Latent Space Model for Road Networks to Predict Time-Varying Traffic

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Dept. of Computer Science
University of Southern California
Motivation

• Traffic congestion is a major problem throughout the world
Motivation

- Traffic congestion is a major problem throughout the world [1]
  - 4.2 billion vehicle-hours of delay
  - 2.8 billion gallons of wasted fuel
  - $87.2 billion in lost productivity (0.7% GDP)

Big Data Opportunity
Big Data Opportunity

Loop detectors
Big Data Opportunity

Loop detectors

Mobile phones
Big Data Opportunity

Loop detectors

Mobile phones

Traffic cameras
Big Data Opportunity

- Loop detectors
- Mobile phones
- Traffic cameras
- Self-driving cars
Sensor Data Collected at IMSC, USC
Sensor Data Collected at IMSC, USC

Highway loop detectors
Sensor Data Collected at IMSC, USC

Arterial loop detectors

Highway loop detectors
Sensor Data Collected at IMSC, USC
Near-Future Traffic Prediction
Near-Future Traffic Prediction

Great, I can arrive on time

use 6pm data
Applications of Traffic Prediction

Route Navigation

Traffic Regulation

Urban Planning
Agenda

• Problem Definition
• Related Work
• Real-time Traffic Prediction with Latent Space Model for Road Network
Traffic Prediction Problem on Road Network

- **Problem Setting**: given series of recent road network snapshots, the goal is to predict the future traffic condition of the whole road network.

\[
G_1 \quad 7:00 \text{ am} \\
G_t \quad 7:55 \text{ am} \\
G_{t+1} \quad 8:00 \text{ am}
\]
Problem Definition

- **Input**: road network snapshots $\langle G_1, G_2, \ldots, G_i \ldots G_t \rangle$,
  - for each $G_i \ (1 \leq i \leq t)$, edge weight is the **travel speed** between two intersections
Problem Definition

- **Input**: road network snapshots \(<G_1, G_2, \ldots, G_i \ldots G_t>,\)
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• **Input**: road network snapshots $<G_1, G_2, \ldots, G_i \ldots G_t>$,
  ✦ for each $G_i \ (1 \leq i \leq t)$, edge weight is the **travel speed** between two intersections

• **Objective**:
  ✦ predict the travel speed of $G_{t+h}$, $h$ is the prediction horizon
Problem Definition

• **Input:** road network snapshots \(<G_1, G_2, \ldots, G_i \ldots G_t>\),
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• **Objective:**
  - predict the travel speed of \(G_{t+h}\), \(h\) is the prediction horizon

Predict the future travel speed for each and every edge on the road network
Challenges

• Data sparsity: missing data
Challenges

• Data sparsity: missing data
  • Missing sensors
Challenges

- Data sparsity: missing data
  - Missing sensors

Skewed distribution of sensors
Challenges

- Data sparsity: missing data
  - Missing sensors
  - Missing values
Challenges

- Data sparsity: missing data
  - Missing sensors
  - Missing values
Challenges

• Complex spatiotemporal relationship
  • Spatial relationship is dictated by topology
Challenges

• Complex spatiotemporal relationship

• Spatial relationship is dictated by topology

Traffic patterns of roads 1 and 2
Challenges

- Complex spatiotemporal relationship
- Spatial relationship is dictated by topology

Traffic patterns of roads 1, 2 and 3
Challenges

- Real-time prediction with streaming data

Sensor data come in a streaming fashion
Challenges

• Large road network (~100k vertices, ~150k edges)

Green lines are sensors, blue lines are network edges
Agenda

• Problem Definition

• Related Work

• Spatiotemporal Traffic Prediction on Road Network
  • Real-time Traffic Prediction with Latent Space Model for Road Network (Prior Work)
  • Blending Real-time and Historical Traffic for Better Traffic Prediction (Proposed Work)
## Related Work in Traffic Prediction

**Without Spatial Correlations**

<table>
<thead>
<tr>
<th>Related techniques</th>
<th>Related papers</th>
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<tbody>
<tr>
<td>(S)ARIMA, Hybrid ARIMA</td>
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The majority of techniques model each sensor/edge independently.
Related Work in Traffic Prediction

With Spatial Correlations
Related Work in Traffic Prediction

With Spatial Correlations

Euclidean Space
Related Work in Traffic Prediction

With Spatial Correlations

Euclidean Space

Vector ARIMA
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Network Space: topology

