} \otimes [\mathbf{b}_0, \mathbf{b}_1, \cdots, \mathbf{b}_{L_d-1}],$$

TABLE I
COMPLEXITY COMPARISON OF THE FREQUENCY-DOMAIN AND THE REGULAR SEMI-BLIND ALGORITHMS

	Frequency-Domain Semi-Blind Algorithm	Regular Semi-Blind Algorithm
Step 1: SOS estimation	Calculate $\mathbf{R}'_y(m)$, $m=0,1,\cdots,L_d-1$: $L_dKN_R^2$ multiplications $L_d(K-1)N_R^2$ additions	Calculate $\mathbf{R}_y(m)$, $m=0,1,\cdots,L-1$: LKN_R^2 multiplications $L(K-1)N_R^2$ additions
Step 2: linear preditor calculation	$\ddot{\mathbf{R}}'_n\tilde{\mathbf{R}}'^{-1}_{n-1}$ $[(L_d-1)N_R]^3$ multiplications	$\ddot{\mathbf{R}}_n\tilde{\mathbf{R}}^{-1}_{n-1}$ $[(L-1)N_R]^3$ multiplications
Step 3 final: channel estimation	$\left(\tilde{\mathbf{A}}'^H\tilde{\mathbf{A}}'+\alpha\hat{\mathbf{B}}'^H\hat{\mathbf{B}}'\right)^\dagger$ $(L_dN_RN_T)^3$ multiplications	$\left(\tilde{\mathbf{A}}^H\tilde{\mathbf{A}}+\alpha\hat{\mathbf{B}}^H\hat{\mathbf{B}}\right)^\dagger$ $(LN_RN_T)^3$ multiplications
In Total	Multiplications: $L_dKN_R^2+\left[(L_d-1)^3+(L_dN_T)^3\right]N_R^3$ Additions: $L_d(K-1)N_R^2$	Multiplications: $LKN_R^2+\left[(L-1)^3+(LN_T)^3\right]N_R^3$ Additions: $L(K-1)N_R^2$

$$\mathbf{d}_{i_R}\triangleq\left[\mathbf{d}^T_{i_R,1},\cdots,\mathbf{d}^T_{i_R,N_T}\right]^T,$$

$$\boldsymbol{\xi}_{i_R,\text{pilot}}\triangleq\left[\boldsymbol{\xi}^T_{i_R,\text{pilot}}(0),\cdots,\boldsymbol{\xi}^T_{i_R,\text{pilot}}(g-1)\right]^T.$$

By defining

$$\bar{\mathbf{Y}}_{\text{pilot}}\triangleq\left[\mathbf{Y}_{1,\text{pilot}},\cdots,\mathbf{Y}_{N_R,\text{pilot}}\right],$$

$$\bar{\mathbf{D}}\triangleq\left[\mathbf{d}_1,\cdots,\mathbf{d}_{N_R}\right],\tag{29}$$

$$\bar{\boldsymbol{\xi}}_{\text{pilot}}\triangleq\left[\boldsymbol{\xi}_{1,\text{pilot}},\cdots,\boldsymbol{\xi}_{N_R,\text{pilot}}\right],$$

one can have

$$\bar{\mathbf{Y}}_{\text{pilot}}=\mathbf{A}_{\text{d}}\bar{\mathbf{D}}+\bar{\boldsymbol{\xi}}_{\text{pilot}}.\tag{30}$$

Subsequently, one can obtain

$$\hat{\bar{\mathbf{D}}}=\mathbf{A}^\dagger_{\text{d}}\bar{\mathbf{Y}}_{\text{pilot}}.\tag{31}$$

Finally, substituting $\hat{\bar{\mathbf{D}}}$ into (23) leads to the enhanced channel estimate $\hat{\mathbf{H}}(n)$. It will be shown in Section 4 that, when the number of multipath L_d is much less than the length L of the composite channel, the performance of the enhanced LS channel estimation is significantly superior to that of the original LS method. Moreover, the complexity of the enhanced LS method is much lower than that of the original version.

