

CSCI 303 Introduction to Algorithms
Spring 2007
February 21, 2007 class notes

Midterm will be on March 5th, during regular class time.

The midterm may cover all of the material taught in class so far, including the current topic: NP-completeness.

We have / will look at five problems:

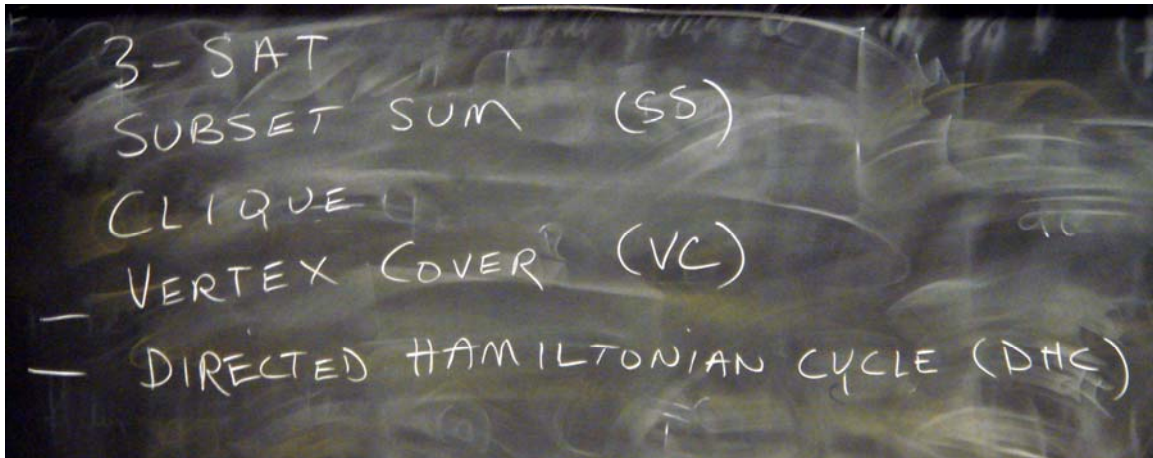
3-SAT

SUBSET SUM

CLIQUE

VERTEX COVER

DIRECTED HAMILTONIAN CYCLE

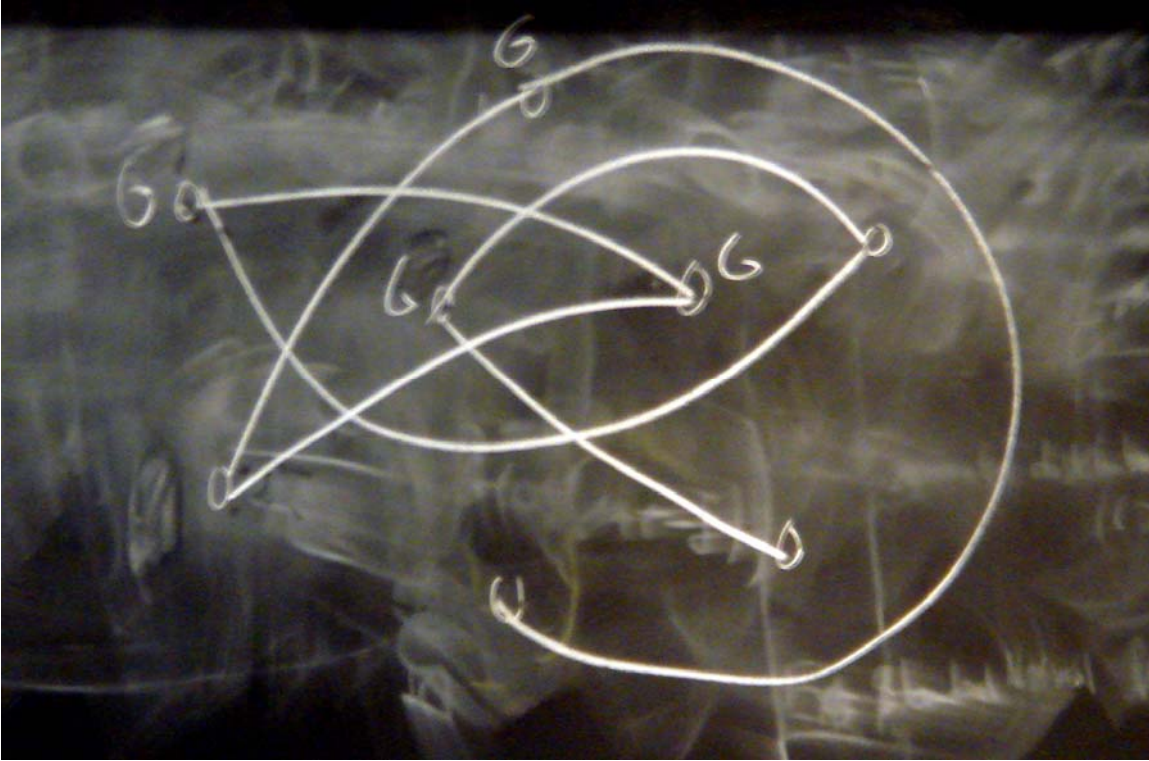


Vertex Cover Problem:

Input: undirected graph G and a number k .

Output: 1 if G has a k -vertex cover and 0 otherwise.

