HW 6 - Counting, Pigeon Hole Principle

1 5.1

40 (a) We first place the bride in any of the six positions. Then, from left to right in the remaining positions, we choose the other five people to be in the picture; this can be done in 9.8.7.6.5 = 15,120 ways. Therefore, the answer is 6.15120 = 90,720.
(b) We first place the bride in any of the six positions, and then place the groom in any of the five remaining positions. Then, from left to right in the remaining positions, we choose the other four people to be in the picture; this can be done in 8.7.6.5 = 1,680 ways. Therefore the answer is 6.5.1680 = 50,400.
(c) From part (a) there are 90,720 ways for the bride to be in the picture. There are (from part (b)) 50,400 ways for both the bride and groom to be in the picture. Therefore, there are 90,720 - 50,400 = 40,320 ways for just the bride to be in the picture. Symmetrically, there are 40,320 ways for just the groom to be in the picture. Therefore, the answer is 40320 + 40320 = 80,640.

42 There are 2^5 strings that begin with two 0's (since there are two choices for each of the last five bits). Similarly, there are 2^4 strings that end with three $1's$. Furthermore, there are 2^2 strings that both begin with two 0's and end with three 1's (since only bits three and four are free to be chosen). By the inclusion-exclusion principle, there are $2^5 + 2^4 - 2^2 = 44$ such strings in all.

52 See the attached pdf file.

58 There are 2^n lines in the truth table, since each of the n propositions can have two truth values. Each line can be filled with T or F, so there are a total of 2^{2^n} possibilities.

2 5.2

16 We can apply the pigeonhole principle by grouping the numbers cleverly into pairs (subsets) that add up to 16, namely $\{1, 15\}, \{3, 13\}, \{5, 11\}, \{7, 9\}$. If we select five numbers from the set $\{1, 3, 5, 7, 9, 11, 13, 15\}$ then at least two of them must fall within the same subset, since there are only four subsets. Two number in the same subset are the desired pair that add up to 16. We also need to point out that choosing four numbers is not enough, since we could choose $\{1, 3, 5, 7\}$, and no pair of them add up to more than 12.

30 There are 99,999,999 possible positive salaries less than one million dollars, i.e., from $0.01 - $999,999.99. By the pigeon hole principle, if there were more than this many people with positive salaries less than one million dollars, then at least two of them must have the same salary.

32 Let $K(x)$ be the number of other computers that computer x is connected to. The possible values for $K(x)$ are 1,2,3,4,5. Since there are six computers, the pigeonhole principle guarantees that at least two of the values $K(x)$ are the same, which is what we wanted to prove.

3 5.4

4 $\binom{13}{8} = 1287$

12 We just add adjacent numbers in this row to obtain the next row (starting and ending with 1, of course):
1 11 55 165 330 462 462 330 165 55 11 1