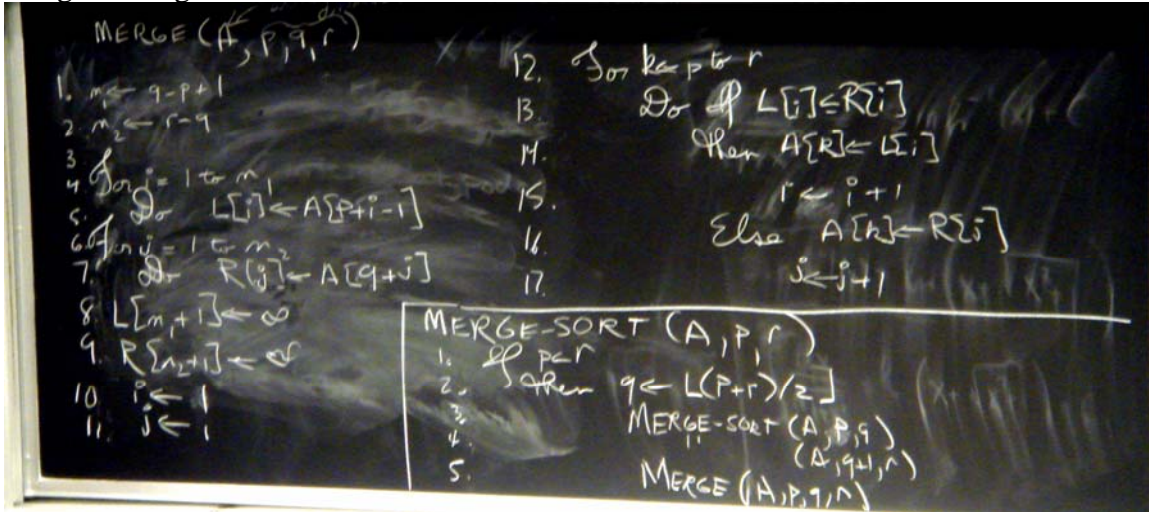


Merge sort algorithm:



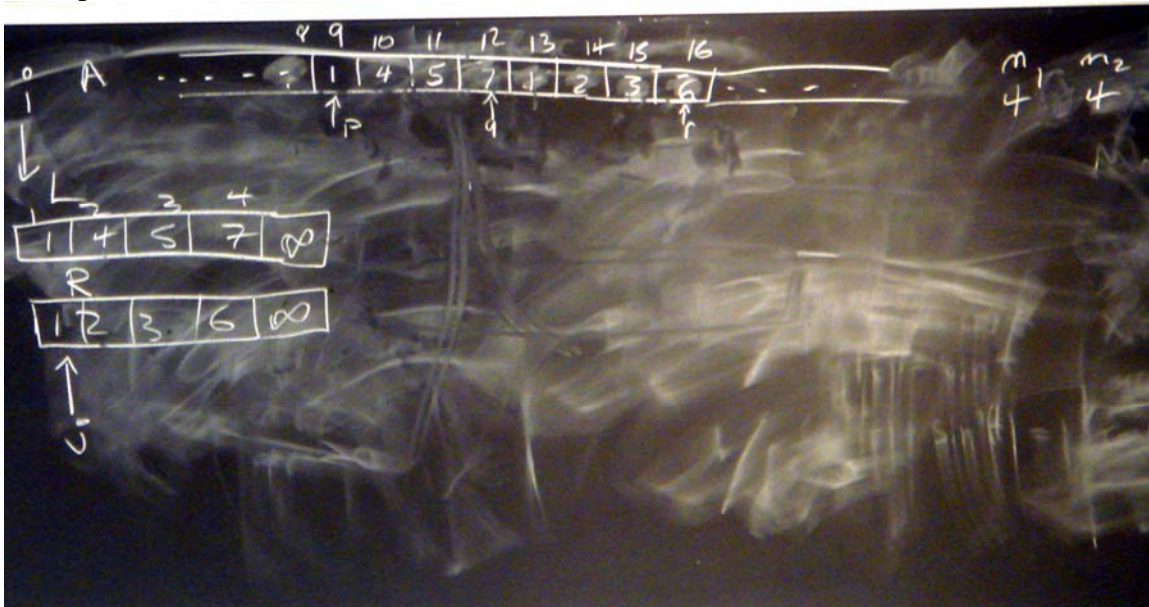
The running time of Merge:

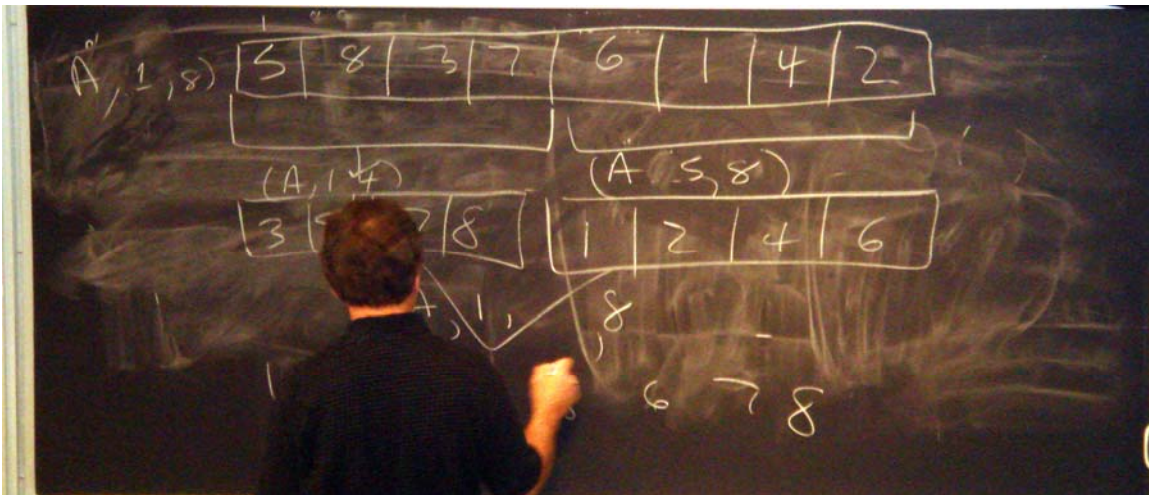
$$T_{MERGE}^{WC}(n) \leq c_1 + c_2 + c_3n + c_4 + c_5 + c_6 + c_7n + c_8 = O(n), \text{ and}$$

