Lecture 3: CS574

Aug 29, 2014

(Makeup for Class on Sept 9, 2014)
Review

- Challenges of Vision
  - Representation
  - Variability due to viewpoint and illumination
  - Occlusions
  - Complexity, role of context
  - ...

- Some recent successes
  - 3-D modeling of earth, 3-D from multiple images
  - Face and smile detection
  - Space robotics
  - Driver assistance tools
  - Medical robotics
  - ...
Today’s Plan

• Study of image formation
• Projection equations
• Role of lenses
• Coordinate Transformations
Simple Eyes

“Eye” for a single cell organism (red spot).
Senses light but poor in determining direction
From Ra’ike: wikipedia

Holes provide some directional sensitivity.
From Alessandro: wikipedia

Nautilus Eye: like a pin hole camera
From Hillewaert: wikipedia
Evolution of Eyes

Human Eye

- Like a camera
  - Lens, pupil (iris), focus by *accommodation*
- Image formed on back of eye (retina)
- Optic nerve sends *data* to brain (cortex)
  - Blind spot (where optic nerve comes out)
Retina

- Two types of photoreceptors
  - Rods: highly sensitive to light, not used for color vision, $\sim 100$M rods
  - Cones: 3 different types with different spectral sensitivities, less sensitive to light, $\sim 5$M cones
    - Explains why color is not seen at night
- Distribution is not uniform
  - High concentration of cones in fovea (0.5 minute visual angle)
  - Fixation (*foveation*) to get high resolution everywhere
Color Sensor Response

• Eyes do not have built in color spectrometer
• Rather, we have 3 sensors with different responses to lights of different color
• Perceived color depends on relative responses of three sensors
Cortex schematic

Optical nerve carries signals from retina to cortex

~100:1 ratio of nerve fibers to receptors: some processing performed at this level

Optical *chiasma*: optic nerve fibers split to two halves of the brain

Many functional areas (V1, V2, ...): knowledge about them is limited
Image Formation

• Geometry
  – Where is the image of a point formed?
• Photometry/Colorimetry
  – How bright is the point?
  – What is its color?
• Ideal camera models
• Real lenses
Pinhole cameras

- Abstract camera model - box with a small hole in it
- Note inverted image
- Pinhole cameras work in practice, ignoring diffraction
The reason for lenses
The thin lens

\[ \frac{1}{z'} - \frac{1}{z} = \frac{1}{f} \]
Thin Lens Properties

• Points at different depth focus at different positions of the image plane
  – With a fixed image plane, not all points will be in focus
  – “Depth of field”, i.e. distance over which focus is acceptable depends on the aperture size
  – Defocus property can be used to infer depth
    • Limited accuracy
    • Not covered in the book or in this course
• Field of view: depends on imaging surface size
Lens Distortions

• Real lenses suffer from various errors/distortions
• Chromatic aberration (not all wavelengths focus at the same point)
• Geometric distortions: complex lens systems used to reduce distortion
• Usually we will assume that complex lenses behave as ideal pinhole models but without the negative effects
  – No diffraction effects, sufficient light collection, all points in focus
The equation of projection

- Note: $k$-axis towards the camera (right handed coordinate system).

- Let $P = (X, Y, Z)$, $p = (x, y, z) = (d/Z*X, d/Z*Y, d)$
  - By similar triangles analysis
Comments on projection equation

• Note: if $x$ is positive, $x'$ is negative as $z$ is negative

• If image plane is in front, image is not inverted; change signs of $x'$ and $y'$.

• Some authors (e.g. RS book) assume that the $z$-axis points towards the object; change signs (again) to accommodate

• To compute image of a curve, project points along the curve
  – Analytical equations may be possible in some cases if the original curve has an analytical equation

• What is projection of a line, a circle, a sphere….?

• Do parallel lines remain parallel?
Converging Lines
Back Projection

• Given an image of an object, what can we infer about the 3-D object casting the image?

• Given a single 2-D image point?
  – A line (orientation) along which the 3-D point must lie, but can not fix its distance

• Given a straight line in the image?
  – Must the object also be a straight line?
    • Not necessarily, but likely (except for accidental viewpoints)
  – Constraints on the object line?
    • Must line in a specific plane (given by pinhole or lens center and the image line)
Homogeneous Coordinates

• Add an extra coordinate
  – $(X, Y, Z) \Rightarrow (X_h, Y_h, Z_h, w) = (wX, wY, wZ, w)$, $w$ is any constant (in the book, $w$ is usually set to 1)

• Advantage: allows perspective transformation to be linearized, i.e. expressed as a matrix equation

$$
\begin{bmatrix}
x_h \\
y_h \\
w_h
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1/d & 0
\end{bmatrix}
\begin{bmatrix}
X_h \\
Y_h \\
Z_h \\
w
\end{bmatrix}
$$

$x_h = X_h, y_h = Y_h, w_h = 1/d* Z_h$

$x = x_h / w_h = d*X_h/Z_h = d * x/z, y = d * x/z$

More common to represent focal length by variable $f$; also to write matrix as

$$
\begin{bmatrix}
f & 0 & 0 & 0 \\
0 & f & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
$$
Next Class

• Chapter 1, sections 1.2 and 1.3