HW 3 - Sets, Functions, and Relations

1 2.1

8 (a) True
(b) True
(c) False
(d) True
(e) True - the one element in the set on the left is an element of the set on the right, and the sets are not equal.
(f) True - similar to part. (e)
(g) False - the two sets are equal.

18 (a) The empty set has no elements, its cardinality is 0.
(b) This set has one element (the empty set), so its cardinality is 1.
(c) This set has two elements, so its cardinality is 2.
(d) This set has three elements, so its cardinality is 3.

22 (a) The power set of every set includes at least the empty set, so the power set cannot be empty. This ϕ is not the power set of any set.
(b) This is the power set of $\{a\}$.
(c) This set has three elements. Since 3 is not the power of 2, this set cannot be the powet set of any set.
(d) this is the power set of $\{a, b\}$.

34 (a) There is a real number whose cube is -1. This is true, since $x = -1$ is a solution.
(b) There is an integer such that the number obtained by adding 1 to it is greater than the integer. This is certainly true - in fact, every integer satisfies this statement.
(c) For every integer, the number obtained by subtracting 1 is again an integer. This is true.
(d) The square of every integer is an integer. This is true.

2 2.2

4 (a) $\{a, b, c, d, e, f, g, h\}$
(b) $\{a, b, c, d, e\}$
(c) There are no elements in A that are not in B, so the answer is ϕ.
(d) $\{f, g, h\}$

18 (a) Suppose that $x \in A \cup B$. Then either $x \in A$ or $x \in B$. In either case, certainly $x \in A \cup B \cup C$. This establishes the desired inclusion.
(b) Suppose that $x \in A \cap B \cap C$. Then x is in all three sets. In particular, it is in both A and B and, therefore, in $A \cap B$, as desired.
(c) Suppose that $x \in (A - B) - C$. Then $x \in A - B$, but $x \notin C$. Since $x \in A - B$, we know that $x \in A$ (we also know that $x \notin B$, but that won't be used here). Since we have established that $x \in A$ but $x \notin C$, we have proved that $x \in A - C$.
(d) To show that the set given on the left-hand side is empty, it suffices to assume that x is some element in that set and derive a contradiction, thereby showing that no such x exists. So suppose that $x \in (A - C) \cap (C - B)$. Then $x \in A - C$ and $x \in C - B$. The first of these statements implies

that by definition that $x \notin C$, while the second statement implies that $x \in C$. This is impossible, so our proof by contradiction is complete.

3 2.3

2 (a) This is not a function because the rule is not well-defined. We do not know whether $f(3) = 3$ or $f(3) = -3$. For a function, it cannot be both at the same time.
(b) This is a function. For all integers n, $\sqrt{n^2 + 1}$ is a well-defined real number.
(c) This is not a function with domain $\mathbb{Z}$, since for $n = \pm 2$, the value of $f(n)$ is not defined by the given rule. In other words, $f(2)$ and $f(-2)$ are not specified since division by 0 makes no sense.

16 (a) $f(n) = n + 17$
(b) $f(n) = \lceil n/2 \rceil$
(c) We let $f(n) = n - 1$ for even values of n, and $f(n) = n + 1$ for odd values of n. Thus we have $f(1) = 2$, $f(2) = 1$, $f(3) = 4$, $f(4) = 3$, and so on. Note that this is just one function, even though its definition used two formulae, depending on the parity of n.
(d) $f(n) = 17$

18 If we can find an inverse, the function is a bijection. Otherwise, we must explain why the function is not one-to-one or onto.
(a) This is a bijection since the inverse function is $f^{-1}(x) = (4 - x)/3$.
(b) This is not one-to-one since $f(17) = f(-17)$, for instance. It is also not onto, since the range is the interval $(\infty, 7]$. For example 344332 is not in the range.
(c) This function is a bijection, but not from $\mathbb{R}$ to $\mathbb{R}$. To see that the domain and range are not $\mathbb{R}$, note that $x = -2$ is not in the domain, and $x = 1$ is not in the range. On the other hand, f is a bijection from $\mathbb{R} - \{-2\}$ to $\mathbb{R} - \{1\}$, since its inverse is $f^{-1}(x) = (1 - 2x)/(x - 1)$.
(d) It is clear that this continuous function is increasing throughout its entire domain ($\mathbb{R}$) and it takes on both arbitrarily large values and arbitrarily small (large negative) ones. So it is a bijection. Its inverse is clearly $f^{-1}(x) = \sqrt[5]{x - 1}$.

4 8.1

4 (a) Being taller than is not reflexive (I am not taller than myself), nor symmetric (I am taller than my sister, but she is not taller than I). It is antisymmetric (vacuously, since we never have A taller than B and B taller than A, even if $A = B$). It is clearly transitive.
(b) This is clearly reflexive, symmetric, and transitive (it is an equivalence relation - see Section 8.5). It is not antisymmetric, since twins, for example, are unequal people born on the same day.
(c) This has exactly the same answers as part (b), since having the same first name is just like having the same birthday.
(d) This is clearly reflexive and symmetric. It is not antisymmetric, since my cousin and I have a common grandparent, and I and my cousin have a common grandparent, but I am not equal to my cousin. This relation is not transitive. My cousin and I have a common grandparent; my cousin and her cousin on the other side of her family have a common grandparent. My cousin's cousin and I do not have a common grandparent.

10 Only the relation in part (a) is irreflexive (the other ones are all reflexive).

SQL select Flight_number from Flights where Departure_time > '09:00'