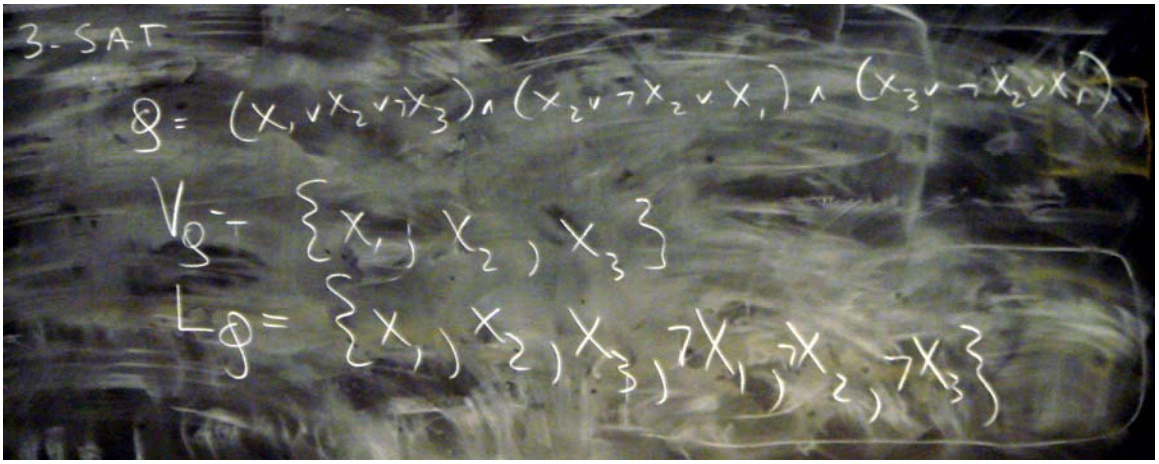
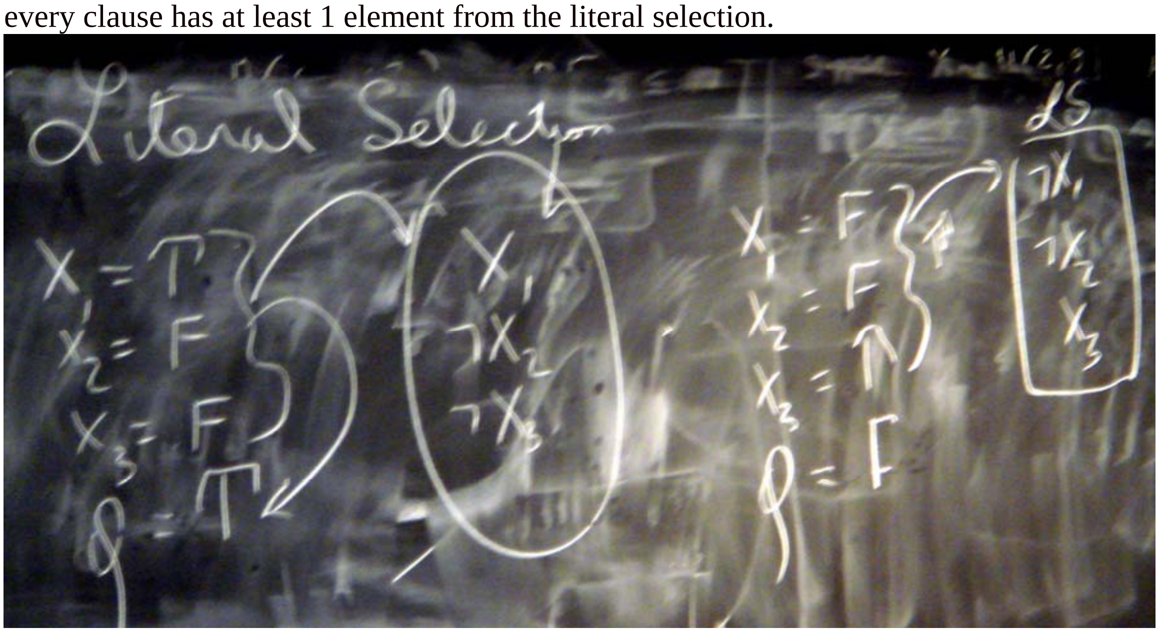
CSCI 303 Introduction to Algorithms
Spring 2007
February 14th, 2007 class notes

