Performance Analysis of Epidemic Routing under Contention

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***Abstract*— Epidemic routing has been proposed as a robust transmission scheme for sparse mobile ad hoc networks. Under the assumption of no contention, epidemic routing has the minimum end-to-end delay amongst all the routing schemes proposed for such networks. The assumption of no contention was justified by arguing that since the network is sparse, there will be very few simultaneous transmissions. Some recent papers have shown through simulations that this argument is not correct and that contention cannot be ignored while analyzing the performance of routing schemes, even in sparse networks.**

Incorporating contention in the analysis has always been a hard problem and hence its effect has been studied mostly through simulations only. In this paper, we find analytical expressions for the delay performance of epidemic routing with contention. We include all the three main manifestations of contention, namely (i) the finite bandwidth of the link which limits the number of packets two nodes can exchange, (ii) the scheduling of transmissions between nearby nodes which is needed to avoid excessive interference, and (iii) the interference from transmissions outside the scheduling area. The accuracy of the analysis is verified via simulations.

I. Introduction

Routing in a mobile ad hoc network is an important and challenging problem and many routing protocols have been proposed and studied (DSDV [1], DSR [2], AODV [3]). These protocols borrowed ideas from wired routing and maintained routing states in the nodes.

In Delay Tolerant Mobile Networks, there does not exist a complete end-to-end path from a source to a destination. Even if such a path exists, it is highly unstable and may change or break soon after it has been discovered (or even while it is being discovered). So, the conventional mobile ad hoc routing protocols will fail to successfully route packets through such networks. The reactive schemes will fail to discover a complete path while proactive protocols will fail to converge. Examples of such networks include sensor networks for wildlife tracking and habitat monitoring [4], [5], military networks [6], inter-planetary networks [7], nomadic communities networks [8], networks of mobile robots [9] etc.

Epidemic routing [10] was proposed as a robust routing scheme for such a network. It adopts a "store-carry-forward" paradigm: every node acts as a relay for other nodes. The algorithm is essentially flooding with some variations to reduce overhead. There has been some effort to theoretically characterize the performance of epidemic routing [11], [12], [13]. But none of these analytical works take contention into account. This is not because contention is not an important problem but because it is very hard to analyze. [14], [15], [16], [17] have taken contention into account through simulations and shown its detrimental effect on the performance.

This paper finds analytical expressions for the delay performance of epidemic routing with contention in the network. Contention manifests itself in three ways: (i) finite bandwidth, which limits the number of packets two nodes can exchange when they are within range, (ii) scheduling of transmissions between nearby nodes which is needed to avoid excessive interference, and (iii) interference from transmissions outside the scheduling area. The reason for taking finite bandwidth into account is easy to see, but taking both scheduling and interference into account might, at first glance, appear unnecessary. However, recent studies [18], [19], [20] have shown that the circular disk model is an inaccurate channel model and multipath fading in a realistic channel can result in significant received power from transmitters which are not nearby. Hence, if a realistic fading channel model is assumed, then the interference from nodes outside the scheduling area cannot be neglected.

The outline of the paper is as follows. Section II presents our notation and assumptions, states some useful results for the random walk mobility model which we use during the course of the analysis and provides a brief introduction to epidemic routing principles. Section III finds the probability that a particular packet is successfully exchanged inspite of contention in the network. Section IV computes the expected delay for epidemic routing in sparse networks and verifies through simulations that the approximations made to simplify the analysis do not effect the accuracy of the analysis. Finally, Section V concludes our paper.

II. Prelimiaries

A. Notation and Assumptions

This section defines our model for the network.

1. M nodes perform independent random walks on a $\sqrt{N} \times \sqrt{N}$ 2-D torus.
2. Each node moves one grid unit in one time unit.

N	Number of grid points
M	Number of nodes in the network
K	The transmission range
Θ	The desirable SIR ratio
S	Average number of distinct packets in the network

TABLE I
NOTATION USED

3. We use manhattan distance (L1 norm) to measure the distance between two nodes.
4. Each node acts as a source sending packets to a randomly selected destination.
5. The average number of distinct packets in the network at a given time is S.
6. Two nodes transmit to each other when the distance between them is less than K. The justification for this scheme can be found in studies which characterize the physical channel [18], [20]. If the distance between the two nodes is less than K (whose actual value depends on the transmitted power), then the packet reception rate is nearly one, that is the probability that a packet is received in error is nearly zero. (This is called the connected region.)
   Note that this does not imply that transmissions from nodes at a distance greater than K are not going to interfere with the ongoing transmission.
7. The signal to interference ratio should be greater than a desired threshold, which we call Θ, for the transmission to be successful.
8. We assume a Rayleigh-Rayleigh fading model (both the desired and the interfering signals are Rayleigh distributed).

B. Useful Results for Random Walk Mobility Model

This section presents results on random walks which we use during the analysis.

Let ET_j denote the expected hitting time until a walk starting from the stationary distribution reaches j. On a symmetric graph, this quantity is independent of j, and we denote it as ET. Let EM denote the expected time until two independent random walks, starting from the stationary distribution, first meet each other. The following lemma states a number of properties of these quantities.

Lemma 2.1: Let independent random walks be performed on a $\sqrt{N} \times \sqrt{N}$ torus. Then,

(i) The steady state distribution of the location of each walk is uniform.
(ii) $ET = cNlogN$, where $c = 0.34$. In theory, this result is valid as $N \to \infty$. However, the result is quite accurate for $N > 25$ [21], [22].
(iii) $EM = \frac{1}{2}ET$.
(iv) The tail of the distribution of the hitting times is exponentially distributed.

