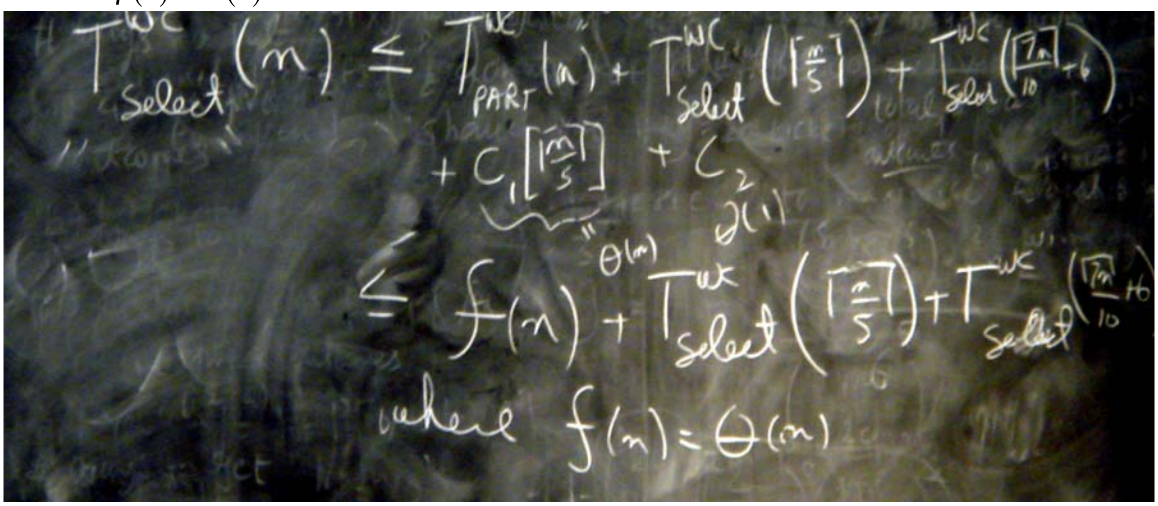
CSCI 303 Introduction to Algorithms
Fall 2006
January 31st, 2007 class notes

The first quiz will be on Wednesday, February 7th, 2007. BRING A BLUE BOOK!
The quiz will cover homeworks 1, 2 and 3.

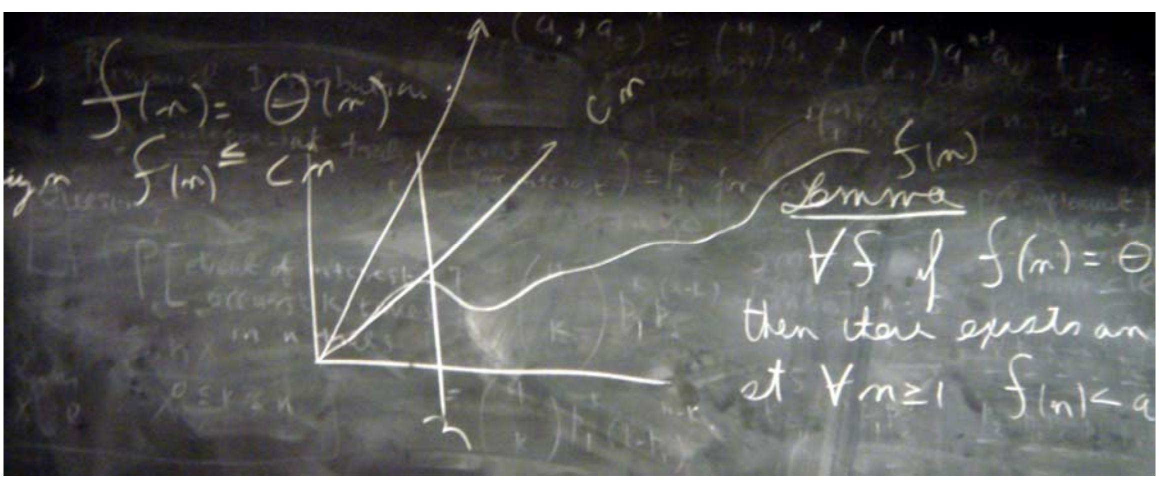
$$T_{SELECT}^{WC}(n) \le T_{PART}^{WC}(n) + \left\lceil \frac{n}{5} \right\rceil c_1 + T_{SELECT}^{WC}\left(\left\lceil \frac{n}{5} \right\rceil\right) + c_2 + T_{SELECT}^{WC}\left(\left\lceil \frac{7n}{10} \right\rceil + 6\right) =$$

$$T_{SELECT}^{WC}\left(\left\lceil \frac{n}{5} \right\rceil\right) + T_{SELECT}^{WC}\left(\left\lceil \frac{7n}{10} \right\rceil + 6\right) + f(n)$$

Where $f(n) = \Theta(n)$.



