CSCI303 Fall 2006
Homework 1 Solutions

2.1-1) last updated fall 2006, Yuriy Brun, ybrun@usc.edu

31	**41**	59	26	41	58
31	41	**59**	26	41	58
31	41	59	**26**	41	58
26	31	41	59	**41**	58
26	31	41	41	59	**58**
26	31	41	41	58	59

2.2-2) last updated fall 2006, Yuriy Brun, ybrun@usc.edu

```
Selection-sort(A)
        for i = 1 to length(A) – 1
            do  min-value = A[i]
                min-index = i
                for j = i + 1 to length(A)
                    do  if A[j] <= min-value
                                    min-value=A[j]
                                    min-index=j
                        A[i] ↔ A[min-index]
```

In the best and worst case, it takes $\Theta(n)$ time to find the minimum (the inner for loop), and we do that $\Theta(n)$ times. So in the best and in the worst case, the algorithm executes in $\Theta(n^2)$ time.

2.2-1) last updated fall 2006, Yuriy Brun, ybrun@usc.edu

By streamlining, $\frac{n^3}{1000} - 100n^2 - 100n + 3 = \Theta(n^3)$

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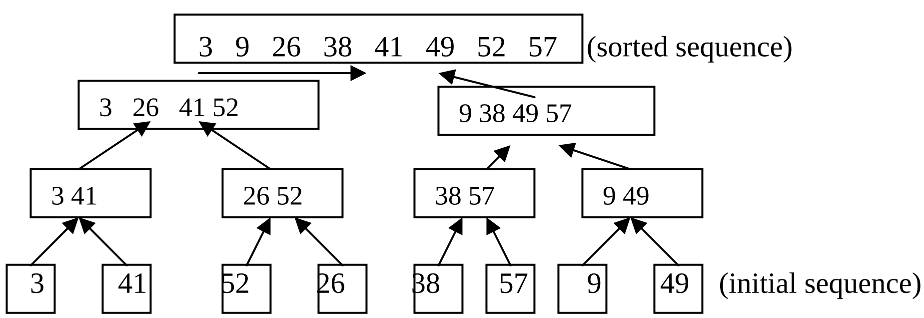

2.3-3) last updated fall 2006, Yuriy Brun, ybrun@usc.edu

Base case: $T(2) = 2\lg 2 = 2$

Induction step: Assume $T(n) = n\lg n$ and show that $T(2n) = 2n\lg 2n$.

By the recurrence: $T(2n) = 2T\left(\frac{2n}{2}\right) + 2n$

By the inductive hypothesis: $T(2n) = 2n\lg n + 2n$

Simplify: $T(2n) = 2n\lg n + 2n = 2n(\lg n + 1) = 2n(\lg n + \lg 2) = 2n(\lg 2n) = 2n\lg 2n$

Thus, by induction, $T(n) = n\lg n$.

1.2-2) last updated fall 2006, Yuriy Brun, ybrun@usc.edu

Insertion sort beats merge sort when

$8n^2 < 64n\lg n$

⇨ $n < 8\lg n$

⇨ which is true when $2 \le n \le 43$

1.2-3) last updated fall 2006, Yuriy Brun, ybrun@usc.edu

To find smallest n such that $100n^2 < 2^n$.

By calculator, n=15.

1-1) last updated fall 2006, Yuriy Brun, ybrun@usc.edu

	1 sec	1 minute	1 hour	1 day	1 month	1 year	1 century
lg n	$2^{1000000}$	$2^{60000000}$	$2^{3600000000}$	$2^{86400000000}$	$2^{2592000000000}$	$2^{31536000000000}$	$2^{31536000000000000}$
$N^{0.5}$	10^{12}	$3.6\text{x}10^{15}$	$1.296\text{x}10^{19}$	$7.46\text{x}10^{21}$	$6.718\text{x}10^{24}$	$9.94\text{x}10^{26}$	$9.94\text{x}10^{30}$
N	10^{6}	$6\text{x}10^{7}$	$3.6\text{x}10^{9}$	$8.64\text{x}10^{10}$	$2.59\text{x}10^{12}$	$3.153\text{x}10^{13}$	$3.153\text{x}10^{15}$
N lg n	62746	2801418	$1.34\text{x}10^{8}$	$2.754\text{x}10^{9}$	$7.2\text{x}10^{10}$	$8.03\text{x}10^{11}$	$6.85\text{x}10^{13}$
N^2	1000	7746	60000	293938	1609347	5615158	56151580
N^3	10^{2}	391	1531	4417	13720	31559	146462
2^n	19.93	25.83	31.74	36.33	41.23	44.84	51.48
N!	9	11	12	13	15	16	17

3.1-2) last updated fall 2006, Yuriy Brun, ybrun@usc.edu

$(n+a)^b = n^b + abn^{b-1} + \cdots + \binom{b}{i} a^i n^{b-i} + \cdots + a^b = \sum_{i=0}^{n} \binom{b}{i} a^i n^{b-i}$, for n large enough, $\frac{n^b}{2} \le \sum_{i=0}^{n} \binom{b}{i} a^i n^{b-i} \le 2n^b$, thus $(n+a)^b = \Theta(n^2)$

3.1-4) last updated fall 2006, Yuriy Brun, ybrun@usc.edu

$2^{n+1} = 2(2^n)$, so $2^{n+1} = O(2^n)$

$2^{2n} = 2^n(2^n)$, so $2^{2n} \ne O(2^n)$

3-3) last updated fall 2006, Yuriy Brun, ybrun@usc.edu

a)

In order:

$1 \approx n^{\frac{1}{\lg n}}$

$\lg \lg^* n$

$\lg^* \lg n$

$\lg^* n \approx 2^{\lg^* n}$

$\ln \ln n$

$\sqrt{\lg n}$

$\ln n$

$\lg^2 n$

$2^{\sqrt{2 \lg n}} = \sqrt{2}^{\lg n} = \sqrt{n}$

$2^{\lg n} = n$

$\lg n! \approx n \lg n$

$n^2 = 4^{\lg n}$

n^3

$(\lg n)!$

$n^{\lg \lg n} = (\lg n)^{\lg n}$

$\left(\frac{3}{2}\right)^n$

2^n

$n2^n$

e^n

$n!$

$(n+1)!$

2^{2^n}

$2^{2^{n+1}}$

b)

The basic idea to find a function f(n) is rather simple. We need this function to oscillate between all its maximal values and minimal values. We observe that sin(x) and cos(x) oscillate between their maximal values and minimal values.

$$f(n) = \begin{cases} n2^{2^{n+2}} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

$f(n)$ grows faster than all functions in part a, so it cannot be bounded above by any one of them, and all functions in part a are greater than 0 for sufficiently large n, so they cannot bound $f(n)$ below either.