Proof: See [22]. □

The above lemma computes the meeting times when the transmission range is zero ($K = 0$). The following lemma extends the previous lemma when $K \geq 0$.

Lemma 2.2: For $K \geq 0$,

$$ET = N\left(clogN - \frac{2^{K+1} - K - 2}{2^K - 1}\right), \qquad (1)$$

$$EM = \frac{1}{2}ET. \qquad (2)$$

Proof: See [12]. □

[22] shows that it is the tails of the distribution of the hitting and meeting times that are exponential. In this paper, we assume that the distribution is itself exponential. According to [11], this is a good approximation for small to moderate K with respect to the total area. Since we are assuming a sparse network (where $K << N$), this approximation does not have a significant impact on the accuracy of the analysis.

EM^+ denotes the expected intermeeting time which is the expected time for two walks to meet if they started from within range of each other. We find its value in Lemma 2.4. But, first we prove a useful result which will be used to derive EM^+.

Lemma 2.3: Let j be a position in the torus, and let $A(K)$ denote the subset of all positions a such that, $\mid x_a - x_j \mid + \mid y_a - y_j \mid = K$. Let a random walk start from $A(K)$ and move towards j in its first step (that is after the first step, the distance between the walk and j is equal to $K-1$). Let further T_j denote the time until this random walk first hits j and T_A^+ denote the time until this random walk first returns to $A(K)$. Then, $P(T_A^+ < T_j) = \frac{1+l_{K-1}}{1+l_K}$, where $l_K = 1 + \sum_{t=1}^{K-1} \prod_{r=1}^{t} \frac{q_r}{p_r}$ and $p_r = \frac{2r+1}{4r}$, $q_r = \frac{2r-1}{4r}$.

Proof: Lets assume that the walk is at one of $4r$ positions at distance $r \leq K$ from node j. Out of these $4r$ positions, 4 are corner positions and the rest $4r - 4$ are non-corner positions. From a corner point, the walk can go to a position at a distance $r + 1$ from node j with probability $\frac{3}{4}$ and to a position at a distance $r - 1$ with probability $\frac{1}{4}$. From a non-corner point, these probabilities are $\frac{1}{2}$ and $\frac{1}{2}$ respectively.

Let $P(r)$ denote the probability that a walk, currently at a distance $r \leq K$ away from node j, will hit $A(K)$ before node j. Then,

$$P(r) = \frac{4}{4r}\left(\frac{3}{4}P(r+1) + \frac{1}{4}P(r-1)\right) + \frac{4r-4}{4r}\left(\frac{1}{2}P(r+1) + \frac{1}{2}P(r-1)\right)$$

$$\Rightarrow P(r) = \frac{2r+1}{4r}P(r+1) + \frac{2r-1}{4r}P(r-1). \qquad (3)$$

Using the boundary conditions $P(0) = 0$ and $P(K) = 1$, the recursive equation (3) can be solved for $P(r)$. $P(T_A^+ < T_j) = P(K-1)$ as after the first step, the walk is $K-1$ distance from node j. Solving (3) gives the desired expression. □

In Lemma 2.4, we first derive the expected time it takes for a random walk to come back to a subset of nodes and then use this value to find EM^+.

Lemma 2.4:

$$EM^+ = \frac{N}{2}\left(\frac{2^{K+1}-K-2}{2^K-1}\right)\left(\frac{1}{2K+1}\right.$$
$$\left.\left(4K-(2K+1)-\left(\frac{1+l_{K-1}}{1+l_K}(2K-1)\right)\right)\right),$$

where $l_K = 1 + \sum_{t=1}^{K-1}\prod_{r=1}^{t}\frac{q_r}{p_r}$ and $p_r = \frac{2r+1}{4r}$, $q_r = \frac{2r-1}{4r}$.

Proof: Let a walk start from $A(K)$. The expected time till it hits the node j, $E_{\pi(A)}T_j$, can be written as

$$E_{\pi(A)}T_j = \frac{4}{4K}\left(\frac{3}{4}\left(E_A T_A^+ + E_{\pi(A)}T_j\right) + \right.$$
$$\left.\frac{1}{4}\left(P(T_A^+ < T_j)E_{\pi(A)}T_j + P(T_A^+ > T_j)g_1(K)\right)\right) +$$
$$\frac{4K-4}{4K}\left(\frac{1}{2}\left(E_A T_A^+ + E_{\pi(A)}T_j\right) + \right.$$
$$\left.\frac{1}{2}\left(P(T_A^+ < T_j)E_{\pi(A)}T_j + P(T_A^+ > T_j)g_1(K)\right)\right), \quad (4)$$

where $g_1(K) = O(K^2)$ and $E_{\pi(A)}T_j = N\left(\frac{2^{K+1}-K-2}{2^K-1}\right)$ [12]. When $N >> K$, Equation (4) can be written as

$$E_{\pi(A)}T_j = \frac{2K+1}{4K}\left(E_A T_A^+ + E_{\pi(A)}T_j\right)$$
$$+\frac{2K-1}{4K}P(T_A^+ < T_j)E_{\pi(A)}T_j.$$

Rearranging the terms gives

$$E_A T_A^+ = N\left(\frac{2^{K+1}-K-2}{2^K-1}\right)\left(\frac{1}{2K+1}\right.$$
$$\left.\left(4K-(2K+1)-P(T_A^+ < T_j)(2K-1)\right)\right).$$