Lemma:
$\forall f$ if $f(n) = \Theta(n)$ then $\exists$ an a such that $\forall n \ge 1 \;\; f(n) < an$.



Theorem:
$(\exists c)(\forall n \geq 1)\left[T_{SELECT}^{WC}(n) \leq cn\right]$

Proof by induction:
We will say T to mean T_{SELECT}^{WC}.
Let $i = \max\{T(1), T(2), \cdots, T(160)\}$. By the lemma, $\exists a\ \forall n \geq 1 \quad f(n) < an$. Let $c = \max\{i, 20a\}$.
If $n \leq 160$, then $T(n) \leq cn$, because $c \geq i = \max\{T(1), T(2), \cdots, T(160)\}$.
If $n > 160$:
Let $T(n) = T_{SELECT}^{WC}(n)$.

$$T(n) \leq T\left(\left\lceil \frac{n}{5} \right\rceil\right) + T\left(\left\lceil \frac{7n}{10} \right\rceil + 6\right) + f(n).$$

$$\leq c\left(\left\lceil \frac{n}{5} \right\rceil\right) + c\left(\left\lceil \frac{7n}{10} \right\rceil + 6\right) + an$$

$$\leq c\left(\frac{n}{5} + 1\right) + c\left(\frac{7n}{10} + 7\right) + an$$

$$= cn - \frac{cn}{20} - \frac{cn}{20} + 8c + an$$

$$= cn + \left(8c - \frac{cn}{20}\right) + \left(an - \frac{cn}{20}\right)$$

Because the quantities $\left(8c - \frac{cn}{20}\right)$ and $\left(an - \frac{cn}{20}\right)$ are nonpositive, $T(n) \leq cn$. Therefore, $T(n) = O(n)$. But Select at least runs Partition and also find n/5 baby medians, so $T(n) = \Omega(n)$. Therefore, $T(n) = \Theta(n)$. □

Pf Let $T = T_{select}$

Let $i = \text{MAX}\,[T(1), T(2), \ldots, T(160)]$

By the lemma $(\exists a)(\forall n \geq 1)\,[f(n) \leq an]$

Let $C = \text{MAX}\,\{i, 20a\}$

If $n \leq 160$ then $T(n) \leq Cn$. Since $C \geq i = \text{MAX}\,[T(1), \ldots, T(160)]$ hence $C \geq T(n)$ when $n \leq 160$

but for $n \geq 1$ $\quad Cn \geq C \geq T(n)$

If $n > 160$

$$\underline{T(n) \leq T\left(\left\lceil \tfrac{n}{5} \right\rceil\right) + T\left(\left\lceil \tfrac{7n}{10} \right\rceil + 6\right) + f(n)}$$

(IH: $T(\lceil n/5 \rceil) \leq C\lceil n/5 \rceil$; IH; $f(n) \leq an$)

$$\leq C\left\lceil \tfrac{n}{5} \right\rceil + C\left(\left\lceil \tfrac{7n}{10} \right\rceil + 6\right) + an$$

$$\leq C\left(\tfrac{n}{5} + 1\right) + C\left(\tfrac{7n}{10} + 7\right) + an$$

$$\leq C\left(\tfrac{9n}{10}\right) + 8C + an$$

$$= Cn - \tfrac{Cn}{20} - \tfrac{Cn}{20} + 8C + an$$

$$= Cn + \underbrace{\left(8C - \tfrac{Cn}{20}\right)}_{\text{I}} + \underbrace{\left(an - \tfrac{Cn}{20}\right)}_{\text{II}}$$

$n > 160$ — I: neg; II: zero or neg

hence $\leq Cn$