What is a vertex cover? Imagine that the graph is several cities with roads between them. You try to place k guards at the vertices (cities) such that all of the edges (roads) are covered by at least one guard. If there is a way to cover all roads with k guards, then the graph has a k -vertex cover.



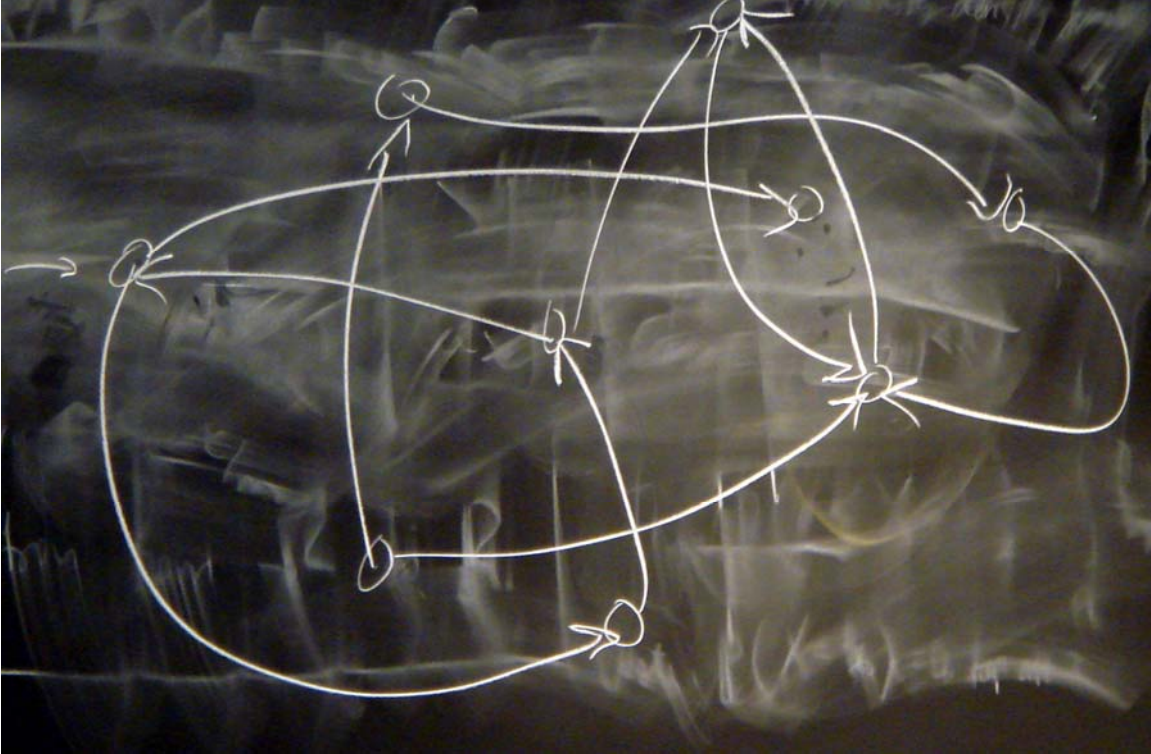
The brute force algorithm works here again. For a given k , there are $\binom{n}{k}$ vertex subsets of size k . Check each vertex subset. When $k = \frac{n}{2}$, $\binom{n}{k} = \Omega(c^n)$, for some c . So, $T_{BF}^{WC}(n) = \Omega(2^n)$.