EM^+, the expected intermeeting time between two walks, is equal to $\frac{1}{2}E_A T_A^+$ as the walks are moving on a symmetric graph. □

C. Epidemic Routing

This section summarizes the basic epidemic routing principles [10]. Each node stores and forwards packets destined for other nodes. Each node maintains a summary vector that indicates the set of packets which it has. When two nodes come within transmission range of each other, they first exchange their summary vectors. Next, based on this information, they exchange packets which they don't have. Thus, the source node copies the packet to every node it meets, and each node in turn forwards the copy to every other node it meets. The packet is delivered when the first node carrying a copy of the packet meets the destination. But, the packet will continue to get copied from one node to the other until all the nodes have a copy of the packet or its TTL expires. We assume that the TTL of the packet is large enough to ensure that all packets are delivered to the destination, that is, there is no packet loss due to TTL expiry.

III. Contention Analysis

First we identify the three mainfestations of contention;

Finite Bandwidth: When two nodes meet, they might have more than one packet to exchange. We assume that two nodes can exchange only one packet in one time unit. They will have to wait until they meet again to transfer more packets.
The number of packets which can be exchanged in a time unit is a function of the packet size and the bandwidth of the links. The following analysis is easily modified if more than one packet can be exchanged in a time unit.

Scheduling: We assume a scheduling mechanism is in place which ensures that no two transmitters are within a distance of $2K$ to each other. This scheduling mechanism is very similar to the CSMA-CA algorithm.

Interference: Even though the scheduling mechanism is ensuring that no simultaneous transmissions are taking place within a distance $2K$ of each other, there is no restriction on simultaneous transmissions taking place seperated by a distance more than $2K$. These transmissions will interfere with each other which can lead to packet loss.

Lets look at a particular packet, label it *Packet A*. Two nodes, i and j at a distance $k \le K$ from each other, want to exchange Packet A. Let $p_{txS}(k)$ define the probability that they will successfully exchange Packet A.

Let there be s other packets (other than Packet A) which i and j want to exchange, $0 \le s \le S-1$ (a packet will be exchanged if and only if one of the nodes has a copy of the packet and the other one doesn't). We label this event as T_s. The packet exchanged is randomly selected from amongst these $s+1$ packets.

Let there be a nodes within a distance $2K$ of the transmitting node (label it event E_a) and let there be c nodes in the $2K < d \le 3K$ ring (label it event E_c) from the transmitter. The nodes in the $2K < d \le 3K$ ring have to be accounted for because a node at the edge of the $2K$ circle can be within the transmission range of these nodes and will contend with the desired transmitter. Let $t(a,c)$ denote the number of possible transmissions whose transmitter lies within $2K$ distance of the desired transmitter. The scheduling mechanism will allow transmission between i and j w.p. $1/t(a,c)$.

There are $M-a$ nodes outside the $2K$ range from the transmitter. If two of these nodes are within the transmission range of each other, then they can exchange packets causing interference with the transmission between i and j. Lets label the event that the packet between i and j is successfully exchanged inspite of the interference caused by these $M-a$ nodes as E_{M-a}.

Then,

$$p_{txS}(k) = \left(\sum_{s=0}^{S-1}\frac{1}{s+1}P(T_s)\right)\times$$
$$\left(\sum_{a,c}\frac{1}{t(a,c)}P(E_a)P(E_c \mid E_a)P(E_{M-a})\right). \quad (5)$$

Equation (5) seperates out the effect of each of the factors on $p_{txS}(k)$. Next, we find expressions for the unknown values in Equation (5).

A. Finite Bandwidth

In this section, we figure out the probability that nodes i and j have s other packets to exchange.

Lemma 3.1: $P(T_s) = \begin{pmatrix} S-1 \\ s \end{pmatrix} (p_{exB})^s (1 - p_{exB})^{S-s-1}$, where $p_{exB} = \sum_{m=1}^{M-1} \frac{2m(M-m)}{M(M-1)} \frac{E(d(m))}{\sum_{l=1}^{M-1} E(d(l))}$ and $d(m)$ is the time it takes for the copies of a packet in the network to increase from m to $m+1$, and $E(d(m))$ is its expected value.

Proof: p_{exB} = P(nodes i and j want to exchange a particular packet B). Nodes i and j will try to exchange packet B only if either i has it and j doesn't or vice versa. Since there are $S-1$ packets other than packet A in the network, $P(T_s) = \begin{pmatrix} S-1 \\ s \end{pmatrix} (p_{exB})^s (1 - p_{exB})^{S-s-1}$.

Now we derive the value for p_{exB}. The probability that there are m copies of a packet in the network is equal to the (expected amount of time there are m copies of the packet in the network)/(expected total time the packet spends in the network before being removed). The packet is removed from the network when no futher transmission of the packet will take place. This occurs either when all nodes have a copy of the packet or its TTL expires. Since we had assumed the TTL is large enough to ensure that all packets are delivered to their destination, the packet will be removed when all nodes have a copy of the packet. Hence, P(m copies of the packet in the network) = $\frac{E(d(m))}{\sum_{l=1}^{M-1} E(d(l))}$.

