HW 11 SOLUTION - Trees

10.1 - 28 This tree has 1 vertex at level 0, m vertices at level 1, m^2 vertices at level $2, \cdots, m^h$ vertices at level h. Therefore, it has

$$1 + m + m^2 + \cdots + m^h = \frac{m^{h+1} - 1}{m - 1} \tag{1}$$

vertices in all. The vertices at level h are the only leaves, so it has m^h leaves.

10.1 - 32 The root of the tree represents the entire book. The vertices at level 1 represent the chapters - each chapter is a chapter of (read "child of") the book. The vertices at level 2 represent the sections (the parent of each such vertex is the chapter in which the section resides). Similarly the vertices at level 3 are the subsections.

10.1 - 44 We choose a root and color it red. Then we color all the vertices at odd levels, blue and all the vertices at even levels red.

10.3 - 8 If vertices have more than two children then we list them in preorder from left to right. The answer is $a, b, d, e, i, j, m, n, o, c, f, g, h, k, l, p$.

10.3 - 14 $d, i, m, n, o, j, e, b, f, g, k, p, l, h, c, a$

10.4 - 20 Since every vertex is connected to every other vertex, the breadth-first search will construct the tree $K_{1,n-1}$ with every vertex adjacent to the starting vertex. The depth-first search will produce a simple path of length $n - 1$ for the same reason.

10.4 - 24 If the edge is a cut edge, then it provides the unique simple path between its endpoints. Therefore, it must be in every spanning tree for the graph. Conversely, if an edge is not a cut edge, then it can be removed without disconnecting the graph and every spanning tree of the resulting graph will be a spanning tree of the original graph not containing this edge. Thus we have shown that an edge of a connected simple graph must be in every spanning tree of this graph if and only if the edge is a cut edge - i.e. its removal disconnects the graph.

10.5 - 6 With Kruskal's algorithm we add at each step the shortest edge that will not complete a simple circuit. Thus we pick edge $\{a, b\}$ first, and then edge $\{c, d\}$ (alphabetical order breaks ties), followed by $\{a, e\}$ and $\{d, e\}$.