Michaelis-Menten kinetics with reversible intermediate step:

$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} ES \underset{k_{-2}}{\overset{k_2}{\rightleftharpoons}} EP \underset{k_{-3}}{\overset{k_3}{\rightleftharpoons}} E+P$$

for initial forward rate, [P]=0

$$v_{forward} = \frac{d[P]}{dt} = k_3[EP]$$

$$\frac{d[EP]}{dt} = -k_3[EP] - k_{-2}[EP] + k_2[ES]$$

$$\frac{d[ES]}{dt} = k_1[E][S] - k_{-1}[ES] - k_2[ES] + k_{-2}[EP]$$

S.S.A. ➔ $\frac{d[EP]}{dt} = 0$; $\frac{d[ES]}{dt} = 0$

Mass balance: $[E] + [ES] + [EP] = [E]_0$

This gives

$$[ES] = \frac{k_1[E]_0[S]}{\frac{-k_2(k_{-2} - k_1[S])}{k_3 + k_{-2}} + (k_2 + k_{-1}) + k_1[S]}$$

$$[EP] = \frac{k_2}{k_3 + k_{-2}}[ES]$$

$$v_{forward} = \frac{d[P]}{dt} = k_3[EP] = \frac{k_3 k_2 [E]_0 [S]}{(k_2 + k_{-2} + k_3)\{\frac{k_{-1}k_{-2} + k_{-1}k_3 + k_2 k_3}{k_1(k_2 + k_{-2} + k_3)} + [S]\}}$$

$$v_{forward} = \frac{v_f[S]}{K_M^f + [S]}$$

with

$$v_f = \frac{k_3 k_2 [E]_0}{(k_2 + k_{-2} + k_3)}$$

$$K_M^f = \frac{k_{-1}k_{-2} + k_{-1}k_3 + k_2 k_3}{k_1(k_2 + k_{-2} + k_3)}$$