- Space-Time ARIMA
  - [Kamarianakis et al., TRB’03]
  - [Min and Wynter, TRC’11]

- Spatiotemporal HMM
  - [Yang et al., VLDB’13]
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✦ not robust to missing data
✦ cannot deal with network dynamics
✦ require complex inference, hard to scale to network
Agenda

• Problem Definition

• Related Work
  • Real-time Traffic Prediction with Latent Space Model for Road Network (Prior Work)
Traffic Prediction Approach

Observation

To be predicted
Traffic Prediction Approach

Observation → Modeling → To be predicted

Pattern
Traffic Prediction Approach

- Observation
  - Modeling
  - To be predicted
  - Pattern
Define a model that can generate the traffic condition of road network
Intuition

• Correlations between adjacent roads
Intuition

- Correlations from adjacent roads are not enough
Intuition

• Correlations from adjacent roads are not enough

Latent correlations deduced from road network
Intuition

- **Latent** correlations deduced from road network
Intuition

- **Latent** correlations deduced from road network
Latent Attributes

- Each vertex has **latent** attributes
- Vertex $i$ has latent attribute vector $u_i$

<table>
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<tr>
<th>$i$</th>
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$u_i \in R_{+}^{1 \times k}$
Latent Attributes

- Each vertex has latent attributes
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**Node attribute matrix** \( U^{nxk} \)

(n is the number of vertices)
Attribute Interaction

- Interaction matrix $B$ between different attributes

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**Attribute interaction matrix $B_{k\times k}$**
Attribute Interaction

- Interaction matrix $B$ between different attributes

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Speed from Latent Attribute

- Traffic speed between vertices $i$ and $j$ ($i \rightarrow j$) is a linear combination of the corresponding traffic patterns

$$d(i, j) = u_i \times B \times u_j^T$$
Speed from Latent Attribute

- Traffic speed between vertices $i$ and $j$ (i -> j) is a linear combination of the corresponding traffic patterns

$$d(i, j) = u_i \times B \times u_j^T$$

$u_i$

$$\begin{pmatrix} 40 & 0 \\ 0 & 10 \end{pmatrix} \times 1 = 50$$

highway  business
Basic Graph Model

Graph matrix: $G^{n \times n}$  
Latent properties: $U^{n \times k}$ and $B^{k \times k}$

Basic Graph Model

Graph matrix: $G^{nxn}$

Latent properties: $U^{nxk}$ and $B^{kxx}$

$$\begin{align*}
\arg \min_{U \geq 0, B \geq 0} & \quad J = \| G - U B U^T \|^2_F
\end{align*}$$

Non-negative Graph Matrix Factorization (NMF) [1]

Outline of LSM

Latent Space Model (LSM) for Road Network

Learning & Inference of LSM

- Global Learning Algorithm
- Incremental Learning Algorithm

Prediction with LSM

- Traffic Prediction with Missing Data
- Batch Mode for Real-time Prediction

Experiment Evaluation
Overview of LSM for Road Network
Overview of LSM for Road Network
Overview of LSM for Road Network
Overview of LSM for Road Network

\[ G_1 \rightarrow G_2 \rightarrow G_3 \rightarrow \ldots \rightarrow G_t \]

Learn

\[ U_1 \rightarrow U_2 \]
Overview of LSM for Road Network

Latent attributes evolve with different timestamps
Overview of LSM for Road Network

Latent attributes evolve with different timestamps
LSM for Road Network

**Basic Graph Model**

**Overcome the Sparsity**

**Temporal Effect**

**Transition Effect**

$$\arg \min_{U \geq 0, B \geq 0} J = \| G - U B U^T \|_F^2$$
LSM for Road Network

Basic Graph Model

Overcome the Sparsity

Temporal Effect

Transition Effect

\[ \arg \min_{U \geq 0, B \geq 0} J = \| G - U B U^T \|_F^2 \]

\[ \arg \min_{U, B} J = \| Y \odot (G - U B U^T) \|_F^2 + \lambda \text{Tr}(U^T L U) \]
LSM for Road Network

**Basic Graph Model**

\[
\arg\min_{U \geq 0, B \geq 0} J = \|G - UBU^T\|_F^2
\]