4. Simulation Results

Here we consider a MIMO-OFDM system with 2 transmit and 4 receive antennas. The number of subcarriers is set to 512, the length of cyclic prefix is 10, and the QPSK modulation is used. A square root raised cosine filter with order 16, oversampling rate 4 and rolloff factor 0.15 is used for the pulse-shaping filter and the matched filter. In our experiments, we simulate the semi-blind channel estimation approach for a sampling duration-based channel and the enhanced LS channel estimation method for an upsampling duration-based channel.

4.1 Sampling Duration-based Channel Scenario

A Rayleigh channel modelled by a 3-tap MIMO-FIR filter is assumed, in which each tap corresponds to a 2×4 random matrix whose elements are i.i.d. complex Gaussian variables with zero mean and unit variance. The length of the linear predictor is $P=4$. As shown in [12], [13], the channel estimation performance is associated with $\mathbf{H}(0)$. Accordingly, we define the metric $\eta\triangleq\frac{\|\mathbf{H}(0)\|_F^2}{\sum_{n=0}^{2}\|\mathbf{H}(n)\|_F^2}$ and conduct a simulation study with respect to different ranges of η.

In the experiments, for the purpose of comparison, the composite channel vector $\mathbf{h}$ is first estimated by the LS method. As for the estimation of the pure discrete-time channel vector $\mathbf{d}$, we consider the time-domain LS, the frequency-domain LS and the proposed frequency-domain semi-blind method, all with pulse shaping. For simplicity, we call these four methods as the basic LS, enhanced time-domain LS (TDLS), enhanced frequency-domain LS (FDLS) and enhanced semi-blind methods. Note that the LS methods can be easily obtained by setting α to zero in the proposed two semi-blind methods with pulse-shaping. The estimation performance is evaluated in terms of the MSE of the estimate of the channel matrix given by

$$\text{MSE}=\frac{1}{N_{\text{MC}}}\sum_{n=1}^{N_{\text{MC}}}\left\|\hat{\mathbf{h}}_n-\mathbf{h}_n\right\|^2$$

$$\left(\text{or MSE}=\frac{1}{\text{N}_{\text{MC}}}\sum_{\text{n}=1}^{\text{N}_{\text{MC}}}\left\|\hat{\mathbf{d}}_{\text{n}}-\mathbf{d}_{\text{n}}\right\|^2\right)$$

where N_{MC} is the number of Monte Carlo iterations, and $\mathbf{h}_n$ ($\mathbf{d}_n$) and $\hat{\mathbf{h}}_n$ ($\hat{\mathbf{d}}_n$) are the true and the estimated channel vectors with respect to the n-th Monte Carlo iteration, respectively.

Experiment A1: MSE versus η

In the first experiment, the channel estimation performance in terms of the MSE versus η is investigated. The simulation is undertaken by 1000 Monte Carlo runs of the transmission of one OFDM symbol under an SNR of 15dB at 512 subcarriers of which 30 are used as pilot for training purpose. Fig. 4 shows the MSE plots resulting from the proposed enhanced

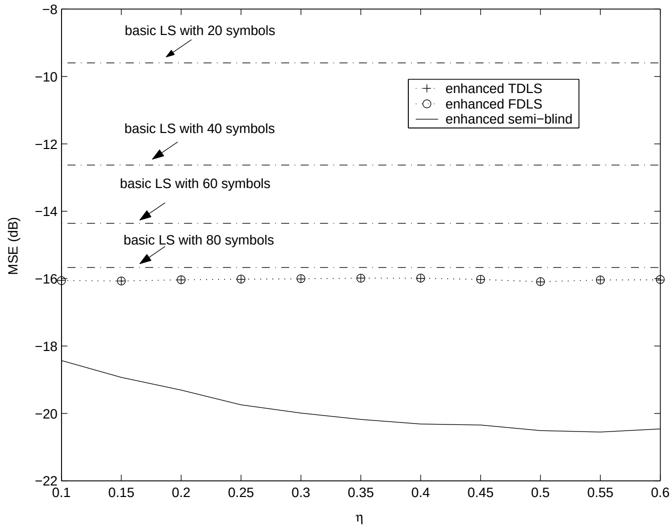

Fig. 4. MSE versus η for a sampling duration-based channel.

