CSCI303 Fall 2006
Homework 4 Solutions

8.1-1) last updated fall 2006, Yuriy Brun, ybrun@usc.edu

Since it takes at least n – 1 comparisons to find any one number's order statistic, the smallest depth of a leaf in a tree must be n – 1.

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Assume that there exists a decision tree of height h to sort this sequence. Since there are $\frac{n}{k}$ subsequences each containing k elements, which are sorted relatively among themselves, there are at least a total of $k!^{\frac{n}{k}}$ possible outcomes at the leaves of the decision tree – since each subsequence can be sorted in k! ways, and there are $\frac{n}{k}$ subsequences. So the number of leaves of the tree, $2^h \geq \left(k!^{\frac{n}{k}}\right)$. Thus, $\lg\left(2^h\right) \geq \lg\left(k!^{\frac{n}{k}}\right) \Rightarrow h \geq n \lg k$ by using Streling inequality: $n! \geq \left(\frac{n}{e}\right)^n$. Therefore, all sorting algorithms must be $\Omega(n \lg k)$ in the worst case.