**Overcome the Sparsity**

\[
\arg\min_{U, B} J = \|Y \odot (G - UBU^T)\|_F^2 + \lambda Tr(U^T L U)
\]

**Temporal Effect**

\[
\arg\min_{U_i, B, A} J = \sum_{i=1}^{t} \|Y_i \odot (G_i - U_i B U_i^T)\|_F^2 + \sum_{i=1}^{t} \lambda Tr(U_i^T L U_i)
\]
**LSM for Road Network**

**Basic Graph Model**

\[ \arg \min_{U \geq 0, B \geq 0} J = \|G - UBU^T\|^2_F \]

**Overcome the Sparsity**

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**Transition Effect**

\[ \arg \min_{U_i, B, A} J = \sum_{i=1}^{t} \|Y_i \odot (G_i - U_iBU_i^T)\|^2_F + \sum_{i=1}^{t} \lambda \text{Tr}(U_i^T LU_i) + \sum_{i=2}^{t} \gamma \|U_i - U_{i-1}A\|^2_F \]
LSM for Road Network

\[
\arg \min_{U \geq 0, B \geq 0} J = \|G - UBU^T\|_F^2
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\[
\arg \min_{U, B} J = \|Y \odot (G - UBU^T)\|_F^2 + \lambda \text{Tr}(U^T LU)
\]
The Sparsity of Graph $G$

- Zero entries dominate the items of the graph matrix
The Sparsity of Graph $G$

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Example graph with 14 vertices and 18 edges
The Sparsity of Graph $G$

- Zero entries dominate the items of the graph matrix

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Most entries of the matrix are zero
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X: edges  
O: observed with sensors

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X: edges
O: observed with sensors

Most entries of the matrix are zero

Missing sensor: 4 out of 18 edges are observed
The Sparsity of Graph $G$

- Zero entries dominate the items of the graph matrix

$X$: edges
$O$: observed with sensors

Most entries of the matrix are zero

Missing sensor: 4 out of 18 edges are observed

Missing values from the 4 existing sensors
Can we learn a model from such a sparse matrix?
Intuition: Missing Value Imputation

• Data completion via nearby edges
  • Find nearby edges with same direction and category
  • Use the weighted average as the estimated readings
Intuition: Missing Value Imputation

- Data completion via nearby edges
  - Find nearby edges with same direction and category
  - Use the weighted average as the estimated readings
Smoothing for Similar Nodes

\[ penalty = \frac{1}{2} \sum_{i,j=1}^{n} \| u_i - u_j \|^2 W_{i,j} \]

Vertices that are close in the road network should be similar in the latent spaces
Smoothing for Similar Nodes

\[
\text{penalty} = \frac{1}{2} \sum_{i,j=1}^{n} \|u_i - u_j\|^2 W_{ij}
\]

\[
= \sum_{i} u_i^T u_i D_{ii} - \sum_{i,j=1}^{n} u_i^T u_j W_{ij}
\]

\[
= \text{Tr}(U^T DU) - \text{Tr}(U^T W U)
\]

\[
= \text{Tr}(U^T L U)
\]
Overcome the Sparsity: Graph Laplacian

\[
\arg\min_{U, B} J = \|G - UBU^T\|_F^2 + \lambda \text{Tr}(U^T L U)
\]

$L$: graph Laplacian matrix induced by the similarity matrix $W$
Overcome the Sparsity: Weighted NMF

- Objective function only considers the non-zero entries

\[
\arg \min_{U, B} J = \|Y \odot (G - UBU^T)\|_F^2 + \lambda Tr(U^T LU)
\]

where \( Y_{ij} = \begin{cases} 
1, & G(i, j) > 0 \\
0, & \text{Otherwise}
\end{cases} \)
LSM for Static Road Network

\[
\arg \min_{U,B} J = \| Y \odot (G - U B U^T) \|_F^2 + \lambda Tr(U^T L U)
\]

**Indication Matrix**

**Graph Laplacian to consider local similarity**
Input Example Graph
Input Example Graph

Grey color represents unknown edges
Completed Graph
Latent Space Model for Road Network

Basic Graph Model

Overcome the Sparsity

Temporal Effect

Transition Effect

arg \min_{U,B} J = \|Y \odot (G - UBU^T)\|_F^2 + \lambda \text{Tr}(U^T LU)