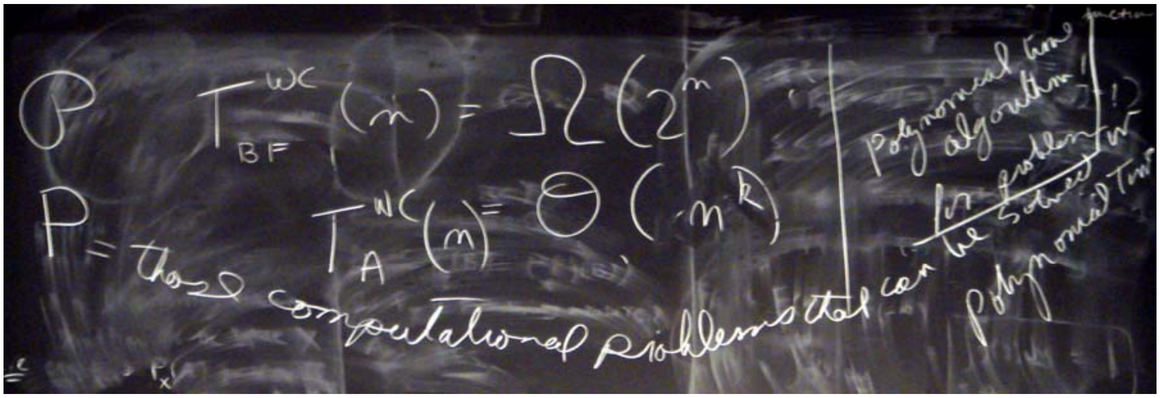
3-SAT:
A Boolean formula $\Phi = (x_1 \vee x_2 \vee \neg x_3) \wedge (x_2 \vee \neg x_2 \vee \neg x_1) \wedge (x_3 \vee \neg x_2 \vee x_1)$
This formula has 3 clauses (C_1, C_2, and C_3).
The variables of Φ are $\{x_1, x_2, x_3\}$.
The literals of Φ are $\{x_1, \neg x_1, x_2, \neg x_2, x_3, \neg x_3\}$.



Literal selection: for each truth assignment, there set of literals that evaluate to T are said to be in the *literal selection*. Φ is satisfiable iff there exists a literal selection such that every clause has at least 1 element from the literal selection.



If a problem has an algorithm that solves it in $O(n^k)$ time, then we say the problem is solvable in polynomial time. P is the set of such problems.

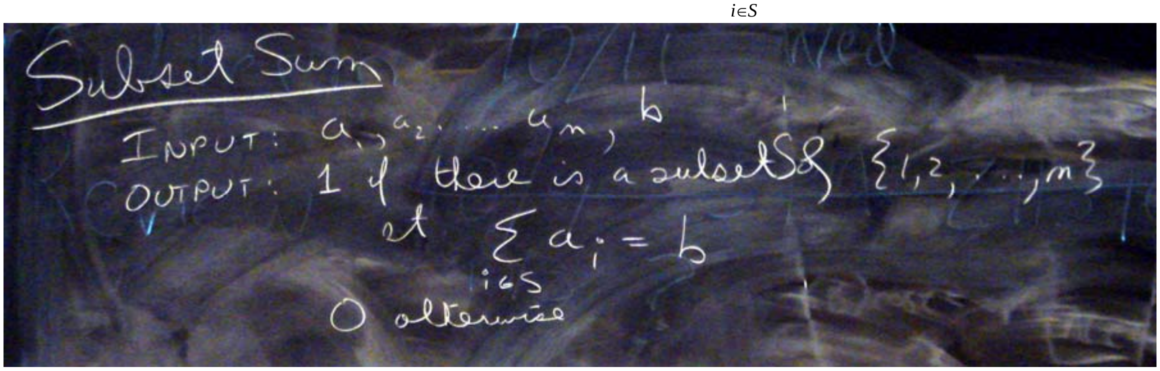


For 3-SAT, there exists an algorithm called the resolution method that tries to solve 3-SAT problems quickly, but in the worst case it is still not polynomial.

