HW 5 SOLUTION - Induction and Recursion

Sec 4.1 - # 4(a) Plugging in $n = 1$ we have that $P(1)$ is the statement $1^3 = [1.(1+1)/2]^2$, which is obviously true.
(b) Both sides of $P(1)$ shown in part (a) equal 1.
(c) The inductive hypothesis is the statement that

$$1^3 + 2^3 + \cdots + k^3 = \left(\frac{k(k+1)}{2}\right)^2 \tag{1}$$

(d) For the inductive step, we want to show that for each $k \geq 1$, that $P(k)$ implies $P(k+1)$. In other words, we want to show that assuming the inductive hypothesis (see part (c)) we can prove

$$[1^3 + 2^3 + \cdots k^3] + (k+1)^3 = \left(\frac{(k+1)(k+2)}{2}\right)^2 \tag{2}$$

(e) Replacing the quantity in brackets on the left-hand side of part (d) by what it equals by virtue of the inductive hypothesis, we have

$$\begin{aligned} \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3 &= (k+1)^2\left(\frac{k^2}{4} + k + 1\right) \\ &= (k+1)^2\left(\frac{k^2+4k+4}{4}\right) \\ &= \left(\frac{(k+1)(k+2)}{2}\right)^2 \end{aligned} \tag{3}$$

(f) We have completed both the basis step and the inductive step, so by the principle of mathematical induction, the statement is true for every positive integer n.

Sec 4.1 - # 18 (a) Plugging in $n = 2$, we see that $P(2)$ is the statement $2! < 2^2$.
(b) Since $2! = 2$, this is the true statement $2 < 4$.
(c) The inductive hypothesis is the statement that $k! < k^k$.
(d) For the inductive step, we want to show for each $k \geq 2$ that $P(k)$ implies $P(k+1)$. In other words we want to show that assuming the inductive hypothesis (see part (c)) we can prove that $(k+1)! < (k+1)^{k+1}$.
(e) $(k+1)! = (k+1)k! < (k+1)k^k < (k+1)(k+1)^k = (k+1)^{k+1}$.
(f) We have completed both the basis step and the inductive step, so by the principle of mathematical induction, the statement is true for every positive integer $n > 1$.

Sec 4.3 - #10 The base case is that $S_m(0) = m$. The recursive part is that $S_m(n+1)$ is the successor of $S_m(n)$ (i.e., integer that follows $S_m(n)$, namely $S_m(n) + 1$).

Sec 4.3 - #38 There are two types of palindromes, so we need two base cases, namely λ is a palindrome, and x is a palindrome for every symbol x. The recursive step is that if α is a palindrome, and x is a symbol, then $x\alpha x$ is a palindrome.

Sec 4.3 - #40 The key fact is that if a bit string of length greater 1 has more 0's than 1's, then either it is the concatenation of two such strings, or else it is the concatenation of two such strings with one 1 inserted either before the first, between them, or after the last. This can be proved by looking at the running count of the excess of 0's over 1's as we read the string from left to right. Therefore, one recursive definition is that 0 is in the set, and if x and y are in the set, then so are the xy, $1xy$, $x1y$, and $xy1$.

(i) $prod(m, n)$, $0 \leq m \leq n$
if $n = 0$ then return 0
else $prod(m, pred(n)) + m$

(ii) flatten(mylist)
if atom(mylist) then return mylist;
else if $\neg$ atom(first(mylist)) then
return append(flatten(first(mylist)), flatten(rest(mylist)));
else return append(list(first(mylist)), flatten(rest(mylist)))

Note that list(x) is equivalent to cons(x, λ)