\arg \min_{U_i,B,A} J = \sum_{i=1}^{t} \|Y_i \odot (G_i - U_iBU_i^T)\|_F^2 + \sum_{i=1}^{t} \lambda \text{Tr}(U_i^T LU_i)
Time-dependent Latent Attribute
Time-dependent Latent Attribute

$$\arg \min_{U_i, B, A} J = \sum_{i=1}^{t} \|Y_i \odot (G_i - U_i B U_i^T)\|_F^2 + \sum_{i=1}^{t} \lambda Tr(U_i^T L U_i)$$

Latent attribute $U_i$ for each timestamp
Latent Space Model for Road Network

Basic Graph Model

Overcome the Sparsity

Temporal Effect

Transition Effect

\[ \arg \min_{U_i, B, A} J = \sum_{i=1}^{t} \|Y_i \odot (G_i - U_i B U_i^T)\|_F^2 + \sum_{i=1}^{t} \lambda \text{Tr}(U_i^T L U_i) \]

\[ \arg \min_{U_i, B, A} J = \sum_{i=1}^{t} \|Y_i \odot (G_i - U_i B U_i^T)\|_F^2 + \sum_{i=1}^{t} \lambda \text{Tr}(U_i^T L U_i) + \]

\[ \sum_{i=2}^{t} \gamma \|U_i - U_{i-1} A\|_F^2 \]
Evolvement of Latent Attribute

How to capture the changes?
Global Transition Matrix

• Learn one global transition matrix $\mathbf{A}$ that captures the state changes

$$U_t \approx U_{t-1} \mathbf{A}$$
Global Transition Matrix

- Learn one global transition matrix $A$ that captures the state changes

$$ U_t \approx U_{t-1} A $$

Arterial: $0.5$  
Highway: $0.8$  

$u_1=0.8$ at $t=1$
Global Transition Matrix

• Learn one global transition matrix $A$ that captures the state changes

$$U_t \approx U_{t-1} A$$

$u_1 = 0.8 \times A = 0.5$

$t=2$

$u_1=0.8$

$t=1$

Arterial

0.5

Highway

0.8
Meaning of Transition Matrix

- $A_{ij}$: How likely a node is to transit from attribute $i$ to attribute $j$

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<tr>
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<th>a2</th>
</tr>
</thead>
<tbody>
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<td>$p(a1\rightarrow a2)$</td>
</tr>
<tr>
<td>a2</td>
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Transition matrix $A_{k\times k}$
Meaning of Transition Matrix

- $A_{ij}$: How likely a node is to transit from attribute $i$ to attribute $j$

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Transition matrix $A_{kxk}$
Meaning of Transition Matrix

- $A_{ij}$: How **likely** a node is to transit from attribute $i$ to attribute $j$

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Transition matrix $A_{kxk}$
Add Global Transition

$$\arg \min_{U_i, B, A} J = \sum_{i=1}^{t} \|Y_i \odot (G_i - U_i B U_i^T)\|_F^2 + \sum_{i=1}^{t} \lambda \text{Tr}(U_i^T L U_i) + \sum_{i=2}^{t} \gamma \|U_i - U_{i-1} A\|_F^2$$

Global transition matrix
Latent Space Model for Road Network

- Basic Graph Model
- Overcome the Sparsity
- Temporal Effect
- Transition Effect
LSM for Road Network

![Mathematical Formulation]

Learn both time-dependent latent attributes and transition matrix
Outline of LSM

Latent Space Model (LSM) for Road Network

Learning & Inference of LSM

- Global Learning Algorithm
- Incremental Learning Algorithm

Prediction with LSM

- Traffic Prediction with Missing Data
- Batch Mode for Real-time Prediction

Experiment Evaluation
Global Learning

Jointly learn all the latent attributes together
While not convergent
for $i = 1$ to $t$

$$U_i \leftarrow U_i \odot \left( \frac{(Y_i \odot G_i)(U_i B^T + U_i B) + \lambda W U_i + \gamma (U_{i-1} A + U_{i+1} A^T)}{(Y_i \odot U_i B U_i^T)(U_i B^T + U_i B) + \lambda D U_i + \gamma (U_i + U_i A A^T)} \right)^{\frac{1}{4}}$$

$$B \leftarrow B \odot \left( \frac{\sum_{t=1}^{T} U_t^T (Y_t \odot G_t) U_t}{\sum_{t=1}^{T} U_t^T (Y_t \odot (U_t B U_t^T)) U_t} \right)$$

$$A \leftarrow A \odot \left( \frac{\sum_{t=1}^{T} U_{t-1}^T U_t}{\sum_{t=1}^{T} U_{t-1}^T U_{t-1} A} \right)$$

