HW1 - Propositional Logic, Logic Equivalences, Predicates and Quantifiers, Scoping and Nested Quantifiers

Problem 1 Let p, q, and r be the propositions:

p**:** You get an A on the final exam

q**:** You do every exercise in the book

r**:** You get an A in the class

Write the following propositions using p, q, r and logical connectives.

(a) You get an A in the class but you do not do every problem in the book

(b) You get an A on the final, you do every problem in the book and you get an A in the class

(c) To get an A in the class, it is necessary for you to get an A on the final

(d) You get an A on the final but you do not do every exercise in the book; nevertheless you get an A in the class

(e) Getting an A on the final and doing every exercise in the book is sufficient for getting an A in the class

(f) You will get an A in the class if and only if you either do every exercise in the book or you get an A on the final

Problem 2 Determine whether these biconditionals are true or false

(a) $2 + 2 = 4$ if and only if $1 + 1 = 2$

(b) $1 + 1 = 2$ if and only if $2 + 3 = 4$

(c) It is winter if and only if it is not spring, summer or fall

(d) $1 + 1 = 3$ if and only if pigs can fly

(e) $0 > 1$ if and only if $2 > 1$

Problem 3 Construct a truth table for $(p \vee \neg q) \rightarrow q$

Problem 4 Use truth tables to proves De Morgan's laws: $\neg(p \wedge q) \equiv \neg p \vee \neg q$ and $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Problem 5 Consider an intelligent alarm system that has the following sentence in its knowledge base: (Motion $\rightarrow$ Intruder) $\vee$ (Noise $\rightarrow$ Intruder)

Assume that the propositions Motion and Noise take on their respective truth values directly from information provided by sensors. Given this sentence, show through a series of logical equivalences, how one can conclude that (Motion $\wedge$ Noise) $\rightarrow$ Intruder.

Problem 6 Given that $p \rightarrow q$, show that the converse $(q \rightarrow p)$ is equivalent to the inverse $(\neg p \rightarrow \neg q)$, but not equivalent to the original.

Problem 7 Show that $p \rightarrow q$ and $\neg q \rightarrow \neg p$ are logically equivalent.

Problem 8 Determine via a truth table that $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology

Problem 9 Express these system specifications using the propositions p, "The message is scanned for viruses" and q, "The message was sent from an unknown system" together with logical connectives.

(a) The message is scanned for viruses whenever the message was sent from an unknown system.

(b) The message was sent from an uknown system but it was not scanned for viruses

(c) It is necessary to scan the message for viruses whenever it was sent from an unknown system

(d) When the message is not sent from an uknown system it is not scanned for viruses

Problem 10 Find the antecedent and consequent of each statement

(a) Healthy plant growth follows from sufficient water

(b) Increased availability of information is a necessary condition for further technological advances

(c) Errors will be introduced only if there is a modification to the program

(d) Fuel savings implies good insulation or storm windows throughout

Problem 11 Write the negation of each statement

(a) If the food is good, then the service is excellent

(b) Either the food is good or the service is excellent

Problem 12 Translate the following compound statements into symbolic notation.

(a) If the prices go up, then housing will be plentiful and expensive; but if housing is not expensive, then it will still be plentiful

(b) Either going to bed or going swimming is sufficient condition for changing clothes; however, changing clothes does not mean going swimming

(c) Either it will rain or snow but not both

(d) If Janet wins, or if she loses, she will be tired

(e) Either Janet will win or, if she loses, she will be tired

Problem 13 What conclusion, if any, can you read from the statements, "If the bill was sent today, you will be paid tomorrow. You will be paid tomorrow."

Problem 14 Use propositional logic to prove $\neg a \wedge (a \vee b) \rightarrow b$