Fundamental mobility properties for realistic performance analysis of intermittently connected mobile networks

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***Abstract*— Traditional mobile ad hoc routing protocols fail to deliver any data in Intermittently Connected Mobile Ad Hoc Networks (ICMN's) because of the absence of complete end-to-end paths in these networks. To overcome this issue, researchers have proposed to use node mobility to carry data around the network. These schemes are referred to as mobility-assisted routing schemes.**

A mobility-assisted routing scheme forwards data only when appropriate relays meet each other. The time it takes for them to first meet each other is referred to as the meeting time. The time duration they remain in contact with each other is called the contact time. If they fail to exchange the packet during the contact time (due to contention in the network), then they have to wait till they meet each other again. This time duration is referred to as the inter meeting time. A realistic performance analysis of any mobility-assisted routing scheme requires a knowledge of the statistics of these three quantities. These quantities vary largely depending on the mobility model at hand. This paper studies these three quantities for the three most popularly used mobility models: random direction, random waypoint and random walk models. Hence, this work allows for a realistic performance analysis of any routing scheme under any of these three mobility models.

I. Introduction

Intermittently connected mobile networks (ICMN's) are networks where most of the time, there does not exist a complete end-to-end path from the source to the destination. Even if such a path exists, it may be highly unstable because of the topology changes due to mobility and may change or break soon after it has been discovered. This situation arises when the network is quite sparse. Examples of such networks include sensor networks for wildlife tracking and habitat monitoring [1], [2], military networks [3], deep-space inter-planetary networks [4], nomadic communities networks [5], networks of mobile robots [6] vehicular ad hoc networks [7] etc.

Traditional mobile ad hoc routing protocols will fail for these networks because they require the existence of complete end-to-end paths to be able to deliver any data. To overcome this issue, researchers have proposed to exploit node mobility to carry messages around the network as part of the routing algorithm [8]–[14]. These routing schemes are referred to as *mobility-assisted routing schemes*.

Since message transmission occurs only when nodes meet each other, the time elapsed between such meetings is the basic delay component. The time it takes for two nodes to first meet starting from their stationary location distribution is called the *meeting time*. A node, which say just received a message, first encounters a given other node that can act as a relay after one meeting time. Once two nodes meet, the duration these two nodes remain in contact with one another will determine the time duration they have to exchange packets. This time duration is referred to as the *contact time*. Contention in the network can cause the transmission between these two nodes to fail. Then the nodes will have to wait till they meet again to get another transmission opportunity. The time till the two nodes, which start from within range of each other and then move out of each other's range, meet again is called the *inter meeting time*. These three quantities constitute the basic components in the realistic performance analysis (any analysis which does not consider finite bandwidth and contention in the network is unrealistic) of any routing scheme, and they vary depending on the specific mobility model in hand. This paper studies these three fundamental quantities for the three most popular mobility models: the random direction, random waypoint and random walk mobility models.

Although, there has been a lot of effort to theoretically characterize the performance of mobility assisted routing schemes for intermittently connected mobile networks [11], [15]–[21], the statistics of these three properties have remained largely unstudied for most mobility models. [11] finds the expected meeting time for the random walk mobility model, and [20] finds the expected meeting time for the random waypoint and random direction mobility models. In addition to studying its expected value, researchers have also studied the tail of the distribution of the meeting times. In particular, [22] proves that the tail of the distribution of the meeting times under random walk is exponential and [15] observes this via simulations. Finally, [21] derives the expected inter meeting time for the random walk mobility model.

In this paper, we compute the expected inter meeting times of

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the random direction and random waypoint mobility models. We also formally prove that the tail of the distribution of the meeting and inter meeting times under random direction and random waypoint mobility is memoryless. We show through simulations that the distribution of the inter meeting time of random walk mobility is Zipfian. Finally, we find the expected contact time for all the three mobility models. Hence, we determine all the necessary quantities to do a realistic performance analysis of mobility-assisted routing under the three most popular mobility models.