Multiplicative Update \cite{1}

While not convergent

for $i = 1$ to $t$

\[ U_i \leftarrow U_i \odot \left( \frac{(Y_i \odot G_i)(U_iB^T + U_iB) + \lambda WU_i + \gamma(U_{i-1}A + U_{i+1}A^T)}{(Y_i \odot U_iBU_i^T)(U_iB^T + U_iB) + \lambda DU_i + \gamma(U_i + U_iAAT)} \right)^{\frac{1}{4}} \]

\[ B \leftarrow B \odot \left( \frac{\sum_{t=1}^{T} U_t^T (Y_t \odot G_t) U_t}{\sum_{t=1}^{T} U_t^T (Y_t \odot (U_tBU_t^T)) U_t} \right) \]

\[ A \leftarrow A \odot \left( \frac{\sum_{t=1}^{T} U_{t-1}^T U_t}{\sum_{t=1}^{T} U_{t-1}^T U_{t-1} A} \right) \]

\[ \text{Update } U_i \]

Multiplicative Update \[1\]

While not convergent
for \(i = 1\) to \(t\)

\[
U_i \leftarrow U_i \odot \left( \frac{(Y_i \odot G_i)(U_i B^T + U_i B) + \lambda W U_i + \gamma(U_{i-1} A + U_{i+1} A^T)}{(Y_i \odot U_i B U_i^T)(U_i B^T + U_i B) + \lambda D U_i + \gamma(U_i + U_i A A^T)} \right)^{1/4}
\]

Update \(U_i\)

\[
B \leftarrow B \odot \left( \frac{\sum_{t=1}^{T} U_t^T (Y_t \odot G_t) U_t}{\sum_{t=1}^{T} U_t^T (Y_t \odot (U_t B U_t^T)) U_t} \right)
\]

Update \(B\)

\[
A \leftarrow A \odot \left( \frac{\sum_{t=1}^{T} U_{t-1}^T U_t}{\sum_{t=1}^{T} U_{t-1}^T U_{t-1} A} \right)
\]

Multiplicative Update \[^1\]

While not convergent

for \( i = 1 \) to \( t \)

Update \( U_i \)

\[
U_i \leftarrow U_i \odot \left( \frac{(Y_i \odot G_i)(U_i B^T + U_i B) + \lambda W U_i + \gamma(U_{i-1} A + U_{i+1} A^T)}{(Y_i \odot U_i B U_i^T)(U_i B^T + U_i B) + \lambda D U_i + \gamma(U_i + U_i A A^T)} \right)^{1/4}
\]

Update \( B \)

\[
B \leftarrow B \odot \left( \frac{\sum_{t=1}^{T} U_t^T (Y_t \odot G_t) U_t}{\sum_{t=1}^{T} U_t^T (Y_t \odot (U_t B U_t^T)) U_t} \right)
\]

Update \( A \)

\[
A \leftarrow A \odot \left( \frac{\sum_{t=1}^{T} U_{t-1}^T U_t}{\sum_{t=1}^{T} U_{t-1}^T U_{t-1} A} \right)
\]

---

Properties of Global Learning

• Jointly consider the spatiotemporal correlations
• Achieve good accuracy in practice
Properties of Global Learning

• Jointly consider the spatiotemporal correlations
• Achieve good accuracy in practice

High time complexity: $O(t(nk^2+kn^2))$ per iteration

$t$: number of snapshots  
$k$: number of dimension for the latent attribute  
$n$: number of vertices in the road network
Outline of LSM

Latent Space Model (LSM) for Road Network

Learning & Inference of LSM

- Global Learning Algorithm
- Incremental Learning Algorithm

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- Batch Mode for Real-time Prediction

Experiment Evaluation
Intuition of Fast Learning

- Prediction with immediate feedback

\[ \hat{G}_{t+1}(\text{Prediction}) \]
Intuition of Fast Learning

• Prediction with immediate feedback

\[ \hat{G}_{t+1} \text{(Prediction)} \quad \text{Feedback} \quad G_{t+1} \text{(Ground truth)} \]
Intuition of Fast Learning