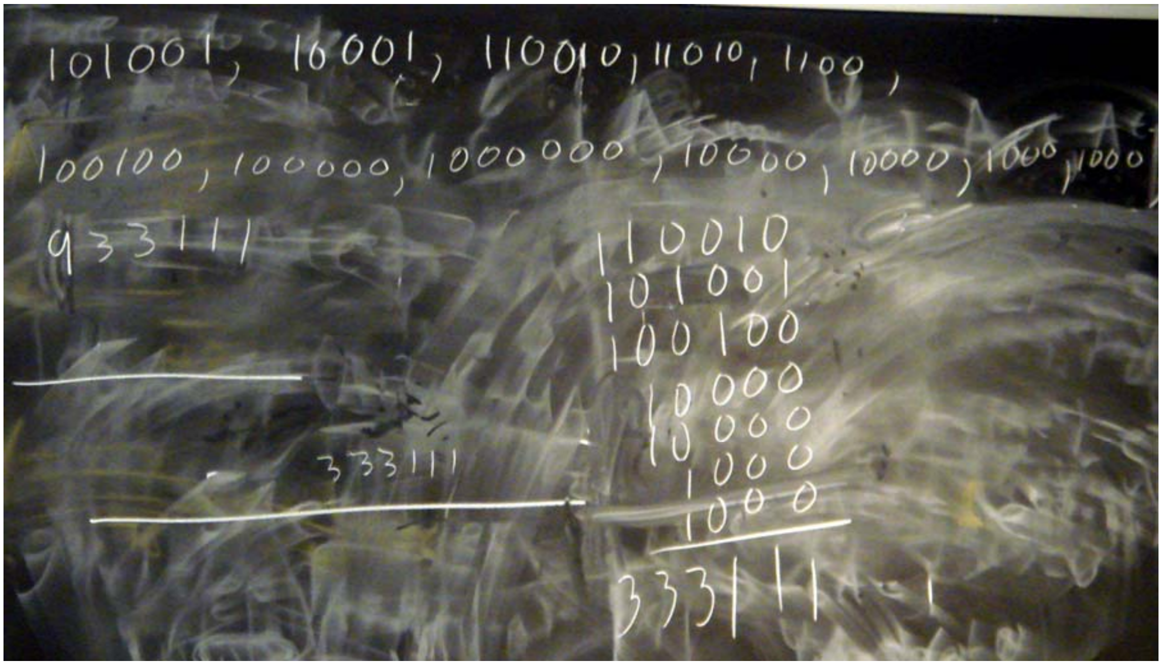
Subset Sum problem:

Input: $a_1, \ldots, a_n, b$

Output: 1 if there exists $S \subseteq \{1,2,\cdots,n\}$ such that $\sum_{i \in S} a_i = b$ and 0 otherwise.

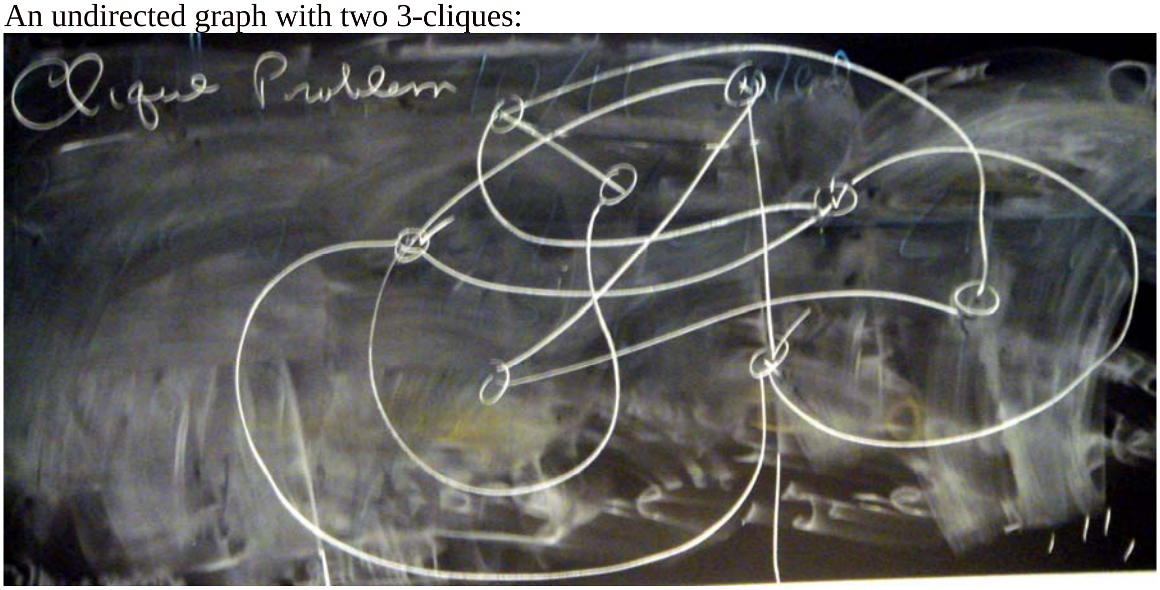


Example:



$T_{BF}^{WC}(n) = \Omega(2^n)$

Clique Problem:
An undirected graph with two 3-cliques:



Input: undirected graph G and a number k
Output: 1 if G has a k-clique and 0 otherwise.