

# CSCI 303 Homework 7

In the following problems,  $\mathbb{N} = \{1, 2, 3, \dots\}$  is the set of natural numbers, the symbol  $\exists$  means “there exists”, the symbol  $\forall$  means “for all”, the symbol  $\downarrow$  means “halts”, and the symbol  $\uparrow$  means “does not halt”.  $B_i(x)$  is the  $i^{\text{th}}$  SBASIC program run on input  $x$ .

**Problem 1 (Not in book):**

For each of the following subsets of  $\mathbb{N}$ , prove whether the subset is decidable or undecidable:

- a.  $X = \{i : B_i(12) = 13\}$
- b.  $Y = \{i : \forall x \in \mathbb{N}, B_i(x) \downarrow\}$
- c.  $Z = \{i : \exists a \in \mathbb{N}, B_i(i) = a \ \& \ a < i\}$

**Problem 2 (Not in book):**

Prove or disprove each of the following statements:

- a.  $\exists i \in \mathbb{N}$  such that  $\forall x \in \mathbb{N}, B_i(x) = i$
- b.  $\exists i \in \mathbb{N}$  such that  $B_i(i) = i^2$
- c.  $\exists i \in \mathbb{N}$  such that  $B_i(i^2) = i$

**Problem 3 (Not in book):**

Let  $S = \{i : B_i(0) \downarrow\}$ .

- a. Is  $S$  recursively enumerable? Prove your answer.
- b. Is  $\bar{S} = \mathbb{N} \setminus S$  recursively enumerable? Prove your answer.