HW 1 - SOLUTIONS

Problem 1 Let p, q, and r be the propositions:

p**:** You get an A on the final exam

q**:** You do every exercise in the book

r**:** You get an A in the class

Write the following propositions using p, q, r and logical connectives.

(a) You get an A in the class but you do not do every problem in the book

(b) You get an A on the final, you do every problem in the book and you get an A in the class

(c) To get an A in the class, it is necessary for you to get an A on the final

(d) You get an A on the final but you do not do every exercise in the book; nevertheless you get an A in the class

(e) Getting an A on the final and doing every exercise in the book is sufficient for getting an A in the class

(f) You will get an A in the class if and only if you either do every exercise in the book or you get an A on the final

Solution 1

(a) $r \wedge \neg q$

(b) $p \wedge q \wedge r$

(c) $r \rightarrow p$

(d) $p \wedge r \wedge \neg q$

(e) $(p \wedge q) \rightarrow r$

(f) $r \leftrightarrow (p \oplus q)$, alternatively $r \leftrightarrow (p \vee q)$ depending on how the sentence is interpreted

Problem 2 Determine whether these biconditionals are true or false

(a) $2 + 2 = 4$ if and only if $1 + 1 = 2$

(b) $1 + 1 = 2$ if and only if $2 + 3 = 4$

(c) It is winter if and only if it is not spring, summer or fall

(d) $1 + 1 = 3$ if and only if pigs can fly

(e) $0 > 1$ if and only if $2 > 1$

Solution 2

(a) True

(b) False

(c) True

(d) True

(e) False

Problem 3 Construct a truth table for $(p \vee \neg q) \rightarrow q$

Solution 3

p	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \rightarrow q$
T	T	T	F
T	F	T	T
F	T	T	F
F	F	F	T

Problem 4 Use truth tables to proves De Morgan's laws: $\neg(p \wedge q) \equiv \neg p \vee q$ and $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Solution 4

p	q	$\neg(p \wedge q)$	$\neg p \vee \neg q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F	F	F
F	F	T	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

Problem 5 Consider an intelligent alarm system that has the following sentence in its knowledge base: (Motion $\rightarrow$ Intruder) $\vee$ (Noise $\rightarrow$ Intruder)

Assume that the propositions Motion and Noise take on their respective truth values directly from information provided by sensors. Given this sentence, show through a series of logical equivalences, how one can conclude that (Motion $\wedge$ Noise) $\rightarrow$ Intruder.

Solution 5 The alarm can conclude that there is an intruder only when both motion and noise are detected. This can be understood directly by recognizing that if either Motion or Noise is false, the value of Intruder is a "don't care" value: the proposition is true regardless of what value it takes. Only when both Motion and Noise are true is it the case that Intruder must also be true if the proposition as a whole is to be true.

Logically, this can be proven by

1. (Motion $\rightarrow$ Intruder)$\leftrightarrow$($\neg$ Motion $\vee$ Intruder)
2. (Noise $\rightarrow$ Intruder)$\leftrightarrow$($\neg$ Noise $\vee$ Intruder)
3. ((Motion $\rightarrow$ Intruder)$\vee$(Noise $\rightarrow$ Intruder))
   $\leftrightarrow$($\neg$ Motion $\vee$ Intruder)$\vee$($\neg$ Noise $\vee$ Intruder)
   $\leftrightarrow$ $\neg$ Motion $\vee$ Intruder $\vee$ $\neg$ Noise $\vee$ Intruder
   $\leftrightarrow$ $\neg$ Motion $\vee$ $\neg$ Noise $\vee$ Intruder
   $\leftrightarrow$ $\neg$(Motion $\wedge$ Noise) $\vee$ Intruder
   $\leftrightarrow$ (Motion $\wedge$ Noise) $\rightarrow$ Intruder

Problem 6 Given that $p \rightarrow q$, show that the converse $(q \rightarrow p)$ is equivalent to the inverse $(\neg p \rightarrow \neg q)$, but not equivalent to the original.

Solution 6 $(q \rightarrow p) \leftrightarrow (\neg q \vee p) \leftrightarrow (\neg p \rightarrow \neg q)$ by the equivalence between $p \rightarrow q$ and $\neg p \vee q$

Problem 7 Show that $\neg(p \leftrightarrow q)$ and $(p \leftrightarrow \neg q)$ are logically equivalent.