• Prediction with immediate feedback

$\hat{G}_{t+1}$ (Prediction)  $G_{t+1}$ (Ground truth)

• Latent attributes of most vertices stay stable
Intuition of Fast Learning

- Prediction with immediate feedback

\[ \hat{G}_{t+1} \] (Prediction) \hspace{1cm} \[ G_{t+1} \] (Ground truth)

- Latent attributes of most vertices stay stable

Arterial \quad 0.2 \quad \hspace{1cm} 0.8 \quad \text{Highway}
Intuition of Fast Learning

• Prediction with immediate feedback

\( \hat{G}_{t+1} \) (Prediction)  \( G_{t+1} \) (Ground truth)

• Latent attributes of most vertices stay stable

\( n_1 \)

\( t=2? \)  \( t=1 \)

Arterial  0.2  0.8  Highway
Incremental Learning

Sequentially Learning from $U_1$ to $U_t$
Incremental Learning

$G_1$ predict $\hat{G}_2$

Sequentially Learning from $U_1$ to $U_t$
Incremental Learning

Sequentially Learning from $U_1$ to $U_t$
Incremental Learning

Sequentially Learning from $U_1$ to $U_t$
Incremental Learning

Sequentially Learning from $U_1$ to $U_t$
Incremental Learning

Sequentially Learning from $U_1$ to $U_t$
Incremental Learning

Sequentially Learning from $U_1$ to $U_t$
Basic Problem

\[ G_t \xrightarrow{?} U_t \]

\[ U_{t-1} \]
Basic Problem

1. predict $G_t$ with $U_{t-1}$
Basic Problem

\[ \hat{G}_t = U_{t-1} B U_{t-1}^T \]
Basic Problem

\[ G_t = U_{t-1} B U_{t-1}^T \]

\[ |G_t(i, j) - \hat{G}_t(i, j)| \geq \delta \]

2. compare
Basic Problem

\[
\hat{G}_t = U_{t-1} B U_{t-1}^T
\]

2. compare

\[
|G_t(i, j) - \hat{G}_t(i, j)| \geq \delta
\]

Candidate node set
Basic Problem

\[ G_t = U_{t-1} B U_{t-1}^T \]

2. compare

\[ |G_t(i, j) - \widehat{G_t}(i, j)| \geq \delta \]

Candidate node set

3. adjust latent attributes to make accurate prediction
Adjust One Vertex

\[ G_t(i, j) = 30 \]

adjust vertex i
Adjust One Vertex

\[ G_t(i, j) = 50 \]

adjust vertex i
Adjust One Vertex

\[ G_t(i, j) = 50 \]

adjust vertex i

\[ U_{t-1}(i) \]

\[ U_{t-1}(j) \]
Adjust One Vertex

$G_t(i, j) = 50$

adjust vertex $i$
Adjust One Vertex

\[ G_t(i, j) = 50 \]

adjust vertex i

\[ U_{t-1}(i) \]

\[ U_{t-1}(j) \]
Adjust One Vertex

$\text{adjust vertex } i$

$G_t(i, j) = 50$

Make minimal changes to adjust the latent attributes
Adjust One Vertex

• Online learning formulation

\[ U_t(i), \xi^* = \arg \min_{U(i) \in \mathbb{R}_+^k} \frac{1}{2} \| U(i) - U_{t-1}(i) \|_F^2 + C \xi \]

s.t. \[ |U(i)BU^T(j) - G_t(i, j)| \leq \delta + \xi \]

make minimal changes to correct the prediction
Adjust One Vertex

- Online learning formulation

\[ U_t(i), \xi^* = \arg \min_{U(i) \in \mathbb{R}_+^k} \frac{1}{2} \| U(i) - U_{t-1}(i) \|_F^2 + C\xi \]

s.t. \[ |U(i)BUT(j) - G_t(i, j)| \leq \delta + \xi \]

**make minimal changes to correct the prediction**
Adjust One Vertex

- Online learning formulation

\[ U_t(i), \xi^* = \arg \min_{U(i) \in R^k_+} \frac{1}{2} \| U(i) - U_{t-1}(i) \|_F^2 + C \xi \]

\[ \text{s.t. } | U(i)BUT(j) - G_t(i, j) | \leq \delta + \xi \]

make minimal changes to correct the prediction
Adjust One Vertex

- Online learning formulation

\[
U_t(i), \xi^* = \arg \min_{U(i) \in \mathbb{R}^k_+} \frac{1}{2} \| U(i) - U_{t-1}(i) \|_F^2 + C \xi
\]

s.t. \[
|U(i)BU^T(j) - G_t(i, j)| \leq \delta + \xi
\]

make minimal changes to correct the prediction
Update Order Matters!