The outline of the paper is as follows: Section II presents our notation, assumptions and then formally defines the meeting time, the inter meeting time and the contact time. Sections III, IV, V finds these statistics for the random direction, random waypoint and random walk mobility models respectively. Finally, Section VI concludes the paper.

II. Notation and Definitions

We first introduce our notation and state the assumptions we will be making throughout the remainder of the paper.

(a) All nodes exist in a two dimensional torus U of area N and have a transmission range equal to K. The position of node i at time t is denoted as $X_i(t)$.
(b) Time is slotted. Two nodes exchange packets only if they are within each other's transmission range at the start of the time slot.

Now we formally define the meeting time, the inter meeting time and the contact time of a mobility model.

Definition 2.1 (Meeting Time): Let nodes i and j move according to a mobility process 'MM' and start from their stationary distribution at time 0. The meeting time (M_{mm}) between the two nodes is defined as the time it takes them to first come within range of each other, that is $M_{mm} = min_t\{t : \|X_i(t)-X_j(t)\| \leq K\}$.

Definition 2.2 (Inter Meeting Time): Let nodes i and j move according to a mobility process 'MM'. Let the nodes start from within range of each other at time 0 and then move out of the range of each other at time t_1, that is $t_1 = min_t\{t : \|X_i(t) - X_j(t)\| > K\}$. The inter meeting time (M^+_{mm}) of the two nodes is defined as the time it takes them to first come within range of each other again, that is $M^+_{mm} = min_t\{t-t_1 : \|X_i(t)-X_j(t)\| \leq K\}$.

Definition 2.3 (Contact Time): Let nodes i and j move according to a mobility process 'MM' and assume they come within range of each other at time 0. The contact time τ_{mm} is defined as the time they remain in contact with each other before moving out of the range of each other, that is $\tau_{mm} = min_t\{t - 1 : \|X_i(t) - X_j(t)\| > K\}$. (Note that t_1 defined in Definition 2.2 is the same $\tau_{mm} + 1$.)

III. Random Direction

Definition 3.1 (Random Direction): In the Random Direction model, each node moves as follows [23]: (i) Choose a direction θ uniformly in $[0, 2\pi)$. (ii) Choose a speed v uniformly in $[v_{min}, v_{max}]$ with $v_{min} > 0$ and $v_{max} < \infty$. Let $\overline{v}$ denote the average speed of a node. (iii) Choose a duration T of movement from a geometric distribution with mean $\overline{T}$. The average distance traveled in a duration $\overline{L}$ is equal to $\overline{T}\overline{v}$. We assume that $\overline{L} = O\left(\sqrt{N}\right)$ to ensure fast mixing [1]. (iv) Move towards θ with speed v for T time slots. (v) After T time slots, pause for T_{stop} time slots where T_{stop} is chosen from a geometric distribution with mean $\overline{T}_{stop}$. (vi) Goto Step (i).

The expected meeting time of the Random Direction model was evaluated in [20]. Here, we will derive the expected inter meeting time in Theorem 3.1, the expected contact time in Theorem 3.2 and finally the distribution of the meeting and the inter meeting times in Theorem 3.3.

Theorem 3.1: The expected inter meeting time $E[M^+_{rd}]$ for the Random Direction model is approximately equal to $E[M_{rd}]$.

Proof: When the nodes move out of the range of each other, they keep moving for a duration which is geometrically distributed. Since we assumed that $\overline{L} = O(\sqrt{N})$, the nodes mix (reach their stationary distribution) after their respective movement duration ends. After the two nodes get mixed, the additional time it will take for them to meet again is equal to the meeting time. In general, since one movement duration is much less than the expected meeting time, $E[M^+_{rd}] = E[M_{rd}]$. □

Now we find the expected contact time for the Random Direction model. To simplify the exposition, we will make a couple of approximations.

