HW 4 SOLUTION - Sequences and Sums

Sec 2.4 - # 10(a) The first term is 3, and the n^{th} term is obtained by adding $2n - 1$ to the previous term. In other words, we successively add 3, then 5 and then 7, and so on. Alternatively, we see that the n^{th} term is $n^2 + 2$; we can see this by inspection if we happen to notice how close each term is to a perfect square, or we can fit a quadratic polynomial to the data. The next three terms are 123, 146, 171.

Sec 2.4 - # 10(b) This is an arithmetic sequence whose first term is 7 and whose difference is 4. Thus the n^{th} term is $7 + 4(n - 1) = 4n + 3$. Thus the next three terms are 47, 51, 55.

Sec 2.4 - # 10(c) The n^{th} term is clearly the binary expansion of n. Thus the next three terms are 1100, 1101, 1110.

Sec 2.4 - # 10(d) The sequence consists of one 1, followed by three 2's, followed by five 3's, and so on. With the number of copies of the next value increasing by 2 each time, and the values themselves following the rule that the first two values are 1 and 2 and each subsequent value is the sum of the previous two values. Thus the next three terms are 8, 8, 8.

Sec 2.4 - # 18(a) We will write out the sum explicitly in each case. In this case the sum turns out to be $(1 - 1) + (1 - 2) + (2 - 1) + (2 - 2) + (3 - 1) + (3 - 2) = 3$.

Sec 2.4 - # 18(b) $(0+0)+(0+2)+(0+4)+(3+0)+(3+2)+(3+4)+(6+0)+(6+2)+(6+4)+(9+0)+(9+2)+(9+4) = 78$.

Sec 2.4 - # 24 From Table 2 we know that $\sum_{k=1}^{200} k^3 = 200^2.201^2/4 = 404,010,000$, and $\sum_{k=1}^{98} k^3 = 98^2.99^2/4 = 23,532,201$. Therefore, the desired sum is $404,010,000 - 23,532,201 = 380,477,799$.