Now, given that there are m copies of packet B in the network, the probability that node i has the packet and j doesn't is equal to $\frac{m(M-m)}{M(M-1)}$. Consequently,

$$p_{exB} = \sum_{m=1}^{M-1} \frac{2m(M-m)}{M(M-1)} \frac{E(d(m))}{\sum_{l=1}^{M-1} E(d(l))}. \square$$

In the subsequent analysis, we will assume that when two nodes meet, the probability that they have no packet to exchange ($= (1-p_{exB})^S$) is negligible. This is a valid assumption if the traffic load is not very low. Since contention will occur only when the traffic load is not very low, this is a valid assumption to make while studying contention.

B. Scheduling

Each of the other $M-2$ nodes (other than i and j) are equally likely to be at any of the N grid points because the random walk mobility model has a uniform stationary distribution. So, we use geometric arguments to figure out how many transmissions does the transmission between i and j contend with.

Lemma 3.2: $P(E_a) = \begin{pmatrix} M-2 \\ a-2 \end{pmatrix} (p_1)^{a-2}(1-p_1)^{M-a}$ where $p_1 = \frac{1+4K(2K+1)}{N}$.

Proof: The node is equally likely to be at any of the N grid points. Consequently, p_1 = P(there is one node within a distance $2K$ of the transmitting node) = (number of grid points within distance $2K$)/(total number of grid points) = $\frac{1+\sum_{r=1}^{2K} 4r}{N} = \frac{1+4K(2K+1)}{N}$. Recall that two nodes, i and j, are already within K distance of the transmitter. So, $P(E_a) = \begin{pmatrix} M-2 \\ a-2 \end{pmatrix} (p_1)^{a-2}(1-p_1)^{M-a}$. $\square$

Corollary 3.1: $P(E_c \mid E_a) = \begin{pmatrix} M-a \\ c \end{pmatrix} (p_2)^c (1-p_2)^{M-a-c}$ where $p_2 = \frac{2(5K^2+K)}{N}$.

Proof: p_2 = P(there is one node within the $2K < d \leq 3K$ ring from the transmitter) = (number of grid points in the $2K < d \leq 3K$ ring)/(total number of grid points) $= \frac{2(5K^2+K)}{N}$. The probability that c nodes out of the $M-a$ nodes are in the $2K < d \leq 3K$ ring is equal to $\begin{pmatrix} M-a \\ c \end{pmatrix} (p_2)^c(1-p_2)^{M-a-c}$. $\square$

Lemma 3.3: $t(a,c) = \left(1 + p_a \left(\begin{pmatrix} a \\ 2 \end{pmatrix} - 1\right)\right) + \left(\frac{acp_c}{2}\right)$ where $p_a = \frac{1}{16} + \frac{A}{4\pi K^2}$, $p_c = \frac{3}{20} - \frac{A}{5\pi K^2}$ and $A = \int_K^{2K} \frac{x}{2K^2} [K^2 cos^{-1}\left(\frac{x^2-3K^2}{2Kx}\right) + 4K^2 cos^{-1}\left(\frac{x^2+3K^2}{4Kx}\right) - \frac{1}{2}\sqrt{(x^2-K^2)(9K^2-x^2)}]dx$.

Proof: It is given that there are a nodes within $2K$ distance of the transmitting node. Hence, there are $\begin{pmatrix} a \\ 2 \end{pmatrix}$ pairs of nodes. Lets choose one such pair and define p_a = P(this pair of nodes are within a distance K of each other).

Out of these a nodes, i and j are already within K distance of each other. The rest of these nodes are within K distance of each other with probability p_a. Hence, the expected number of possible transmissions amongst these a nodes is $1 + p_a \left(\begin{pmatrix} a \\ 2 \end{pmatrix} - 1\right)$.

Next we figure out the value of p_a. Lets choose a pair of nodes amongst these a nodes and label the nodes u_1 and u_2. The probability that a node u_1 is at a distance x away from the transmitter is $\frac{2\pi x dx}{4\pi K^2}$. Conditioned over the fact that node u_1 is at a distance x from the transmitter, the probability that node u_2 is within K distance from u_1 is equal to the common area Y between the two circles in Figure 1 divided by total area where node u_2 can lie ($= 4\pi K^2$) . The common area to both the circles is

$$\begin{array}{ll} K^2 cos^{-1}\left(\frac{x^2-3K^2}{2Kx}\right) + 4K^2 cos^{-1}\left(\frac{x^2+3K^2}{4Kx}\right) \\ -\frac{1}{2}\sqrt{(x^2-K^2)(9K^2-x^2)} & , \quad \text{if } x > K, \\ \text{and } \pi K^2, & \text{if } x \leq K \ . \end{array}$$

Hence,

$$p_a = \int_0^K \frac{2\pi x \times \pi K^2}{4\pi K^2 \times 4\pi K^2} dx + \frac{1}{4\pi K^2} \int_K^{2K} \frac{x}{2K^2}$$

$$\left[K^2 cos^{-1}\left(\frac{x^2-3K^2}{2Kx}\right) + 4K^2 cos^{-1}\left(\frac{x^2+3K^2}{4Kx}\right)\right.$$

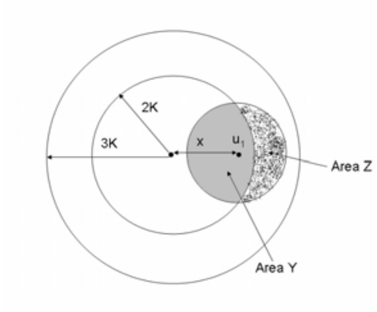


Fig. 1. Node u_1 is at a distance x from the transmitter. If another nodes lies in the area Y, transmissions between them contend with the desired transmission. If another node lies in the area Z, the transmission emanating from u_1 to this node contends with the desired transmission

$$-\frac{1}{2}\sqrt{(x^2-K^2)(9K^2-x^2)}\Bigg]\,dx = \frac{1}{16}+\frac{A}{4\pi K^2}.$$