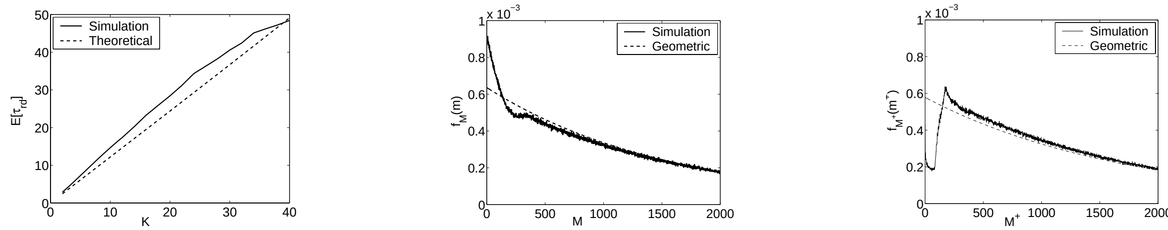
(a) We approximate the geometric distribution with an exponential distribution. In other words, we assume that both movement and pause durations are exponentially distributed. Exponential distribution is the equivalent continuous version of geometric distribution. Assuming a continuous distribution simplifies the analysis because we don't have to worry about the corner cases where two time durations expire at the same time.
(b) Let $T = \frac{L}{v}$. In general, $E[T] \neq \frac{E[L]}{\overline{v}}$, but for the ease of analysis, we will assume that they are equal.

When two nodes come within range of each other, one of the following is true: (a) Both the nodes are moving or (b) Only one of the nodes is moving and the other is paused. Let $E[\tau^1_{rd}]$ denote the expected contact time given both nodes were moving when they came within range of each other and let $E[\tau^2_{rd}]$ denote the expected contact time given only one of the nodes was moving when they came within range. We derive their values in Appendix A.

Theorem 3.2: The expected contact time $E[\tau_{rd}]$ for the Random Direction model is equal to $\frac{p_m^2}{p_m^2+2p_m(1-p_m)}E[\tau^1_{rd}] + \frac{2p_m(1-p_m)}{p_m^2+2p_m(1-p_m)}E[\tau^2_{rd}]$, where $p_m = \frac{\overline{T}}{\overline{T}+\overline{T}_{stop}}$ is the the probability that a node is moving at any time.

Proof: The probability that both nodes are moving is equal to p_m^2. The probability that only one of the nodes is moving is equal to $2p_m(1 - p_m)$. For two nodes to come within range from out of range, at least one of the nodes has to be moving. Hence, to find $E[\tau_{rd}]$, we have to condition over the fact that at least one of the two nodes is moving. Applying the law of total probability gives the result. □

[1] The mixing time of a mobility model is the time it takes for a node to come back to its stationary distribution after starting from any arbitrary initial distribution.



(a) (b) (c)

Fig. 1. Random Direction Mobility Model: (a) Comparison of the theoretical and simulation results for the expected contact time for parameters $N = 100 \times 100, \overline{T} = 400, \overline{v} = 1, \overline{T}_{stop} = 150$ (b) Meeting time distribution with parameters $N = 300 \times 300, K = 30, \overline{T} = 160, \overline{v} = 1, \overline{T}_{stop} = 150$ (c) Inter Meeting time distribution with parameters $N = 300 \times 300, K = 30, \overline{T} = 160, \overline{v} = 1, \overline{T}_{stop} = 150$

We made a few approximations while deriving the expected contact time to keep the analysis tractable. Since all the approximations were easily justifiable, we do not expect that they would drastically effect the accuracy of the analysis, which we verify in Figure 1(a) where we compare the analytical and simulation results for the expected contact time of the Random Direction model. We compared the analytical and simulation results for a number of scenarios and both the values were always close to each other. We show only one of these plots here due to lack of space.

Theorem 3.3: The tail of the distribution of the meeting time and the inter meeting time for the Random Direction mobility model is geometric.

