CSCI 303 Homework 5

Problem 1 (Not in book):
Using the polynomial time reduction given in class, give an instance of the Directed Hamiltonian Cycle problem that corresponds to the instance of the 3-SAT problem where

$$\phi = (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee \neg x_1) \wedge (x_2 \vee x_2 \vee \neg x_3).$$

Problem 2 (34.1-5):
Show that an otherwise polynomial time algorithm that makes at most a constant number of calls to polynomial time subroutines runs in polynomial time, but that a polynomial number of calls to polynomial time subroutines may result in an exponential time algorithm.

Problem 3 (Derived from 34.3-2):
Show that if Decision Problem 1 is polynomial time reducible to Decision Problem 2 and Decision Problem 2 is polynomial time reducible to Decision Problem 3, then Decision Problem 1 is polynomial time reducible to Decision Problem 3.

Problem 4 (Not in book):
Prove that the 3-SAT problem is polynomial time reducible to the Vertex Cover problem.

Problem 5 (Derived from 34.3-3):
Let D be a decision problem, then the complement of D, denoted $\overline{D}$, is the decision problem such that positive instances of D are negative instances of $\overline{D}$ and negative instances of D are positive instances of $\overline{D}$. For example,

$\overline{\text{The 3-SAT Problem}}$:
INPUT: ϕ, a Boolean Formula in 3-CNF form
OUTPUT: 1 if ϕ is *not* satisfiable
0 if ϕ is satisfiable

$\overline{\text{The Search Problem}}$:
INPUT: $(a_1, a_2, \ldots, a_n; b)$ are numbers
OUTPUT: 1 if there does *not* exist an i such that $a_i = b$
0 if if there does exist an i such that $a_i = b$

a. Prove or disprove:
The search problem is polynomial time reducible to $\overline{\text{the search problem}}$.

b. Prove or disprove:
For all decision problems D, D is polynomial time reducible to $\overline{D}$.

c. Prove or disprove:
The 3-SAT problem is polynomial time reducible to $\overline{\text{the 3-SAT problem}}$.

Problem 6 (Derived from 34.2-3):
Consider the following problem

The Hamiltonian Cycle Identification Problem:
INPUT: G, a directed graph
OUTPUT: a list of nodes, in order, of a Hamiltonian cycle of G, if one exists

0 if G does not contain a Hamiltonian cycle

Suppose that someone gives you a polynomial time algorithm to solve the Hamiltonian Cycle problem. Describe how to use this algorithm to find a polynomial time algorithm that solves the Hamiltonian Cycle Identification problem.

Problem 7 (34.5-5):

The Set-Partition problem takes as input a set S of numbers. The question is whether the numbers can be partitioned into two sets A and $\overline{A} = S - A$ such that $\sum_{x \in A} x = \sum_{x \in \overline{A}} x$. Show that the Set-Partition problem is NP-complete.