The Problem:
5. Consider an intelligent alarm system that has the following sentence in its knowledge base:
((Motion => Intruder) v (Noise => Intruder))

Assume that the propositions Motion and Noise take on their respective truth-values, directly from information provided by the agent's sensors. Given this sentence, show precisely the circumstances under which this agent can conclude the presence of an intruder (i.e. conclude whether or not the proposition Intruder is true).

One Good Solution :

1. (Motion => Intruder) ↔ (¬Motion v Intruder)

2. (Noise => Intruder) ↔ (¬Noise v Intruder)

3. ((Motion => Intruder) v (Noise => Intruder)) ↔ (¬Motion v Intruder) v (¬Noise v Intruder)
↔ (¬Motion v Intruder v ¬Noise v Intruder)
↔ (¬Motion v ¬Noise v Intruder)
↔ (¬(Motion & Noise) v Intruder)
↔ ((Motion & Noise) => Intruder)

Therefore, we can see by a simple application of Modus Ponens that there has to be Noise and Motion for the Alarm to conclude that there is an Intruder!

Most students tried to build a truth table of the formula as follows:

Let Motion = M Noise = N Intruder = I

M	N	I	M=>I	N=>I	(M=>I)V(N=>I)
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	1	1	1

Answers (all erroneous) given by students:
(1) In every case ie. whether or not there is motion (and/or) noise – an intruder may / may not be there
(2) An intruder is there is every case except when M and N are TRUE and I is FALSE
(3) The proposition Intruder may / may not be true (for various combinations of noise and motion).

My Analysis:

As I mentioned, the above truth table analysis is bogus. First, let's look at the first proof a little more closely. Lines 1 and 2 just took the individual disjuncts of the alarm system's rule and put then into conjunctive normal form. Line 3 did a substitution, then the rest is a few trivial manipulations to the conclusion: ((Motion & Noise) => Intruder).

And what does that formula mean in the context of this problem ? We were asked under what circumstances the alarm would answer that Intruder was true. Well, under what circumstances would Intruder be True ? With (Motion&Noise) and modus ponens, we could conclude Intruder was true. No other combination of Motion and Noise would allow this deduction. So, what the hell is that truth table telling us ? (The answer is, in fact, in the truth table, but it is hiding. Be patient.)

First, let us consider a simpler problem. Suppose you were given the formula (P =>Q) and asked under what circumstances could you conclude Q. Note first that this is simply a restatement of our problem with simpler terms. Well, the answer is obvious. If P were true, you could conclude Q using Modus Ponens. Under no other circumstances could you conclude Q. Right ? You must convince yourself of this before you go on.

Now, lets do an analogous truth table on this problem like the one students showed above :

P	Q	P =>Q
0	0	1
0	1	1
1	0	0
1	1	1

Well, there it is. What do you think ? So, does the answer fall out ? P =>Q is true in all cases except the 1,0 case and so therefore, what ? What indeed ? Again, convince yourself that this truth table is as confused as the student's. Right ?

What's missing is four more columns for our original problem. The question asks under what conditions [of the percepts] can you conclude Q. There is only one percept in the simplified problem and that is P. Let's do the analysis 2 columns at a time. Our question asks, what something do we need such that : ((P =>Q) & something) =>Q.
The something can only be P or ~P. Let's try them both. First ~P

P	Q	P =>Q	((P =>Q) & ~P)	((P =>Q) & ~P)=>Q
0	0	1	1	0
0	1	1	1	1
1	0	0	0	1
1	1	1	0	1

So, it is not true that ((P =>Q) & ~P)=>Q in all models. See, the 0th row is false. Let's try the analysis with P being true.

P	Q	P =>Q	((P =>Q) & P)	((P =>Q) & P)=>Q
0	0	1	0	1
0	1	1	0	1
1	0	0	0	1
1	1	1	1	1

Aha!!! It's true in all models. Therefore, given the rule P =>Q, under what conditions can you always conclude that Q is true. The ANSWER : when P is true. That proves modus ponens with a truth table.

Now to do this truth table analysis with the student's matrix would take a long time, because with two percepts, Motion and Noise , there would four extra tables with each with the two extra columns. I'll do one and then show how to solve it by inspection of The student's matrix. Look at the case where both percepts are off : (~M&~N)

M	N	I	(M=>I)V(N=>I)=x	x&(~M&~N)=y	y=>I
0	0	0	1	1	0
0	0	1	1	1	1
0	1	0	1	0	1
0	1	1	1	0	1
1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	0	0	1

1	**1**	**1**	**1**	**0**	**1**

Not true in all models. Row 0 results in False. If you do the same thing for each of (M&~N), (~M&N), (M&N), only with M&N will it be true all the way down.

Now, let's see how we could have deduced this result directly from the student's matrix. Look at his rows 0 and 1. Those rows correspond to the case where we have (~M&~N). Notice that the detector rule (last column) is true in both rows, but in row 0 Intruder is false and in row 1 Intruder is True. THEREFORE, it is not the case that if both Motion and Noise are off that we can conclude anything about Intruder.

Now, look at rows 2 and 3. Those rows correspond to the case where we have (~M&N). Notice that the detector rule (last column) is true in both rows, but in row 2 Intruder is false and in row 3 Intruder is True. THEREFORE, it is not the case the if Motion if off and Noise is on that we can conclude anything about Intruder.

Now, look at rows 4 and 5. Those rows correspond to the case where we have (M&~N). Notice that the detector rule (last column) is true in both rows, but in row 4 Intruder is false and in row 5 Intruder is True. THEREFORE, it is not the case the if Motion if on and Noise is off that we can conclude anything about Intruder.

Now, finally look at rows 6 and 7. Those rows correspond to the case where we have (M&N). Notice that the detector rule (last column) is FALSE in row 6, but TRUE in row 7, the first time we have variation. Fortunately in row 6 Intruder is false so therefore the y=>I implication is still TRUE. And in row 7 I is true, so the implication is TRUE. THEREFORE, IT IS the case the if Motion and Noise are on that we can conclude Intruder is TRUE.

I'll do it in full for the truth table:

M	N	I	(M=>I)V(N=>I)=x	x&(M&N)=y	y=>I
0	0	0	1	0	1
0	0	1	1	0	1
0	1	0	1	0	1
0	1	1	1	0	1
1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	0	0	1
1	1	1	1	1	1

TRUE in all models. Therefore M&N and the rule imply an Intruder. Aren't you glad you asked ?

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