Proof: Let node A and node B start from their stationary distribution at time 0. Lets define one time epoch as the time duration at the end of which one of the two nodes change their state (either from moving to paused or from paused to moving). Let N_{meet} denote the number of epochs until node A meets node B, and $Pr[N_{meet} > n]$ denote the probability that node A and node B do not meet after n epochs.

Although consecutive epochs are not independent (the end of one epoch is the beginning of the next one), the random process describing the lengths and end points of the sequence of epochs drawn is ergodic [24]. Thus, we can use the statistics of a single epoch to describe the whole process, as if the epochs were drawn independently (the argument is similar to the one made in [20] and [24]). Thus, $Pr[N_{meet} > n] = Pr[$A and B do not meet in the first n time epochs$] = (Pr[$A and B do not meet in a time epoch$])^n$. Consequently, the number of epochs needed till A meets B is geometrically distributed. Thus the distribution for the meeting time when the meeting time is much larger than one epoch time, is also geometric. Thus the tail of the distribution of the meeting time is geometric.

A similar argument holds for the inter meeting time also. □

Theorem 3.3 makes no comment on the body of the distribution of the meeting and the inter meeting time. The papers which theoretically analyze the performance of mobility assisted routing schemes invariably assume that these distributions are exponential (not just the tails) [11], [15], [17], [20]. We use simulations to study the accuracy of this assumption. We plot the distribution of the meeting time and the inter meeting time of the Random Direction mobility model for a representative set of parameters [2] in Figures 1(b) and 1(c) respectively. These plots show that the tail contains most of the distribution and hence the loss in accuracy in assuming the entire distribution to be exponential is not significant.

IV. Random Waypoint

Definition 4.1 (Random Waypoint): In the Random Waypoint model, each node moves as follows [25]: (i) Choose a point X in the network uniformly at random. (ii) Choose a speed v uniformly in $[v_{min}, v_{max}]$ with $v_{min} > 0$ and $v_{max} < \infty$. Let $\overline{v}$ denote the average speed of a node. (iii) Move towards X with speed v along the shortest path to X. (iv) When at X, pause for T_{stop} time slots where T_{stop} is chosen from a geometric distribution with mean $\overline{T}_{stop}$. (v) Go to Step (i). One iteration of these steps is referred to as an epoch.

The expected meeting time of the Random Waypoint model was evaluated in [20]. Here, we will derive the expected inter meeting time in Theorem 4.1, the expected contact time in Theorem 4.2 and finally the distribution of the meeting and the inter meeting times in Theorem 4.3.

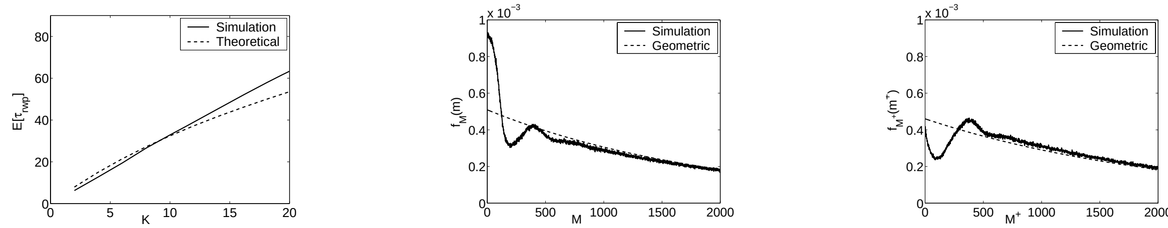
Theorem 4.1: The expected inter meeting time $E[M^+_{rwp}]$ for the Random Waypoint model is approximately equal to $E[M_{rwp}]$.

Proof: When the nodes move out of the range of each other, they pick up a destination uniformly at random in the torus. After reaching their destination, they are fully mixed (back in their stationary distribution) and the additional time it takes for them to meet again is equal to the meeting time. In general, since an epoch time is much less than the expected meeting time, $E[M^+_{rwp}] = E[M_{rwp}]$. □

Now we find the expected contact time for the Random Waypoint mobility model. The approach is exactly the same as for the Random Direction model. Also, we will make the same two approximations as we made to find the expected contact time for the Random Direction model.