• Adjust many correlated vertices together, different with traditional online learning
Update Order Matters!

• Adjust many correlated vertices together, different with traditional online learning

Reverse topological order: for given edge \((i,j)\), node \(j\) should be updated before node \(i\)

Update i and j, then k? Or reversely?
Time Complexity of Incremental Learning

$O(t(nk^2 + kn^2))$ per iteration

$t$: number of snapshot

$k$: number of dimension for the latent attribute

$\Delta n$: number of nodes in the candidate set

$\Delta m$: number of edges adjacent to the candidate nodes

global learning

$O(tk(\Delta n + \Delta m))$ per iteration

incremental learning
Summary of Algorithms

Global Learning

- jointly consider the spatiotemporal correlation
- achieve good accuracy

Incremental Learning

- sequentially learning
- online, streaming
- efficient, scalable
Outline of LSM

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Experiment Evaluation
Historical graph snapshots

\(G_1\) \(G_2\) \(G_i\) \(G_t\)

\(U_1\) \(U_2\) \(U_i\) \(U_t\)

A

Time
Historical graph snapshots

\[ G_1 \rightarrow G_2 \rightarrow G_i \rightarrow G_t \]

\[ U_t + 1 = U_t A \]
Historical graph snapshots

\[ G_1, G_2, G_i, G_t \]

Prediction

\[ G_{t+1} = U_{t+1} B U_{t+1}^T \]

\[ U_{t+1} = U_t A \]
Traffic prediction at future $h$ step

\[ U_{t+h} = U_t A^h \]

\[ G_{t+h} = U_{t+h} B U_{t+h}^T \]

latent attribute at $t+h$

prediction at $t+h$
Traffic prediction at future $h$ step

\[ U_{t+h} = U_t A^h \]

\[ G_{t+h} = U_{t+h} B U_{t+h}^T \]

latent attribute at $t+h$

prediction at $t+h$

Missing data completion for snapshot $G_i$ ($1 \leq i \leq t$)
Traffic prediction at future $h$ step

$$U_{t+h} = U_t A^h$$

latent attribute at $t+h$

$$G_{t+h} = U_{t+h} B U_{t+h}^T$$

prediction at $t+h$

Missing data completion for snapshot $G_i$ $(1 \leq i \leq t)$

$$G_i = U_i B U_i^T$$
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Experiment Evaluation
Full-batch Mode

Learning for all recent graph snapshots
Full-batch Mode

Learning for all recent graph snapshots
Full-batch Mode

Learning for all recent graph snapshots
Full-batch Mode

Learning for all recent graph snapshots
Full-batch Mode

Learning for all recent graph snapshots
Mini-batch Mode

Learning for one most recent graph snapshot
Mini-batch Mode

Learning for one most recent graph snapshot
Mini-batch Mode

\[ G_1, G_2, \ldots, G_t, G_{t+1}, G_{t+2} \]

Learning for **one** most recent graph snapshot
Mini-batch Mode

Learning for one most recent graph snapshot
Proposed Batch Setting

- Incremental update during one time window
Proposed Batch Setting

- Incremental update during one time window
Proposed Batch Setting

- Incremental update during one time window
- Global learning at the end of one window
Proposed Batch Setting

- Incremental update during one time window
- Global learning at the end of one window
Proposed Batch Setting

- **Incremental update** during one time window
- **Global learning** at the end of one window
Proposed Batch Setting

- Incremental update during one time window
- Global learning at the end of one window
Proposed Batch Setting

Global learning

- Incremental update **during** one time window
- Global learning **at the end** of one window
Proposed Batch Setting

- Incremental update during one time window
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Outline of LSM

Latent Space Model (LSM) for Road Network

Learning & Inference of LSM
  ➡ Global Learning Algorithm
  ➡ Incremental Learning Algorithm