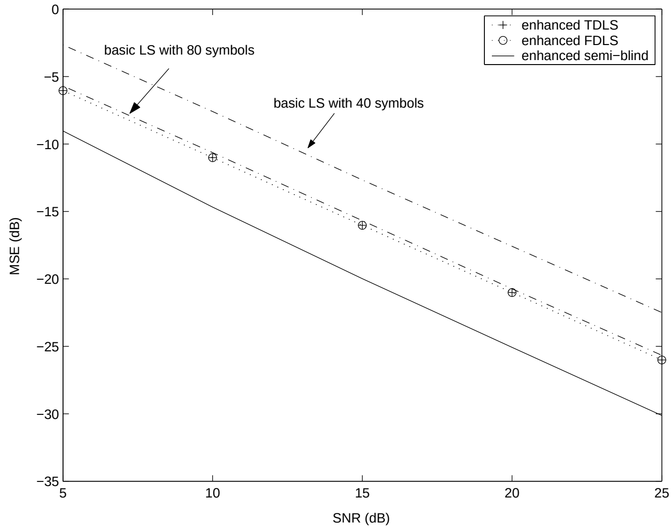

Fig. 5. MSE versus SNR for a sampling duration-based channel

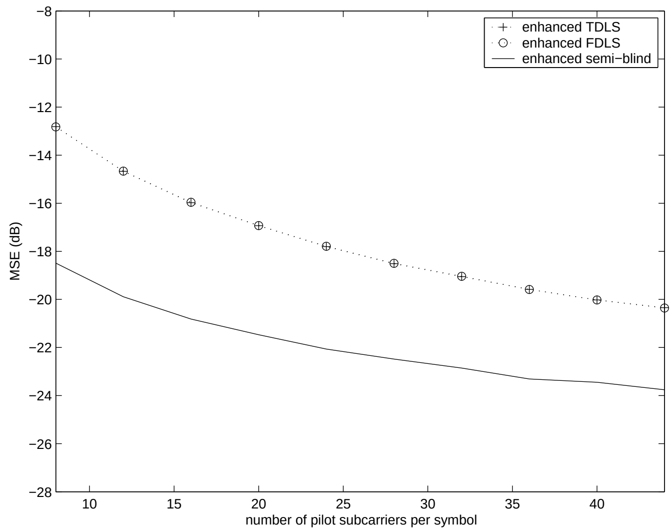

Fig. 6. MSE versus pilot length for a sampling duration-based channel

TDLS, FDLS and semi-blind methods as well as the basic LS estimation with 20, 40, 60 and 80 symbols, respectively. Obviously, the MSE performance of the proposed three enhanced methods are significantly better than that of the basic LS. In particular, the performance of the enhanced TDLS and FDLS methods with only one symbol is still better than that of the basic LS method with 80 symbols, indicating that the new approach focusing on the pure multipath channel vector significantly outperforms that for composite channel vector irrespective of pulse-shaping. One can find a high consistency between the enhanced TDLS and the enhanced FDLS methods. Also, the enhanced semi-blind method significantly outperforms the two enhanced LS methods. In addition, the performance of the semi-blind method improves with the increasing value of η when $\eta < 0.3$, and remains almost the same when η is in the range of 0.3 to 0.6, which represents typical mobile communication scenarios where the first arrived path is comparable to or stronger than other paths.

Experiment A2: MSE versus SNR

Now we investigate the channel estimation performance versus the SNR. The simulation involves 5000 Monte Carlo runs of the transmission of one OFDM symbol. Fig. 5 shows the channel estimation results of the three enhanced methods and that of the basic LS method with 40 and 80 symbols when $\eta > 0.2$. It is seen that the performances of the two LS methods are almost the same, which consistently outperform the basic LS method with 80 symbols. In addition, it is observed that the enhanced semi-blind can achieve nearly 3$\sim$4 dB gains over the two enhanced LS methods, when the SNR varies from 5 to 25 dB, respectively.