Theorem 4.2: The expected contact time $E[\tau_{rwp}]$ for the Random Waypoint model is equal to $\frac{p_m^2}{p_m^2+2p_m(1-p_m)}E[\tau^1_{rwp}] + \frac{2p_m(1-p_m)}{p_m^2+2p_m(1-p_m)}E[\tau^2_{rwp}]$, where $p_m = \frac{\frac{0.3826\sqrt{N}}{\overline{v}}}{\frac{0.3826\sqrt{N}}{\overline{v}}+\overline{T}_{stop}}$ is the the probability that a node is moving at any time, $E[\tau^1_{rwp}]$ is the expected contact time given both nodes were moving when they

[2]Plots for other sets of parameters also result in the same observation. We omit them here due to lack of space.



(a) (b) (c)

Fig. 2. Random Waypoint Mobility Model: (a) Comparison of the theoretical and simulation results for the expected contact time for parameters $N = 150 \times 150, \overline{v} = 1, \overline{T}_{stop} = 150$ (b) Meeting time distribution with parameters $N = 300 \times 300, K = 30, \overline{v} = 1, \overline{T}_{stop} = 150$ (c) Inter Meeting time distribution with parameters $N = 300 \times 300, K = 30, \overline{v} = 1, \overline{T}_{stop} = 150$

came within range of each other and $E[\tau^2_{rwp}]$ is the expected contact time given only one of the nodes was moving when they came within range. Appendix B discusses how to find their values.

Proof: The proof runs along similar lines as the proof of Theorem 3.2. □

We made a few approximations while deriving the expected contact time to keep the analysis tractable. Since all the approximations were easily justifiable, we do not expect that they would drastically effect the accuracy of the analysis, which we verify in Figure 2(a) where we compare the analytical and simulation results for the expected contact time of the Random Waypoint model. We compared the analytical and simulation results for a number of scenarios and both the values were always close to each other. We show only one of these plots here due to lack of space.

Theorem 4.3: The tail of the distribution of the meeting time and the inter meeting time of the Random Waypoint model is geometric.

The proof of Theorem 4.3 runs along the same line as the proof of Theorem 3.3. Now we use simulations to study the accuracy of the assumption that these distributions are exponentially distributed (not just the tails). We plot the distribution of the meeting time and inter meeting time of the Random Waypoint mobility model for a representative set of parameters in Figures 2(b) and 2(c) respectively. These plots show that the tail contains most of the distribution and hence the loss in accuracy in assuming the entire distribution to be exponential is not significant. (Plots for other sets of parameters also result in the same observation. We omit them here due to lack of space.)

V. Random Walk

We now assume that nodes are moving on a $\sqrt{N} \times \sqrt{N}$ grid in a 2-D torus. Each node moves one grid unit in one time unit.

Definition 5.1 (Random Walk): In the Random Walk mobility model, each node moves as follows: (i) Choose one of the four neighboring grid points uniformly at random. (ii) Move towards the chosen grid point during that time slot. (iii) Goto Step (i).

The expected meeting time and the expected inter meeting time of the Random Walk model was evaluated in [20] and [21] respectively. [22] proves that the tail of the distribution of the meeting time is exponentially distributed. Here, we will derive the expected contact time in Theorem 5.1. Then we study the distribution of the inter meeting time using simulations.