Directed Hamiltonian Cycle Problem:

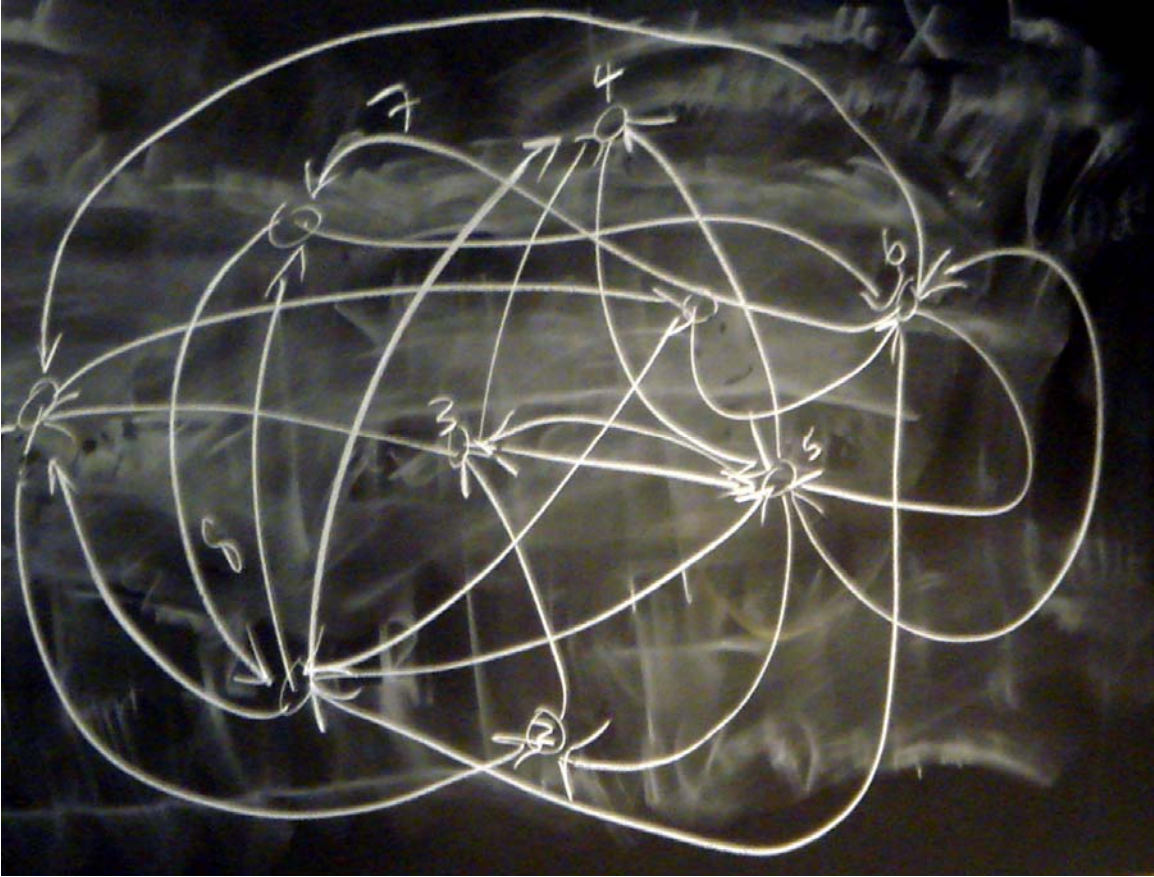
Input: directed graph G

Output: 1 if G has a directed Hamiltonian cycle and 0 otherwise.

What is a Hamiltonian cycle? Imagine that the graph is cities with one-way non-stop flights. A Hamiltonian cycle is a directed cycle in the graph that contains all the cities exactly once.



Brute force works again: try every permutation of nodes to see if it's a Hamiltonian cycle. There are $n!$ such permutations, so $T_{BF}^{WC}(n) = \Omega(n!)$.