$$T_{MERGE}^{WC}(n) \geq n = \Omega(n), \text{ therefore,}$$

$$T_{MERGE}^{WC}(n) = \Theta(n).$$

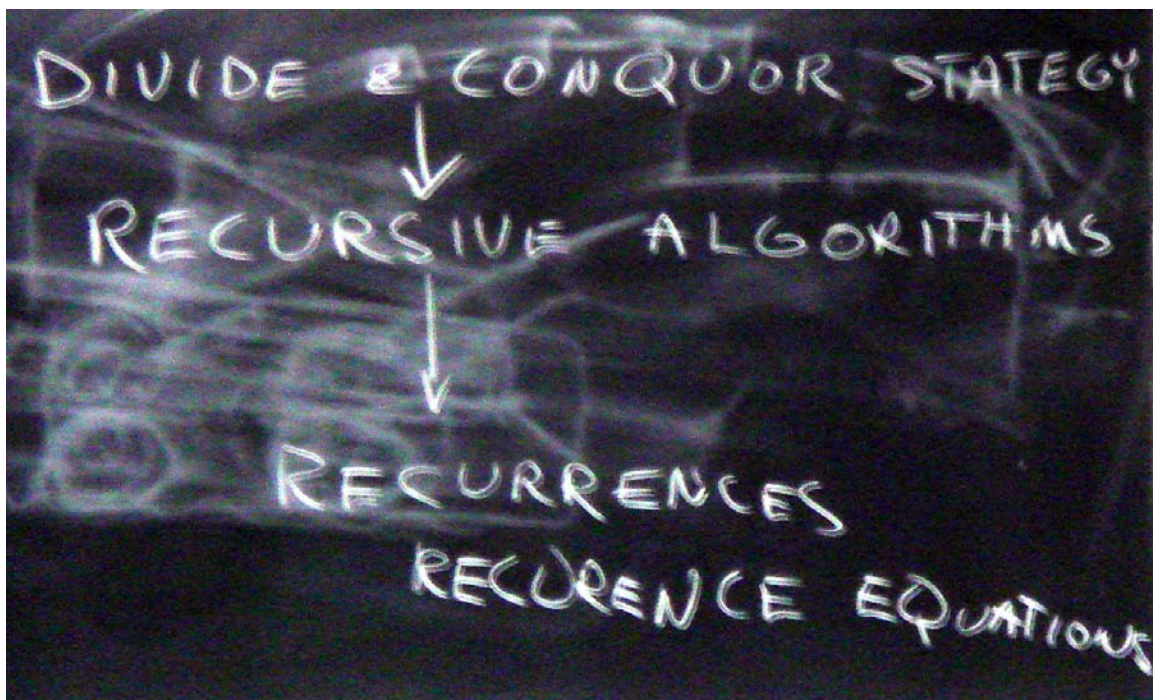
Example:





So the running time of Merge Sort is:

$$T_{MS}^{WC}(n) \leq c_1 + c_2 + 2T_{MS}^{WC}\left(\frac{n}{2}\right) + T_{MERGE}^{WC}(n) = 2T_{MS}^{WC}\left(\frac{n}{2}\right) + f(n), \text{ where } f(n) = \Theta(n).$$



Fundamental theorem of arithmetic: All natural numbers have unique prime number factorizations. This theorem changed the field of numerology to number theory.

Fundamental theorem of chemistry (atomic hypothesis): All matter is made out of atoms. This theorem changed the field of alchemy to chemistry.

Fundamental theorem of algebra: Every polynomial of degree n with complex number coefficients has n roots.

Fundamental theorem of calculus: Integration is the inverse of differentiation.

These are **important** theorems. The “Master theorem” is **not** an important theorem.

Master Theorem:

Let $a \geq 1, b > 1$ be constants, let $f(n)$ be a function, let $T(n)$ be defined on the non-negative integers by the recurrence $T(n) = aT\left(\frac{n}{b}\right) + f(n)$, where we interpret $\frac{n}{b}$ to mean either

$\left\lfloor \frac{n}{b} \right\rfloor$ or $\left\lceil \frac{n}{b} \right\rceil$. Then $T(n)$ can be bounded asymptotically as follows:

1. If $f(n) = O(n^{\log_b a - \varepsilon})$ for some $\varepsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$
2. If $f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \lg n)$
3. If $f(n) = \Omega(n^{\log_b a + \varepsilon})$ for some $\varepsilon > 0$, then $T(n) = \Theta(f(n))$