Theorem 5.1: Let $E[CM]_k$ denote the expected additional time two nodes will remain in contact with each other when the distance between them is equal to $k \leq K$. By definition, the expected contact time $E[\tau_{rw}]$ for the Random Walk mobility model is equal to $E[CM]_K$. The $E[CM]_k$'s can be found by solving the following set of linear equations:

$$E[CM]_k = 1 + \frac{16K-20}{64K} E[CM]_{k-2} + \frac{16K+12}{64K} E[CM]_{k+2} + \frac{32K+8}{64K} E[CM]_k \qquad 3 < k < K$$

$$E[CM]_k = 1 + \frac{7}{48} E[CM]_{k-2} + \frac{15}{48} E[CM]_{k+2} + \frac{26}{48} E[CM]_k \qquad k = 3$$

$$E[CM]_k = 1 + \frac{3}{32} E[CM]_{k-2} + \frac{11}{32} E[CM]_{k+2} + \frac{18}{32} E[CM]_k \qquad k = 2$$

$$E[CM]_k = 1 + \frac{9}{16} E[CM]_k + \frac{7}{16} E[CM]_{k+2} \qquad k = 1$$

$$E[CM]_k = 1 + \frac{4}{16} E[CM]_k + \frac{12}{16} E[CM]_{k+2} \qquad k = 0$$

$$E[CM]_k = 1 + \frac{16K-20}{64K} E[CM]_{k-2} + \frac{32K+8}{64K} E[CM]_k \qquad k = K$$

Proof: The theorem can be proved using elementary combinatorics. The proof is omitted here due to limitations of space. Please refer to [26] for details. □

Now we look at the distribution of the inter meeting time of the Random Walk model. We plot the inter meeting time distribution obtained from simulations and the distribution of a geometric random variable in Figure 3(a). Unlike random direction and random waypoint mobility models, here the body contains most of the distribution. Its easy to see that the geometric distribution does not match the body of the inter meeting distribution. Hence, assuming a geometric distribution for the inter meeting time will lead to significant inaccuracies.

The curve for the inter meeting time distribution in Figure 3(a) shows that the probability that the two nodes meet again in the first few time slots after moving out of range, is very high. Still, the expected value of the inter meeting time is very large ($O(N)$ where N is the area of the grid). We also observe that the probability density function is close to a straight line on the log-log scale. So we use a Zipfian distribution to fit the body of the inter meeting time distribution. (The probability density function of a Zipfian distribution is proportional to $\frac{1}{i^{\alpha}}$.)

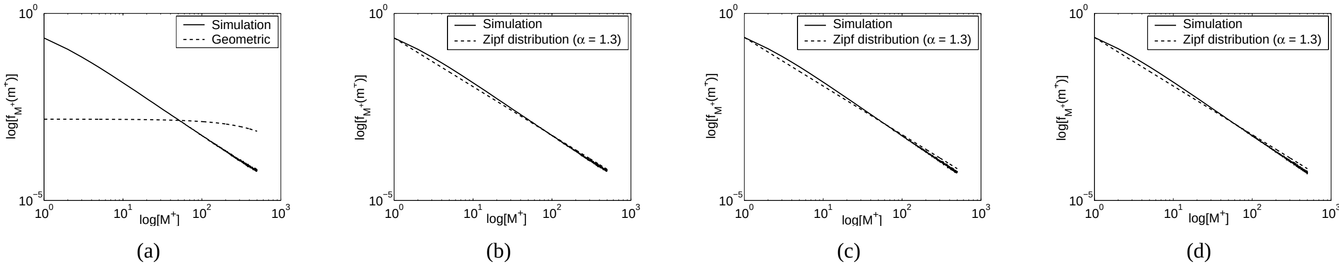


Fig. 3. (a) Inter Meeting distribution for Random Walk Mobility model (with parameters $N = 80 \times 80, K = 4$) and a geometric distribution having the same mean. (b)-(d) Inter Meeting distribution for Random Walk Mobility model with parameters (b) $N = 80 \times 80, K = 4$ (c) $N = 120 \times 120, K = 6$ (d) $N = 150 \times 150, K = 6$

We plot the inter meeting distribution for some sample network parameters and a Zipfian distribution with $\alpha = 1.3$ in Figures 3(b)-3(d). Its easy to see that a Zipfian distribution fits the inter meeting time distribution pretty well.