Now, we quantify the contention due to the c nodes in the $2K < d \leq 3K$ ring. Contention arises when one of the a nodes can transmit to one of the c nodes. There are ac such pairs. Lets choose one pair and label the corresponding nodes u_1 and u_3, where u_1 lies within $2K$ distance of the transmitter while u_3 lies in the $2K < d \leq 3K$ ring. Define p_c = P(u_1 and u_3 are within a distance K of each other). Though both the nodes can transmit to each other, contention with the desired transmitter will arise only when u_1 transmits to u_3. u_3 can transmit to u_1 wihtout contending with the transmitter as u_3 is not within $2K$ distance of the transmitter. Hence, the expected number of transmissions contending are $\frac{acp_c}{2}$.

To find p_c, notice that, conditioned over the fact that node u_1 is at a distance x from the transmitter, the probability that node u_3 is within K distance from u_1, is equal to the Area Z in Figure 1 divided by the total area where node u_3 can lie ($= 5\pi K^2$). In a manner similar to that used for the deriviation of p_a, p_c is derived to be $\frac{3}{20} - \frac{A}{5\pi K^2}$. □

C. Interference

The interference caused by other nodes depend on the number of simultaneous transmissions and the distance between these transmitters and the desired receiver. Both these quantities are not deterministic, and hence we use the Law of Total Probability to find $P(E_{M-a})$.

Given that there are $M-a$ nodes outside the scheduling area (event E_a), let there be x interfering transmissions at a distance of $d_1, d_2, \ldots, d_x$ from the desired transmitter. Then,

$$P(E_{M-a}) = \sum_{x}\sum_{d_1,d_2,\ldots,d_x} P(E_{M-a} \mid x, d_1, d_2, \ldots d_x)\; P(d_1, d_2, \ldots, d_x \mid x)P(x). \quad (6)$$

It is possible to calculate both $P(x)$ and $P(d_1, d_2, \ldots, d_x \mid x)$ to substitute in Equation (6), but, the resulting expressions will be very complicated. So, we will replace x and the d_i's with their expected values in the expression for $P(E_{M-a})$.

Since each node is moving independently of each other, $E[d_1] = E[d_2] = \ldots = d_{avg}$. We label $E[x]$ as $\overline{x}$.

First, we figure out d_{avg}. Let $P(d)$ denote the probability that the distance between any two nodes on the grid equals d. Then,

$$P(d) = \begin{cases} 1/N & d = 0 \\ 4d/N & 1 \leq d < n/2 \\ 2(n-1)/N & d = n/2 \\ 4(n-d)/N & n/2 < d < n \\ 1/N & d = n \end{cases}.$$

Hence, $d_{avg} = E$[distance between two simultaneous transmissions] $= \sum_{d=2K+1}^{n} d \times P(d)$ which can be computed using algebra.

Next, we figure out $\overline{x} = E[x]$.

Lets define p_m to be the probability that two nodes meet. Then, p_m is just the inverse of the intermeeting time, that is, $p_m = \frac{1}{EM^+}$. There are $\binom{M-a}{2}$ possible pairs of nodes. Hence, the expected number of interfering transmissions $= \overline{x} = p_m \binom{M-a}{2}$.

Lemma 3.4: $P(E_{M-a}) \approx \left(\frac{1}{1+\frac{\Theta k^2}{d_{avg}^2}}\right)^{\overline{x}}$, where k is the distance between nodes i and j.

Proof: $P(E_{M-a} \mid x, d_1, d_2, \ldots d_x)$ is the complement of the outage probability and depends on the channel model. Kandukuri et al [23] evaluated the outage probability to be $1 - \prod_{r=1}^{x} \frac{1}{1+\frac{\Theta P_r^R}{P_0^R}}$ for the Rayleigh-Rayleigh fading channel, where P_0^R is the received power from the desired signal and P_r^R is the received power from the r^{th} interferer. Assuming all the nodes are transmitting at the same power level and $\alpha = 2$ in the distance attenuation model, $P(E_{M-a} \mid x, d_1, d_2, \ldots d_x) = \prod_{r=1}^{x} \frac{1}{1+\frac{\Theta d_0^2}{d_r^2}}$, where $d_0 = k$ (the distance between nodes i and j). Replacing x and the d_i's with their expected values gives $P(E_{M-a}) \approx \left(\frac{1}{1+\frac{\Theta k^2}{d_{avg}^2}}\right)^{\overline{x}}$. □

Note that the preceeding analysis neglects the inteference due to transmissions between a node outside the $2K$ circular region and a node inside it. Since the number of such transmissions are very few in number as compared to the other transmissions, even though they are closer than other transmissions, their effect on interference is negligible.

Now, we have all the components to put together to find $p_{txS}(k)$ in Equation (5).

IV. Delay analysis of epidemic routing for sparse networks

A number of papers [11], [12], [13] have analyzed epidemic routing but none of them take contention into account. These papers argue that since the network is pretty sparse, there won't be a lot of contention in the first place and hence, nothing much is lost by analyzing the protocol without taking contention into account. [14], [16], [17] have shown through simulations that this assumption is not valid. In Section IV-A, we reconfirm through simulations that contention has a significant impact on the system performance, which implies that an analysis that does not take contention into account is inaccurate.