Back to the 5 problems:

3-SAT

SUBSET SUM

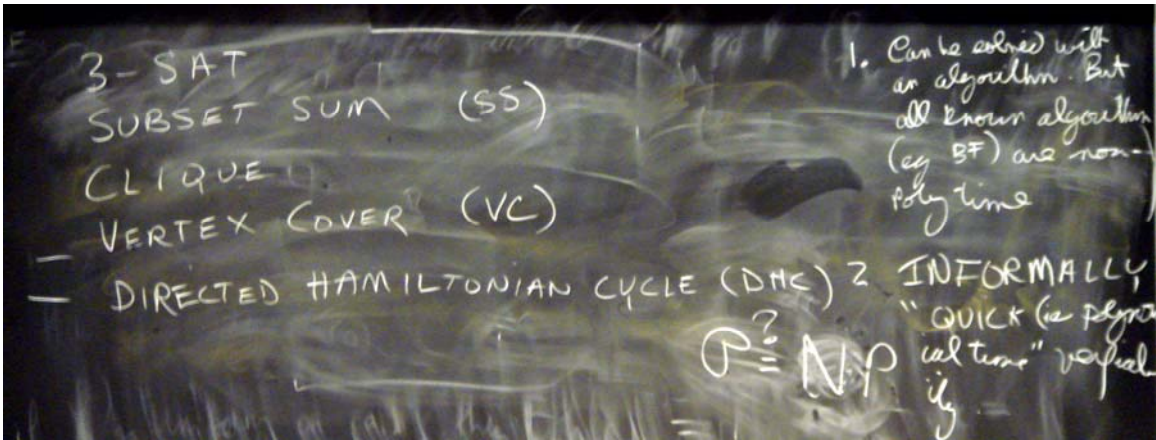
CLIQUE

VERTEX COVER

DIRECTED HAMILTONIAN CYCLE

They all can be solved using algorithms, but all known algorithms are non-polynomial time algorithms.

They can be verified quickly (e.g. I give you the truth assignment for a 3-SAT problem – you can quickly check that the formula evaluates to T). This property is called quick- (i.e. polynomial time) verifiability. It is these types of problems that are known as the class NP.

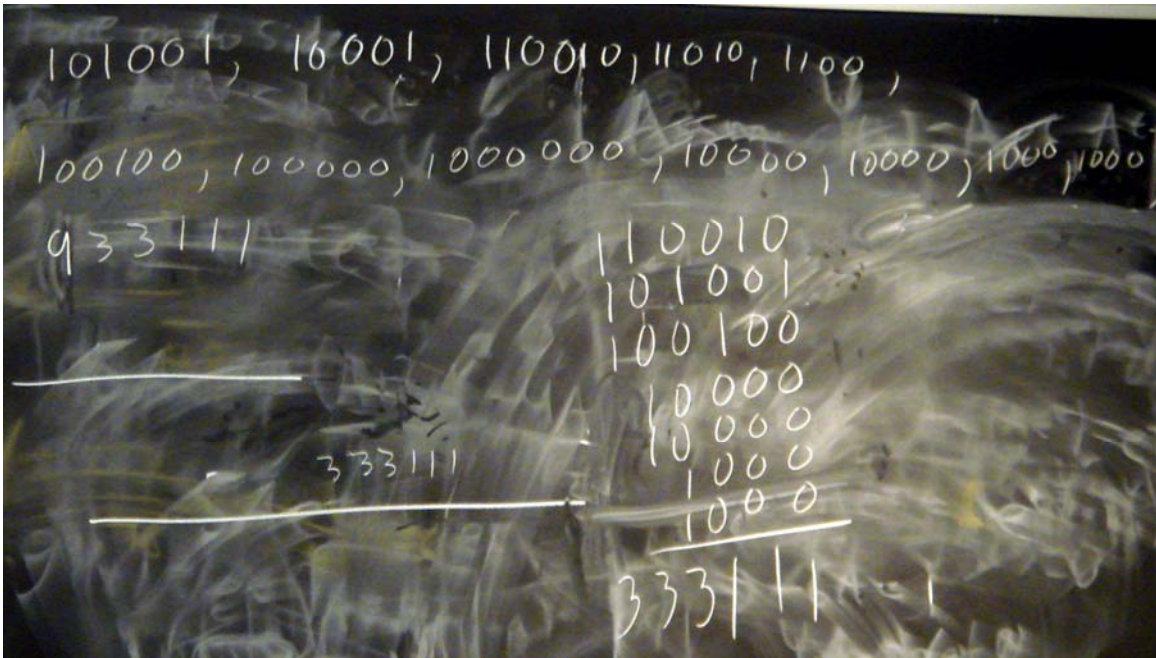


Cook, Kard, Levin proved that all "these" (NP-Complete) problems are actually the same problem!