VI. Conclusions and Discussion

Realistic performance analysis of mobility-assisted routing with contention in the network requires a knowledge of the statistics of the meeting time, inter meeting time and the contact time of the mobility model. These quantities vary largely depending on the mobility model in hand. In this paper, we compute the expected inter meeting times of the random direction and random waypoint mobility models. We also prove that the tail of the distribution of the meeting and inter meeting time for random direction and random waypoint is memoryless. We show through simulations that the inter meeting time distribution for random walks is Zipfian. Finally, we find the expected contact time for all the three mobility models. In future, we plan to study these quantities for other mobility models too, for example, the community based mobility model [20].

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Appendix

A. Expected Contact Time for the Random Direction Mobility model

Lemma 1.1: The expected contact time given both nodes were moving when they came within range of each other, $E[\tau_{rd}^1]$, is equal to $(1-p_1)\frac{4K}{1.27\pi\bar{v}} + p_1\left(\frac{0.6366K}{1.27\bar{v}} + E[\tau_1^{add}]\right)$, where $p_1 \approx \int_0^\pi \frac{1}{\pi}\left(1 - e^{\left(-\frac{4K sin(\phi)}{1.27\bar{v}\bar{T}}\right)}\right) d\phi$ is the probability that one of the two nodes pause while they are within range of each other and

$E[\tau_1^{add}]$ is the expected additional time the two nodes remain within range after one of the nodes paused.

Proof: When both the nodes are moving when they come within range of each other, either they move out of each other's range before any of them pauses or one of them pauses before they move out of range.

(a) They move of each other's range before pausing: Let one node be static and let the other node move at a speed $\vec{v}_i - \vec{v}_j$. This model is equivalent to the model when both nodes are moving at speeds $\vec{v}_i$ and $\vec{v}_j$ respectively. We will work with the former model during this proof as well as all the subsequent proofs. The angle of $\vec{v}_i - \vec{v}_j$ is uniformly distributed between $[0, 2\pi)$.
So, when these two nodes come within range of each other, the angle ϕ in Figure 4 will be uniformly distributed within $[0, \pi)$. They will remain in contact with each other while the first node travels along the chord AB in Figure 4. The length of the chord AB is equal to $2Ksin(\phi)$. E[distance for which the nodes remain in contact with each other] = E[length of chord AB] = $\int_0^{\pi} \frac{1}{\pi} 2Ksin(\phi) d\phi = \frac{4K}{\pi}$. The expected speed of moving node is $E[\|\vec{v}_i - \vec{v}_j\|] = 1.27\overline{v}$. Thus the expected time they remain in contact with each other is approximately equal to $\frac{4K}{1.27\pi\overline{v}}$.

(b) One of the nodes pauses before they move out of each other's range: The moving node is equally likely to pause anywhere on the chord AB in Figure 4 since the distribution of movement duration is memoryless. Let the node stop at point C which is $0 \leq x \leq 2Ksin(\phi)$ distance away from A. $f_{X|\Phi}(x \mid \phi)$ is uniformly distributed between 0 and $2Ksin(\phi)$. Multiplying by $f_\Phi(\phi)$ and integrating over ϕ gives us $f_X(x)$. The expected distance node travels before pausing can then be evaluated to $0.6366K$. The expected time the node travels before pausing is equal to $\frac{0.6366K}{1.27\overline{v}}$. $E[\tau_1^{add}]$ is the additional time spent within range of each other.

Now we find p_1 to complete the proof. p_1 is the probability that one of the two nodes pause before moving out of range. Since the movement duration of both the nodes is exponential with mean $\overline{T}$, p_1 given ϕ and $\|\vec{v}_i - \vec{v}_j\|$ is equal to $1 - e^{-\frac{2Ksin(\phi)}{\frac{\overline{T}}{2}\|\vec{v}_i - \vec{v}_j\|}}$. To simplify exposition, we replace $\|\vec{v}_i - \vec{v}_j\|$ by its expected value. Hence, $p_1 \approx \int_0^{\pi} \frac{1}{\pi} \left(1 - e^{\left(-\frac{4Ksin(\phi)}{1.27\overline{v}\overline{T}}\right)}\right) d\phi$ which can be evaluated numerically. □

The next lemma evaluates the expected contact time when only one node was moving when they came within range of each other, $E[\tau_{rd}^2]$. When only one node is moving, either they will move out of each other's range before the paused node restarts again and the moving node pauses, or the moving node pauses or the paused node restarts before they move out of each other's range. The derivation has to account for all the three scenarios.