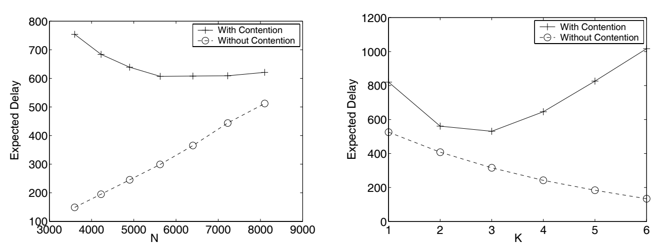
A. Motivation

This section shows through simulations that inspite of the network being sparse, since epidemic routing is a flooding based protocol, contention will have a big impact on the system performance. System performance is measured using end to end delay which is defined as the time needed to transfer a packet between a source and a destination. The TTL is assumed to be large enough to ensure that all packets get delivered to their respective destinations. We assume that the average number of distinct packets in the network at a given snapshot in time is equal to the number of nodes, that is $S = M$. We choose this value because this is the highest value S can have without making the system unstable: as soon as a distinct packet is serviced, that is, everybody has a copy of it, a new distinct packet is introduced. Clearly, choosing the highest possible value of S will create the worst-case contention.

A realistic simulator written in C++ is used to evaluate the performance of epidemic routing. The simulator allows the user to choose from different physical layers and mobility models, simulates MAC layer collision avoidance and allows the user to build any routing scheme on top of it. We simulate epidemic routing with Rayleigh fading channel and random walk mobility model. The scheduling scheme disallows any other transmission upto two hops from the transmitter.

Figure 2(a) plots the delay as a function of N for $M = 100$, $K = 2$ and $\Theta = 4$ while Figure 2(b) plots the delay as a function of K for $N = 4900$, $M = 100$ and $\Theta = 4$.

The plots show that ignoring contention in the analysis is not only grossly underestimating the delay but also predicting incorrect trends. For example, the delay as a function of K achieves a minimum with contention but it is strictly decreasing without contention. Even though the networks are very sparse, contention has a significant impact on the system performance.



(a) (b)

Fig. 2. Comparison of delay with and without taking contention into account (a) Delay vs N for $M = 100$, $K = 2$ and $\Theta = 4$ (b) Delay vs K for $M = 100$, $N = 4900$, $\Theta = 4$.

B. Delay Analysis

We first derive an expression for $E(d(m))$ which is the expected time it takes for the copies of a packet in the network to increase from m to $m+1$. (We define the m^{th} time epoch as the duration during which there are m copies of the packet in the network.)

Two nodes can start talking as soon as they come within K distance of each other. If the first exchange of the given packet is unsuccessful, then in the next time step, either the nodes might move away from each other, in which case they will wait for another intermeeting time, or else they might remain in contact with each other, in which case they have another chance to exchange the packet. In the second case, they will keep on trying until either they successfully exchange the packet or they move out of range of each other. Let $p_{success}$ denote the probability that the nodes successfully exchange the given packet before going out of range of each other.

When the packet is generated at the source, the source node can be uniformly distributed anywhere on the 2-D torus. So, the expected time to meet another node is equal to EM, the expected meeting time. Now, if the source meets another node and is unable to successfully exchange the given packet, then the expected time till it meets this node again is equal to EM^+, the intermeeting time. Consequently at the start of each time epoch, there will be certain pair of nodes whose expected time to meet is EM and for the other pairs, it will be EM^+. This number will have to be tracked as time evolves to find $E(d(m))$.

Lemma 4.1:

$$E(d(m)) = \sum_{k=1}^{q2_m} \left(\frac{1}{\frac{(1-p_{success})(q2_m-k)}{EM} + p_{success}\left(\frac{q1_m+k}{EM^+} + \frac{q2_m-k}{EM}\right)} \left(\prod_{l=0}^{k-1} \frac{\frac{(1-p_{success})(q2_m-l)}{EM}}{\frac{(1-p_{success})(q2_m-l)}{EM} + p_{success}\left(\frac{q1_m+l}{EM^+} + \frac{q2_m-l}{EM}\right)} \right) \right) \quad (7)$$

where $q1_m = (m-1)(M-m)$ and $q2_m = (M-m)$.

Proof: We first derive $E(d(m))$ as a function of $p_{success}$ and then compute $p_{success}$.

When there are m copies of a packet in the network, if any of the nodes that carry one of these copies meet any of the other $M - m$ nodes, a packet exchange is possible. The packet exchange will take place successfully with probability $p_{success}$. The m^{th} time epoch starts when a transmission occurs that increases the number of copies of the packet to m. The new node can now exchange the packet with the rest of the $M - m$ nodes. When this new node receives the packet, it can be anywhere on the 2-D torus with the same probability. Hence, the meeting time for these new $M - m$ pairs is exponentially distributed with mean EM. The rest of the $(m-1)(M-m)$ pairs are carried forward from the previous time epochs. Depending on whether these pairs have met or not in the previous time epochs, some of these will have exponential meeting times with mean EM (implying they have not met before in any of the previous time epochs) and the rest will have exponential meeting times with mean EM^+ (implying they have met at least once in some previous time epoch). We assume that all these $(m-1)(M-m)$ pairs have met at least once in some previous time epoch, that is, their expected meeting times are equal to EM^+. This is a valid assumption if there is significant contention in the network and the value of $p_{success}$ is small. Since epidemic routing is a flooding based protocol, there is enough contention in the network to make the assumption accurate.

