CSCI 545 – Summer 2009

Homework 4 Due Date:08/05/09 5:00 PM

Please submit your homework at the class time or at SAL 245.

5-5 Describe the configuration space for the three-link manipulator shown in figure 3.26.

5-8 Derive the equations needed to compute the shortest distance from a point p to the line segment in the plane with vertices a_1 and a_2.

5-11 Verify Equation (5.6).

5-12 Consider a simple polygonal robot with four vertices, such that at q=(0,0,0) the vertices are located at $a_1(0)=(0,0)$, $a_2(0)=(1,0)$, $a_3(0)=(1,1)$, and $a_4(0)=(0,1)$. If two point obstacles are located at $o_1=(3,3)$ and $o_2=(-3,-3)$, determine the artificial workspace and configuration space forces that act on the robot. (Note: Choose P0 yourself (the distance of influence of an obstacle. For instance sqrt(18))

7-2 Consider a rigid body undergoing a pure rotation with no external forces acting on it. The Kinetic energy is then given as

$$K=(1/2)\ (I_{xx}w_x^2+ I_{yy}w_y^2+ I_{zz}w_z^2)$$

With respect to a coordinate frame located at the center of mass and whose coordinate axes are principal axes. Take as generalized coordinates the Euler angles $\emptyset, \theta, \psi$ and show that the Euler-Lagrange equations of motion of the rotating body are

$$I_{xx}\dot{w}_x + \left(I_{zz} - I_{yy}\right)w_y w_z = 0$$

$$I_{yy}\dot{w}_y + (I_{xx} - I_{zz})w_z w_x = 0$$

$$I_{zz}\dot{w}_z + \left(I_{yy} - I_{xx}\right)w_x w_y = 0$$

7-7 Consider a 3-Link Cartesian manipulator,

(a) Compute the inertia tensor J_i for each link i=1,2,3 assuming that the links are uniform rectangular solids of length 1, width 1/4, and height ¼ , and mass 1.
(b) Show that the Christoffel symbols c_{ijk} are all zero for this robot. Interpret the meaning of this for the dynamic equations of motion.
(c) Derive the equations of motion in matrix form:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u$$