Lemma 1.2: The expected contact time given only one of the nodes was moving when they came within range of each other, $E[\tau_{rd}^2]$, is equal to $(1 - p_2)\frac{4K}{\pi\overline{v}} + p_2\left(\frac{0.6366K}{\overline{v}} + p_{21}E[\tau_2^{add}] + p_{22}E[\tau_3^{add}]\right)$,

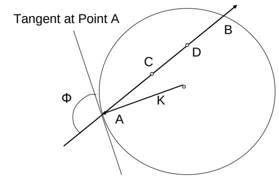

Fig. 4. The first node enters the transmission range of the second node at an angle ϕ to the tangent at A and moves along the chord AB.

where $p_2 \approx \int_0^{\pi} \frac{1}{\pi} \left(1 - e^{\left(-\frac{2Ksin(\phi)}{\frac{1}{\frac{1}{\overline{T}} + \frac{1}{\overline{T}_{stop}}}}\right)}\right) d\phi$ is the probability that the paused node restarts again or the moving node pauses before moving out of each other's range, $p_{21} = \frac{\frac{1}{\overline{T}}}{\frac{1}{\overline{T}} + \frac{1}{\overline{T}_{stop}}}$ is the probability that the moving node pauses before the paused node restarts and $p_{22} = \frac{\frac{1}{\overline{T}_{stop}}}{\frac{1}{\overline{T}} + \frac{1}{\overline{T}_{stop}}}$ is the probability that the paused node restarts before the moving node pauses. $E[\tau_2^{add}]$ and $E[\tau_3^{add}]$ are the expected additional times the two nodes remain within range after both of them are paused and after both of them start moving respectively.

Proof: See [26]. □

The derivation of $E[\tau_1^{add}]$, $E[\tau_2^{add}]$ and $E[\tau_3^{add}]$ is also similar to the proof of Lemma 1.1. We omit the derivation here due to limitations of space. Please refer to [26] for details.

B. Expected Contact Time for the Random Waypoint Mobility model

Lemma 1.3: Let $p = Pr[$node A pauses within the transmission range of node $B \mid$ node A is passing through the transmission range of node $B]$. Then $p = \frac{\frac{\pi K^2}{N}}{\frac{\pi K^2}{N} + p_{r1} + p_{r2}}$ where $p_{r1} = \frac{1}{N}\int_K^{\frac{\sqrt{N}}{2}} \frac{2\pi l}{N} \int_{\sqrt{l^2-K^2}}^{\frac{\sqrt{N}}{\sqrt{2}}} 2rsin^{-1}\left(\frac{K}{l}\right) drdl$ and $p_{r2} = \frac{1}{N}\int_{\frac{\sqrt{N}}{2}}^{\frac{\sqrt{N}}{\sqrt{2}}} \frac{4l}{N}\left(\frac{\pi}{2} - 2cos^{-1}\left(\frac{\sqrt{N}}{2l}\right)\right) \int_{\sqrt{l^2-K^2}}^{\frac{\sqrt{N}}{\sqrt{2}}} 2rsin^{-1}\left(\frac{K}{l}\right) drdl$.
$p_{r1} + p_{r2}$ is the probability that node A will pass through the transmission range of node B but not pause within node B's transmission range.

Proof: See [26]. □

$E[\tau_{rwp}^1]$ and $E[\tau_{rwp}^2]$ can be derived in a manner similar to the derivation of $E[\tau_{rd}^1]$ in Lemma 1.1. The only difference is that the probability that a moving node pauses within range of the other node is equal to p (derived in Lemma 1.3).