Let $q1_m$ and $q2_m$ denote the number of pairs, at the start of the m^{th} time epoch, whose meeting times are exponential with means EM^+ and EM respectively. Then, $q1_m = (m-1)(M-m)$ and $q2_m = (M-m)$.

For the m^{th} time epoch, the set of states (number of pairs with meeting time exponential with mean EM^+, number of pairs with meeting time exponential with mean EM) along with a state TS (transmission successful) (jumping to this state denotes the end of the time epoch) form a continuous time Markov chain. The chain will have transient states $(q1_m + k, q2_m - k) : 0 \leq k \leq q2_m$ and an absorbing state TS. The CTMC jumps from state $(q1_m + k, q2_m - k)$ to $(q1_m + k + 1, q2_m - k - 1)$ when any of the $q2_m - k$ pairs meet but they are unable to exchange the packet and the corresponding rate is $\frac{(1-p_{success})(q2_m-k)}{EM}$. The CTMC jumps from state $(q1_m + k, q2_m - k)$ to state TS when any of the $m(M-m)$ pairs meet and they successfully exchange the given packet and the corresponding rate is $p_{success}\left(\frac{q1_m+k}{EM^+} + \frac{q2_m-k}{EM}\right)$.

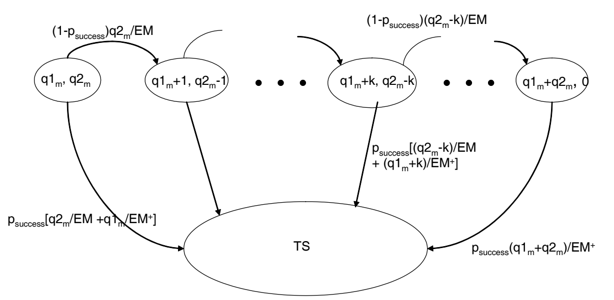


Fig. 3. The Markov chain describing the evolution of (number of pairs with expected meeting times $= EM$, number of pairs with expected meeting times $= EM^+$) when there are m copies of the packet in the network.

The CTMC is shown in Figure 3. $E(d(m))$ is equal to the expected time it takes the Markov chain to end up in the TS state. Solving the Markov chain using standard Markov chain theory [24] gives the desired result.

To complete the derivation, we now derive $p_{success}$. Two nodes meet if they come within a distance K of each other. If the packet exchange is unsuccessful, then the two nodes will both move before trying to transmit again. This can be modeled as if one node does not move and the other node moves two steps before another transmission is attempted. These attempted transmissions will have to be tracked to find $p_{success}$.

Let p_k denote the probability that the two nodes move out of range of each other without being able to exchange the packet given that the current distance between them is $k \leq K$. Then,

$$p_k = \begin{cases} (1 - p_{txS}(k))\left(\frac{2k+1}{4k}p_{k+1} + \frac{2k-1}{4k}p_{k-1}\right) & K-k \text{ is even} \\ \left(\frac{2k+1}{4k}p_{k+1} + \frac{2k-1}{4k}p_{k-1}\right) & K-k \text{ is odd,} \end{cases} \tag{8}$$

where $p_{txS}(k)$ is the probability that they successfully exchange the given packet if the distance between them is k (its value was derived in Section III (see Equation (5))).

The boundary conditions are,

$$p_0 = \begin{cases} (1 - p_{txS}(0))p_1 & K \text{ is even} \\ p_1 & K \text{ is odd,} \end{cases}$$

and

$$p_{K+1} = 1.$$

After the first unsuccessful exchange, the two nodes move within a distance $K-1$ with probability $\frac{2K-1}{4K}$ or move to a distance $K+1$ with probability $\frac{2K+1}{4K}$. Hence,

$$p_{success} = 1 - \left((1 - p_{txS}(K))\left(\frac{2K+1}{4K} + \left(p_{K-1}\frac{2K-1}{4K}\right)\right)\right). \tag{9}$$

p_{K-1} can be evaluated by solving the linear set of Equations (8) and then $p_{success}$ can be evaluated using Equation (9). □

The value of $E(d(m))$ depends on $p_{success}$ which in turn depends on $p_{txS}(k)$ whose value depends on $E(d(m))$ (through Lemma (3.1)). Since the functions defining these dependencies are not straightforward (for example $p_{success}$ depends on $p(k)$ through the linear set of Equations (8)), we recursively solve for $E(d(m))$. The recursive algorithm is as follows:

1. First assume $p(k)^0 = 1 \ \forall k$, where the superscript represents the iteration number in the recursion.
2. Lemma 4.2 below determines the value of $E(d(m))^0$ for $p(k) = 1$.
3. Using $E(d(m))^{l-1}$, determine $p(k)^l$.
4. Determine $E(d(m))^l$ using $p(k)^l$.
5. If the change in the value of $\sum_{m=1}^{M-1} E(d(m))^l$ is less than 5% in the l^{th} iteration, stop else goto Step 3.

In all the cases that we studied, the recursive loop always converged after 4 iterations.

As mentioned above. the recursive algorithm requires the value of $E(d(m))$ for $p(k) = 1$ at the second step. The following lemma states its value.

