HW 2 - Transposing and Negating Quantifiers; Rules of Inference; Introduction to Proofs

1 Section 1.3

Problem 8 Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the universe of discourse consists of all animals.

- **(a)** $\forall x(R(x) \to H(x))$
- **(b)** $\forall x(R(x) \land H(x))$ (what is wrong with this statement?)
- **(c)** $\exists x(R(x) \to H(x))$
- **(d)** $\exists x(R(x) \land H(x))$

Problem 16 Determine the truth value of each of these statements if the universe of discourse of each variable consists of all real numbers

- **(a)** $\exists x(x^2 = 2)$
- **(b)** $\exists x(x^2 = -1)$
- **(c)** $\forall x(x^2 + 2 \geq 1)$
- **(d)** $\forall x(x^2 \neq x)$

Problem 20 Suppose that the universe of discourse of the propositional function $P(x)$ consists of -5,5,-3,-1,1,3, and 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

- **(a)** $\exists x P(x)$
- **(b)** $\forall x P(x)$
- **(c)** $\forall x((x \neq 1) \to P(x))$
- **(d)** $\exists x((x \geq 0) \land P(x))$
- **(e)** $\exists x(\neg P(x)) \land \forall x((x \leq 0) \to P(x))$

Problem 32 Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (The universe of discourse is "koalas")(Do not simply use the words "It is not the case that")

- **(c)** Every koala can climb
- **(d)** No monkey can speak French
- **(e)** There exists a pig that can swim and catch fish

Problem 50 Show that $\forall x P(x) \lor \forall x Q(x)$ and $\forall x(P(x) \lor Q(x))$ are NOT logically equivalent.

2 Section 1.4

Problem 6 Let $C(x, y)$ mean that student x is enrolled in class y, where the universe of discourse for x consists of all students at USC and the universe of discourse for y is the set of all classes being given at USC. Express each of these statements by a simple English sentence

- **(a)** C(Randy Goldberg, CS 252)

(b) $\exists x C(x, \text{Math 695})$

(c) $\exists y C(\text{Carol Sitea}, y)$

(d) $\exists x(C(x,\text{Matt 222}) \wedge C(x, \text{CS 252}))$

(e) $\exists x \exists y \forall z((x \neq y) \wedge (C(x, z) \rightarrow C(y, z)))$

(f) $\exists x \exists y \forall z((x \neq y) \wedge (C(x, z) \leftrightarrow C(y, z)))$

Problem 10 Let $F(x, y)$ be the statement "x can fool y", where the universe of discourse consists of all the people in the world. Use quantifiers to express each of these statements.

(a) Everybody can fool Fred.

(b) Everybody can fool everybody.

(c) Everybody can fool somebody.

(d) There is no one who can fool everybody.

(e) No one can fool both Fred and Jerry.

(g) Nancy can fool exactly two people.

(h) There is exactly one person whom everybody can fool.

(i) No one can fool himself or herself.

(j) There is someone who can fool exactly one person besides himself.

Problem 22 Use predicates, quantifiers, logical connectives, and mathematical operators to expres the statement that there is a positive integer that is not the sum of three squares.

Problem 40 Find a counterexample, if possible, to these universally quantified statements, where the universe of discourse for all variables consists of all integers

(b) $\forall x \exists y(y^2 - x < 100)$

3 Section 1.5

Problem 5 Use rules of inference to show that hypotheses, " Randy works hard," "If Randy works hard, then he is a dull boy," and "If Randy is a dull boy, then he will not get the job" imply the conclusion "Randy will not get the job."

Problem 6 Construct an argument using rules of inference to show that the hypotheses, "If it does not rain or it it is not foggy, then the sailing race will be held and the life saving demonstration will go on, " "If the sailing race is held, then the trophy will be awareded, " and "The trophy was not awarded" imply the conclusion "It rained."

Problem 16 For each of these arguments determine whether the argument is correct of incorrect and explain why.

(a) Everyone enrolled in the university has lived in a dormitory. Mis has never lived in a dormitory. Therefore, Mia is not enrolled in the university.

(b) A convertible car is fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive.

4 Proofs

Note: Proof steps should be numbered and justified

Problem 1 Do two proofs. One forward and one by contradiction:

1. $p \to q$
2. $r \to s$
3. $p \vee r$

Prove $q \vee s$

Problem 2 First order logic proof.

1. $\exists x R(x)$
2. $\neg \exists x (R(x) \wedge S(x))$

Prove: $\exists x \neg S(x)$