Experiment A3: MSE versus pilot length

Here we investigate the channel estimation performance of the enhanced semi-blind approach versus the number of pilot subcarriers per symbol, in comparison with that of the two enhanced LS methods. The number of OFDM symbols used in the three methods is set to 2, and the number of pilot subcarriers per OFDM symbol varies from 8 to 48. Fig. 6 shows the MSE plots from 500 Monte Carlo iterations at an SNR of 15 dB when $\eta > 0.2$. It is seen that the performance of all the algorithms is improved with an increasing number of pilot subcarriers. Again, the performance of the proposed TDLS and FDLS methods are almost the same, and the semi-blind method is superior to both TDLS and FDLS nearly by 6 and 4dB when the number of pilot subcarriers is 8 and 48, respectively. It implies that the proposed semi-blind method is more advantageous for pilot signals of a shorter length.

Experiment A4: BER versus SNR

In this experiment, the BER performance of the MIMO-OFDM system is investigated by using the estimated channel matrix and an ordered vertical-Bell laboratories layered space time (V-BLAST) decoder. The simulation involves 5000 Monte Carlo runs of the transmission of one OFDM symbol

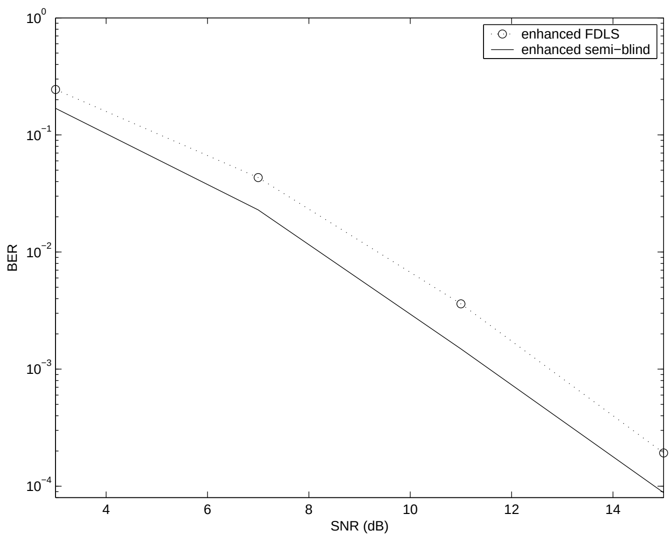

Fig. 7. BER versus SNR for a sampling duration-based channel

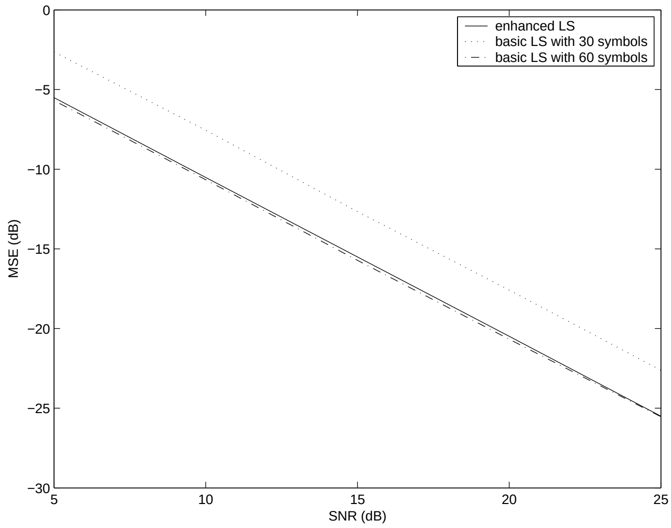

Fig. 8. MSE versus SNR for an upsampling duration-based channel.

with 8 pilot subcarriers. Fig.7 shows the BER performance versus the SNR for the FDLS method and the semi-blind method, when $\eta > 0.2$. It is seen that the performance of the semi-blind method is superior to the LS method by 1.6∼3.4 dB when the SNR varies in the range of $3 \sim 15\mathrm{dB}$.