We will show that 3-SAT and SUBSET SUM are the same problem. Remember the formula and subset sum examples from previous lectures:

$$3\text{-SAT: } \Phi = (x_1 \vee x_2 \vee \neg x_3) \wedge (x_2 \vee \neg x_2 \vee \neg x_1) \wedge (x_3 \vee \neg x_2 \vee x_1)$$

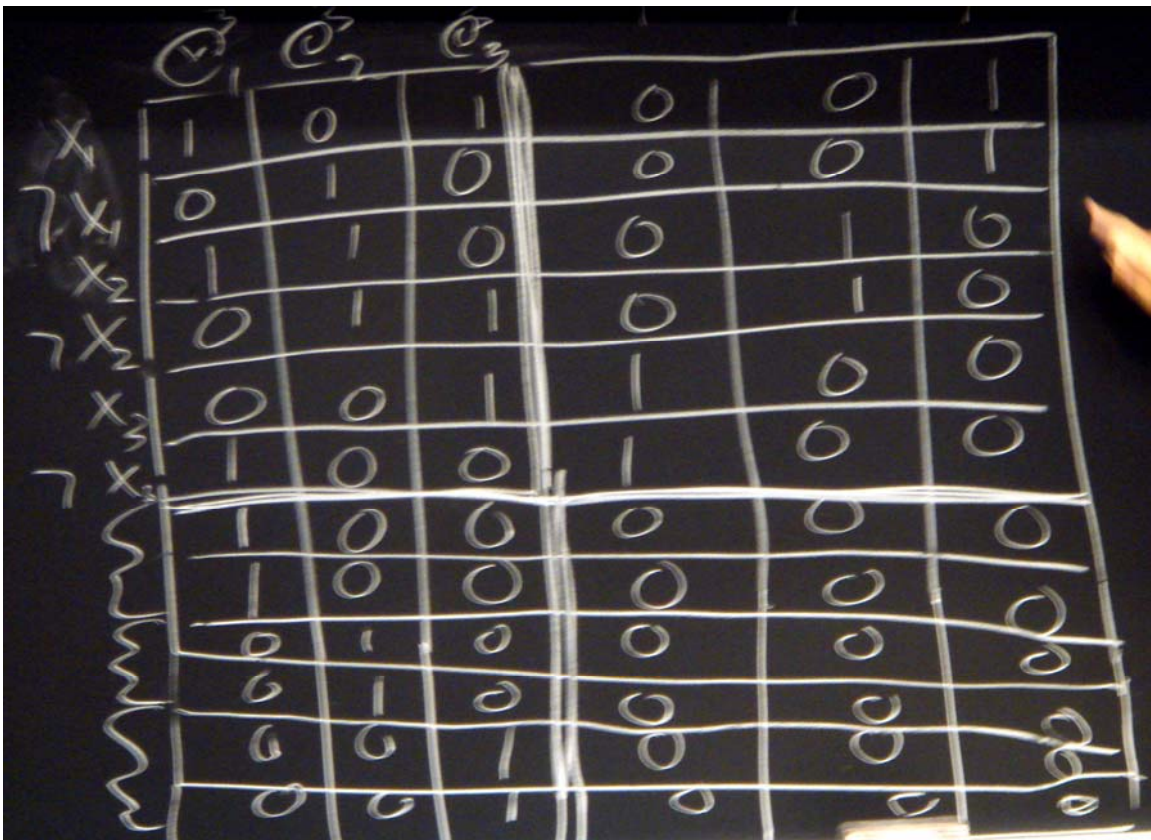
SUBSET SUM:



Those are actually related. Let's look at the numbers we used to SUBSET SUM.

	C_1	C_2	C_3	$x_3 \neg x_3$	$x_2 \neg x_2$	$x_1 \neg x_1$
x_1	1	0	1	0	0	1
$\neg x_1$	0	1	0	0	0	1
x_2	1	1	0	0	1	0
$\neg x_2$	0	1	1	0	1	0
x_3	0	0	1	1	0	0
$\neg x_3$	1	0	0	1	0	0
C_1	1	0	0	0	0	0
C_1	1	0	0	0	0	0
C_2	0	1	0	0	0	0
C_2	0	1	0	0	0	0
C_3	0	0	1	0	0	0
C_3	0	0	1	0	0	0

The area in yellow encodes Φ and the rest of the matrix is straight forward.



Next class, we'll continue talking about this reduction.