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Homework 7 Solutions

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Set-Partition is in NP because we can verify a given partition in polynomial time. To show that it is NP-Complete we reduce Subset-Sum to Set-Partition. The Subset-Sum problem is: given a set A={a_1, a_2, …, a_n} of positive integers and a target t, determine if there exists $S \subseteq A$ such that $\sum_{a \in S} a = t$. We want to convert that problem to a Set-Partition problem.

Let $k = \sum_{a \in A} a$, and let l be some positive integer such that l > k. Create a new set $A' = A \cup \{l - t, l - k + t\}$. The sum of the elements of A' is 2l. We claim that there exists a subset of the numbers in A that add up to t if and only if there exists a partition of A' such that the elements of the two halves add up to the same value.

Assume there exists a subset of A whose elements' sum is t. So there is another subset S' of A whose elements add up to k – t. So by adding the element l – t to S and the element l – k + t to S', we divide the set A' into two parts which add up to the same number l.

Assume that there is a partition of A' such that the sum of the elements of the two halves add up to the same value. That value must be l because $\sum_{a \in A'} a = 2l$. Note that both l – t and l – k + t cannot be in the same subset because they add up to more than l. Examine the subset which contains l – t. Without l – t, the elements of that set add up to t. And all of those elements are in A, so you have a subset of A which adds up to t. □