4.2 Upsampling Duration-based Channel Scenario

A 3-path Rayleigh channel is assumed, in which each path corresponds to a 2×4 random matrix whose elements are i.i.d. complex Gaussian variables with zero mean and unit variance. The delays of the three paths are set to $0,\ \frac{3}{4}T,\ \mathrm{and}\ 2T$.

Experiment B1: MSE versus SNR

At first, we investigate the channel estimation performance versus the SNR. The simulation involves 5000 Monte Carlo runs of the transmission of one OFDM symbol. Fig. 8 shows the channel estimation result of the enhanced LS method and that of the basic LS method with 30 and 60 symbols. It is seen that the enhanced LS method with only one symbol can achieve 2.9 dB gain over the basic LS method with 30 symbols. Moreover, it is observed that 60 symbols are needed for the basic LS method to achieve the same performance as given by the enhanced LS method.

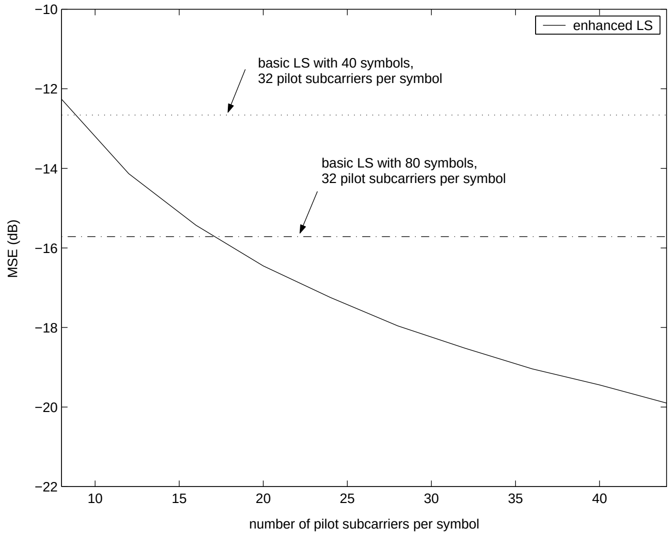

Fig. 9. MSE versus pilot length for an upsampling duration-based channel.

Experiment B2: MSE versus pilot length

Now we examine the channel estimation performance of the enhanced LS method versus the number of pilot subcarriers per symbol. The number of OFDM symbols used is set to 2, and the number of pilot subcarriers per OFDM symbol varies from 8 to 48. Fig. 9 shows the MSE plots from 5000 Monte Carlo iterations at an SNR of 15 dB when $\eta > 0.2$. It is seen that the performance of the enhanced LS algorithm is improved with the increase of the number of pilot subcarriers. In particular, the enhanced LS algorithm with 18 pilots (2 symbols with 9 pilot subcarriers) can achieve a similar result of the basic LS method with 1280 pilots (40 symbols with 32 pilot subcarriers). In addition, 36 pilots in the enhanced LS method serves as 2560 pilots in the basic LS method.

5. Conclusions

In this paper, the channel estimation issue of MIMO-OFDM systems with pulse shaping has been thoroughly studied. By taking into account the effect of both pulse-shaping filter in the transmitter and the matched filter in the receiver, a new channel estimation problem focusing on the estimation of the pure multipath channel has been formulated for two typical multipath scenarios corresponding to the sampling- and upsampling-duration based channels. Then, two novel channel estimation approaches, i.e., the semi-blind and the pilot-aided least-square methods, have been developed, respectively, for the two multipath cases. The effectiveness of the new channel estimation methods in terms of the MSE of the channel estimate has been confirmed by computer simulations with comparison to the basic LS method as well as its enhanced versions with pulse-shaping. Furthermore, it has been shown that the enhanced semi-blind technique significantly outperforms the enhanced time-domain or frequency-domain LS method.

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