Lemma 4.2: For $p(k) = 1 \ \forall k$,

$$E(d(m)) = \frac{EM}{m(M-m)}$$

Proof: The proof follows along the same line as in [12]. Let the state of a Markov Chain be defined as the number of copies the packet has. The rate of going from state m to state $m+1$ is $\frac{m(M-m)}{EM}$. (The markov chain is shown in Figure 4.) The

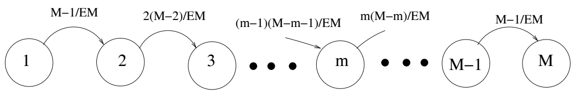


Fig. 4. The Markov Chain describing the evolution of the number of copies of a packet when $p(k) = 1$.

expected amount of time the MC spends in state m is equal to $E(d(m)) = \frac{EM}{m(M-m)}$. □

Now we derive an analytical expression for the delivery delay of epidemic routing.

Theorem 4.1: Let ED denote the expected delay until a packet is delivered to its destination. Then,

$$ED = \sum_{i=1}^{M-1} \frac{1}{M-1} \sum_{m=1}^{i} E(d(m)). \tag{10}$$

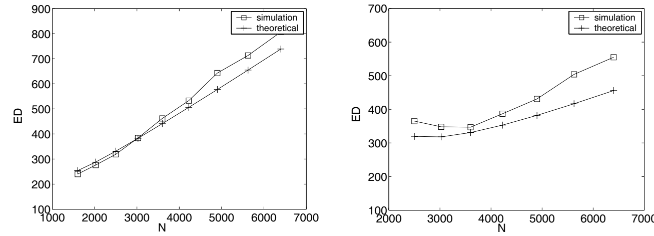
Proof: At the start of the m^{th} time epoch, m of the M nodes have the packet. When one of the remaining $M - m$ nodes gets the packet, the copies of the packet increase to $m+1$ and the m^{th} time epoch ends. Since the destination was chosen randomly, it is equally likely to obtain a copy of the packet at the end of any of the $1 \leq m \leq M-1$ time epochs. Consequently, $ED = \sum_{i=1}^{M-1} \frac{1}{M-1} \sum_{m=1}^{i} E(d(m))$. □

C. How accurate are the approximations

Some approximations were made during the course of the analysis to keep it tractable. Since all the approximations were easily justifiable, we do not expect that they would drastically effect the accuracy of the analysis. To validate our claim, we use simulations to evaluate their effect on the accuracy of the analysis. In this section, we plot the theoretical results against the simulation results for a number of representative cases.

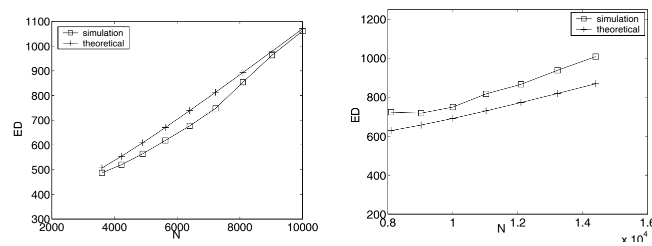
Figures 5-7 plot ED against N for different values of M, K and Θ. Since both the curves in all the figures are close to each other, we can conclude that the analysis is fairly accurate.

Comments on which approximations are causing more error than others are in order. Except for two approximations, all the other have a negligible impact. Both the approximations were made in Section III-C while analyzing interference. The assumption to neglect the interference from transmissions between nodes within a distance $2K$ from the transmitter and the nodes in the $2K < d \leq 3K$ ring becomes worse as K and Θ increase. This approximation results in an overestimate of $p(K)$ and hence gives a lower value of ED than the actual value. This explains the curves in Figures 5(b), 6(b) and 7(b). The second approximation is replacing x with its expected value in Equation (6). Since the outage probability is a convex function of x for the Rayleigh fading channel, Jensen's inequality implies that $p(k)$ will be an underestimate of the actual value resulting in a higher value of ED. The effect of this approximation becomes more visible for lower values of K. This explains the curves in Figures 5(a), 6(a) and 7(a).



(a) (b)

Fig. 5. Analysis vs Simulation: Delay vs N for $M = 50$(a) $K = 2, \Theta = 4$ (b) $K = 4, \Theta = 6$.

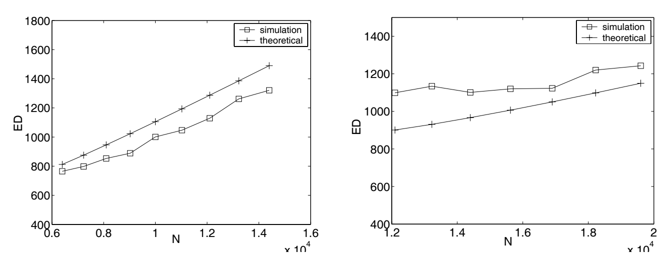


(a) (b)

Fig. 6. Analysis vs Simulation: Delay vs N for $M = 100$(a) $K = 2, \Theta = 4$ (b) $K = 4, \Theta = 6$.

V. Conclusions

The paper provides analytical expressions for the delay performance of epidemic routing under contention. All three mainfestations of contention, finite bandwidth, scheduling and interference are incorporated in the analysis. Simulations are used to verify that the approximations made to simplify the analysis do not have a significant effect on the accuracy of the analysis.



(a) (b)

Fig. 7. Analysis vs Simulation: Delay vs N for $M = 150$(a) $K = 2, \Theta = 4$ (b) $K = 4, \Theta = 6$.

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