Solution 7 $\neg(p \leftrightarrow q) \equiv \neg((p \rightarrow q) \wedge (q \rightarrow p)) \equiv \neg(p \rightarrow q) \vee \neg(q \rightarrow p) \equiv \neg(\neg p \vee q) \vee \neg(\neg q \vee p) \equiv (p \wedge \neg q) \vee (q \wedge \neg p) \equiv p \leftrightarrow \neg q$ (By identity 3 in Table 8, pg. 25)

Problem 8 Determine via a truth table that $(\neg q \wedge (p \to q)) \to \neg p$ is a tautology

Solution 8 - By Contradiction The only way this implication can be false if $\neg q \wedge (p \to q)$ is true and $\neg p$ is false. Therefore, p must be true if this statement is not a tautology. The only way for $\neg q \wedge (p \to q)$ to be true is that q be false, and so again that must be the case if this statement is not a tautology. With these values, $(p \to q)$ becomes true $\to$ false, which can not be true. Therefore, the original statement is a tautology, since assuming otherwise results in a contradiction.

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg q \wedge (p \to q)$	$\neg q \wedge (p \to q) \to \neg p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

Problem 9 Express these system specifications using the propositions p, "The message is scanned for viruses" and q, "The message was sent from an unknown system" together with logical connectives.

- **(a)** The message is scanned for viruses whenever the message was sent from an unknown system.
- **(b)** The message was sent from an uknown system but it was not scanned for viruses
- **(c)** It is necessary to scan the message for viruses whenever it was sent from an unknown system
- **(d)** When the message is not sent from an uknown system it is not scanned for viruses

Solution 9

- **(a)** $q \to p$
- **(b)** $q \wedge \neg p$
- **(c)** $q \to p$
- **(d)** $\neg q \to \neg p$

Problem 10 Find the antecedent and consequent of each statement

- **(a)** Healthy plant growth follows from sufficient water
- **(b)** Increased availability of information is a necessary condition for further technological advances
- **(c)** Errors will be introduced only if there is a modification to the program
- **(d)** Fuel savings implies good insulation or storm windows throughout

Solution 10

- **(a)** Antecedent: sufficient water. Consequent: healthy plant growth
- **(b)** Antecedent: further technological advances. Consequent: increases availability of information
- **(c)** Antecedent: errors will be introduced. Consequent: there is a modification of the program
- **(d)** Antecedent: fuel savings. Consequent: good insulation or storm windows throughout

Problem 11 Write the negation of each statement

(a) If the food is good, then the service is excellent

(b) Either the food is good or the service is excellent

Solution 11

(a) The food is good but the service is poor

(b) The food is poor and so is the service

Problem 12 Translate the following compound statements into symbolic notation. You need not attempt to simplify them or prove that they are valid.

(a) If the prices go up, then housing will be plentiful and expensive; but if housing is not expensive, then it will still be plentiful

(b) Either going to bed or going swimming is sufficient condition for changing clothes; however, changing clothes does not mean going swimming

(c) Either it will rain or snow but not both

(d) If Janet wins, or if she loses, she will be tired

(e) Either Janet will win or, if she loses, she will be tired

Solution 12

(a) (PricesIncrease $\rightarrow$ PlentifulHousing $\wedge$ ExpensiveHousing) $\wedge$ ($\neg$ ExpensiveHousing $\rightarrow$ PlentifulHousing)

(b) ((GoingBed $\vee$ GoingSwimming) $\rightarrow$ ChangingClothes) $\wedge$ $\neg$(ChangingClothes $\rightarrow$ GoingSwimming)

(c) (Rain $\vee$ Snow) $\wedge$ $\neg$(Rain $\wedge$ Snow)

(d) (JanetWins $\vee$ JanetLoses) $\rightarrow$ JanetTired ($\oplus$ is also an acceptable relation between JanetWins and JanetLoses)

(e) JanetWins $\vee$ (JanetLoses $\rightarrow$ JanetTired) ($\oplus$ is also an acceptable relation between the clauses of this statement)

Problem 13 What conclusion, if any, can you read from the statements, "If the bill was sent today, you will be paid tomorrow. You will be paid tomorrow."

Solution 13 The hypotheses have the form $(b \rightarrow p) \wedge p$. Only p, "you will be paid tomorrow," can be concluded, using simplification. You can not determine whether the bill was sent.

Problem 14 Use propositional logic to prove $\neg a \wedge (a \vee b) \rightarrow b$

Solution 14 Proof by a series of equivalences:

1. $\neg a \wedge (a \vee b) \rightarrow b$	
2. $(\neg a \wedge a) \vee (\neg a \wedge b) \rightarrow b$	1, distribution
3. $F \vee (\neg a \wedge b) \rightarrow b$	2, negation
4. $(\neg a \wedge b) \rightarrow b$	3, identity
5. $\neg(\neg a \wedge b) \vee b$	
6. $a \vee \neg b \vee b$	5, deMorgan
7. $a \vee T$	6, negation
8. True	