Prediction with LSM
  ➡ Traffic Prediction with Missing Data
  ➡ Batch Mode for Real-time Prediction

Experiment Evaluation
Experiment Setting

• Dataset
  • March and April, 2014 sensor data with more than 60 million records
  • Two subgraphs of Los Angeles road network

<table>
<thead>
<tr>
<th></th>
<th># of Nodes</th>
<th># of Edges</th>
<th># of Sensors</th>
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<tbody>
<tr>
<td>SMALL</td>
<td>3984</td>
<td>12536</td>
<td>1642</td>
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<tr>
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Algorithms

• Proposed algorithms: **LSM-Global, LSM-Inc**

• Baselines:
  • LSM-Naive [Zhang et al. KDD’12]
  • ARIMA [Williams et al. TRB’98], ARIMA-SP
  • SVR[Wu et al. ITS’04], SVR-SP
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  - SVR [Wu et al. ITS’04], SVR-SP

• Measurement: MAPE, RMSE

\[
MAPE = \left( \frac{1}{N} \sum_{i=1}^{N} \frac{|y_i - \hat{y}_i|}{y_i} \right)
\]
Prediction Eval. (h=1)

- LSM-Naive
- LSM-Global
- LSM-Inc
- ARIMA
- ARIMA-SP

Rush hour
Prediction Eval. (h=1)

LSM-Naive  LSM-Global  LSM-Inc  ARIMA  ARIMA-SP

Rush hour

LSM-Global performs slightly better than LSM-Inc
Prediction Eval. \((h=1)\)

- **LSM-Naive**
- **LSM-Global**
- **LSM-Inc**
- **ARIMA**
- **ARIMA-SP**

**MAPE (%)**
- 5%
- 10%
- 15%
- 20%
- 25%
- 30%
- 35%

**Time (am)**
- 7:00
- 7:05
- 7:10
- 7:15
- 7:20
- 7:25
- 7:30
- 7:45
- 8:00
- 8:05

**Rush hour**

**LSM-Global** are two times better than **ARIMA-SP** and **LSM-Naive**
Prediction Eval. (h=1)

ARIMA performs much worse than ARIMA-SP

Rush hour
Prediction Eval. (h=1)

- LSM-Naive
- LSM-Global
- LSM-Inc
- ARIMA
- ARIMA-SP

Non-Rush hour
Prediction Eval. (h=6)

- LSM-Naive
- LSM-Global
- LSM-Inc
- ARIMA
- ARIMA-SP

Rush hour
The error of all methods increases for long-term prediction.
## Running Time

<table>
<thead>
<tr>
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<tr>
<td><strong>train + pred</strong></td>
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<tr>
<td>LSM-Naive</td>
<td>1353 ms</td>
<td>29439 ms</td>
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<tr>
<td>LSM-Global</td>
<td>869 ms</td>
<td>14247 ms</td>
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<tr>
<td>LSM-Inc</td>
<td><strong>407 ms</strong></td>
<td><strong>4145 ms</strong></td>
</tr>
<tr>
<td>ARIMA</td>
<td>484,000 ms</td>
<td>987,000 ms</td>
</tr>
<tr>
<td>SVR</td>
<td>47420,000 ms</td>
<td>86903,000 ms</td>
</tr>
</tbody>
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Online Setting

• **Batch**: combination of incremental learning and global learning
• Mini-batch: only use the current snapshot
• Full-batch: always use all snapshots
• No-batch: Ignore all the feedbacks
Online Eval. (Accuracy)

![Graph showing the relationship between time and MAPE for different batch sizes]
Online Eval. (Running Time)

Running time (ms)

Time (am)

Mini-batch
Full-batch
Batch
Other Experiments

• Comparison of missing value completion
• Effect of varying t, i.e., number of snapshots
• Effect of varying span, the time gap between two snapshots
• Effect of varying other hyper parameters
• Effect of missing data for ARIMA and SVR
• Experiment results based on RMSE

See details in the paper [Deng et.al, KDD’16]
Contributions

• Propose LSM-RN, a **unified framework** for traffic prediction with missing observations

• Develop efficient **incremental online learning** algorithm, that enables real-time traffic prediction

• Scale to **large road network**, train and predict as data arrive


Reference


Q&A
Thank you!